CHAPTER THREE

 **Estimation**

**Statistical Inference . . .**

Statistical inference is the process by which we infer population properties from sample properties.



There are two types of statistical inference:

 Estimation Hypotheses Testing

3-1:Estimation . . .

The objective of estimation is to approximate the value of a population parameter on the basis of a sample statistic. For example, the sample mean X¯ is used to estimate the population mean µ.

**Parameter**: is unknown constant which include in every probability distribution, each distribution has at least one parameter.

**There are two types of estimators:**

1- Point Estimator



2- Interval Estimator



1-Point Estimator . . .

A point estimator draws inferences about a population by estimating the value of an unknown parameter using a single value or point.



2-Interval Estimator . . .

 An interval estimator draws inferences about a population by estimating the value of an unknown parameter using an interval. Here, we try to construct an interval that “covers” the true population parameter with a specified probability.



**Good estimation:** any function t( is to be good estimation if it satisfy some property .

A- Unbiasedness.

B-minimum variance.

**3-2: Properties of point estimation:**

1**-Un biasedness . . .**

**Definition1:** let be a function of f(x,) and let an estimate T=t(x1 , x2, ...,xn)is said unbased estimate for iff:

**Remark: special case :**

**If**

**E(T)=**

**Definition 2 :** An unbiased estimator of a population parameter is an estimator whose expected value is equal to that parameter. Formally, an estimator ˆµ for parameter µ is said to be unbiased if:

E(ˆµ) = µ .

Example28: is unbiased estimation for = of

1- Ber( 2- poisson (

ans:

1- Ber(

=

2- poisson (

Example29: is unbiased estimation for = of uniform

Solution:

=

=

Example 30/ H.w/ from Normal(

1- is

2-

Example 31/ let be the order statistics of a r.s.s.3 from uniform(0,are

1) 2) 3) unbiased est. for

Solution//

E(T1=4y1)=

E(T1=4y1)=

=

=

2-

=

3- Home work

Not: if not unbiased we must find (Biased part) The biased part in an estimation is:

Example(31)// from N() find biased part for if T=

Sol//

)==

)

Example : from

SOL//

)==

=

Unbiased in limit :

T is said be unbiased est. in limit for

Example//

from

sol//

)==

Example : from

 2-

Sol//

)==

2-

)==

Mean square error:-

Let T be an estimate for

Theorm:-

Proof:-

=

=V(T)+

Example: In a r.v.s.s.2 from

Fisher information :-

Fisher information x about for may be defined by:-

Where:

Or

 is fisher information of single observation.

Example:- let be a r.s.s.n.from

Example:-

Let be a r.s.s.n. taken from ,find the fisher information of the sample and show that I the equation

SOL//

=

2- Efficiency

Efficiency (for a single estimate)

Definition(cramer):-

An unbiased estimate T for of is said to be efficient estimate iff:-

Where:

And

=

 Example//In a r.s.s.n. show that is efficient est. for

Sol//

=1

C.R.L.B. =

Example//In a r.s.s.n. show that is efficient est. for

Sol//

=1

C.R.L.B.=

C.R.L.B.==

Remark:-

Let be two estimators for of and if both estimates are unbiased estimates for , then we say that is more efficient estimate than iff

Example//

Show that is more efficient estimate than from

Sol//

)== unbiased est. for

 unbiased est. for

Relative Efficiency:

Let be two unbiased est.for of ,then we defined the efficiency of relative by:

Where: if are not unbiased est. for then

Example//

In a r.s.s.2 from find Relative efficiency for for .

Sol//

)== unbiased est. for

/x)=

than

**Consistent estimator: -**

**Theorem :** An estimator T is said to be consistent estimator for parameters iff

1-

2-

Example:

Is consistent est.for from 1-

Solution//

)==

) is unbiased est. for

 is consistent estimate .

2-

)== unbiased est. for

 is consistent estimate.

Example//

Let be to an s.s.n from binomial distribution show that are unbiased and consistent estimator for .

SOL//

E(T)=E=

*=*

**3-Sufficient estimation:**

Let be a random sample from a distribution with p.d.f. and let T be an estimator for parameter we say that T is sufficient estimator for if the fisher information in a random sample size(n) is same as in (T).

Example :

let be a r.s.s.n. from N( show that is sufficient estimator for .

Factorization theorem :-

Definition : let be random sample with j.p.d.f. sufficient est.for parameter

h(

Where:

h( is a function of (x) only

not:-

like lihod function

=

Example//

In a r.s.s.n. from poisson ,show that T= is sufficient est. for

 by factorization theorem

Solution//

 , x=0,1,…

=

=

T=

= \*

=

 is suff. Est. for .

Example// In a r.s.s.n. from1- exp 2- ber ,show that T= is sufficient est. for by factorization theorem

Jointly sufficient estimators

Definition // let be random sample from a distribution with p.d.f. then we say that the be jointly sufficient est. for respectively iff:-

Example// let be a r.s.s.n. from , show that , are jointly suff. Est. for respectively.

5- completeness:-

Let T be an estimate for of we say that is complete if

example//

let x be a r.v. from ber( ,show that is complete?