**Chapter two**

**Transformation**

**2-1:Transformation Technique (For Discrete Random Variables)**

**Definition:-**

 Let X be a r.v. of the discrete type, having p.d.f. f(x), and assume that A denoted set of discrete points at each of which f(x)>0. Let Y=h(X), define a one to one transformation that maps the space A onto the space B which is obtained by transforming each point in A accordance with Y=h(X). If we solve Y=h(X) for X in terms of Y. say X = h-1(Y),then the p.d.f. of Y is given by :

**Example 17//** let

Sol:

Let g(y) denoted the p.d.f. of Y, then g(y)=p(Y=y)

)

**Example18//** If the p.d.f. of X is given by :

Find the p.d.f. of Y=2X+1

Solution:

**Definition:**

 define a one to one transformation that maps space A (the set of points at which onto B, then the j.p.d.f of Y1,Y2,…,Yk is as follows:

=

**Example19:**

if and independent. Let Y1=X1+X2 and Y2=X2 find the j.p.d.f of Y1 and Y2?

Solution:

=)=

 )

**2-2: transformation technique (for continuous Random Variables).**

Definition: Let X be a continuous r.v. with p.d.f.f(x) and the set of possible values A. For the invertible function be a r.v. with the set of possible values B=h(A)=(. Suppose that the inverse of Y=h(X) is the function X = h\_1(Y), which is differentiable for all values of Y€B. the p.d.f of Y, is given by:

**Example 20//** If find the p.d.f. of

**Solution:**

Y=h(X)=

**Example21:-**

If the p.d.f. of X is

Find the p.d.f.of =

**Solution:**

Since

***Theorem:***

Let X1,X2....,Xn be jointly continuous r.vs. with p.d.f

Let A = {x: f(x) > 0} Suppose that A can be decomposed into sets A1,...,Am, such that Yj = h1(Xl,X2, ...,Xn),Y2 = h2(X1,X2,....Xn),…,Yn = hn(X1,X2,...,Xn) is a one to one transformation of onto B, i=1,2,…,m Define Jacobean:

For i=1,2,…,m where Ji

Thene for (

Remark:

The previous theorem is a general case, we will take just the case when m=1 i.e.

Where

**Example22//** Let X1 and X2 have independent gamma distribution with parameters respectively. If and Z2 = X1 + X2, find the j.p.d.f. of Z1 , and Z2.

**Solution//**

Since x1 and x2 are independents, then:

And

**2-1: Order Statistics**

**Definition//** The order statistics of the r.v. x1,x2,…,xn are the sample values placed inascending order and denoted

The distribution of individual order statistical is :-

**if**

**Remark:**- The j.p.d.f. of two order statistics

**Example 23//** let be the order statistics , of a random sample size 5 with p.d.f.

Find:

Solution//

=

**Example 24//**

Let a order statistics of a r.v.s.4 taken from p.d.f. and let

Find :

Not:

1- the largest value means

2-the smaller value means

3- the sample range is ()

4-the sample mid rang (

5- the sample median is

**Example25: H.W**: let be the order statistics from )

**Example26:** let be the order statistics from

1- largest value and expected

2-find smallest value

3-find pdf of median

4- the J.p.d.f. of g(

Solution//

Since

Where

F(x)=2x 0<x<1

The largest value means

i=5

2- The smaller valu means

i=1

3-

n=5 odd number use

=

 H.W

**Example27**: ( H.W): let be rss3 with p.d.f. p(x)=2x , 0<x<1 , then show , , are independent.

**2-4:T-distribution:**

Let W be arandom variable having p.d.f. W and V random variable with p.d.f. V and Wand V are independent random variable , then the p.d.f. of is known as distribution and given by

**Proof:-**

W

 ,

 V

Since v and w are independent

V=u

 . |J|

 .

To find mean and variance

Mean=

Since

 Mean=

To find variance

since

since

Not:

**2-5:The F-distribution:**

 Is the distribution of the ration of two independent chi-square random variable divided by their respection degrees of freedom let u be a chi-square random variable with m degree of freedom, let v be a chi-square random variable with n degree of freedom and let u and v be independent, then the random variable is distributed as an F distribution with m and n degree of freedom the density of x is