**Chapter one**

1-1: methods of Estimation .

The following are familiar methods of estimation: -

1)- moments method.

2)-maximum likelihood method.

3)-Least(or minimum) variance method.

4)-Bayesian method.

------------------------------------------------------------------------------

1)- Moments Method:-

In this method we equate population moments and sample moments about the origin. This means that we solve:

Where is the number of parameters to be estimated ,and

=

=→sample 𝑚𝑜𝑚𝑒𝑛𝑡 𝑎𝑏𝑜𝑢𝑡 𝑡ℎ𝑒 𝑜𝑟𝑖𝑔𝑖𝑛

Example//let be a r.s.s.n use moment method to an estimate for parameters of the following distribution:-

1-Ber()
2-unif()
3-Geomtric()
4-poisson(𝜃)

Sol//

1-Bernoulli:-

Because this distribution has only one parameter then

Where:-

 == =

=  where

= =

= moment estimate for



3-Geomtric

where

== =

==

 =

+ =1

4-poisson(𝜃)

 ,

Because this distribution has only one parameter then

== =

= where

==

= moment estimate for





2-Maximum Likelihood Estimation Method(m.l.e.).























3-Minimum Variance Method (Least Variance Method) (m.v.u.e.):-

 In this method the unbiased estimate , which has the minimum (or least)variance should satisfy the following relation

=

Example1// In a r.s.s.n from

1- Ber(

2-poi( ,Find m.v.u.e. for

Solution//

=

 =

=

 )

 =

 =

 =

V()==

=

= is (m.v.u.e.)

*pois*

*=* *\* \** \*…\*

*=*

=

 =

= is m.v.u.e.

4- Bayesian Estimation Method(BEM) :-

Philosophy observed data x is fixed , and the unknown parameter is random (certainty about depends on both empirical information x and prior knowledge about )

In Bayesian Estimation method the parameters treats as random variable with prior probability p() .or we have prior information about the parameter .

Let A and B be two event , then the conditional probability of A given B is :-

P(A/B)= =

Let A= and B=X, Then in arss with p.d.f. f(x;) and prior probability p() :

) =

P(x) does not contain , we can write it as:

) p(x/) p()

 L() p()

Where:

) is called posterior probability and Bayes estimator denote Bayes  is the mean of posterior probability E(/X)

L() : Is likehood function

P() : is prior probability .

We have two types of prior probability

1. Non Informative prior probability .
2. Informative prior probability .
3. Non Informative prior probability

Is proportional to the square root of fisher information :

P() , F.I=-E[

Example//

Estimate Bayes estimator for parameter of

1. Exp() 2- ber()
2. Informative prior probability .

The form of prior probability for parameters of some dist. As follows:

|  |  |  |
| --- | --- | --- |
| ID | Probability distriution | Informative prior probability |
| 1- | Bern() | Beta() |
| 2- | Bino(n,) | Beta() |
| 3- | Geo() | Beta() |
| 4- | Poisson() | Gamma() |
| 5- | Expo() | Gamma() |
| 6- | Expo( | Inverse Gama() |
| 7- | Normal()( | Inverse Gama() |
| 8- | Normal()( | Normal() |

Example//estimate the parameter of

1. Geo() 2- poisson()

Using Bayesian information prior probability

**1-2:The Exponential family of probability dist. Fun.**

Let x has a p.d.f. f(x;) then the family of f(x;) is be longs to exponential class of dist. If it can be written in the following :-

f(x;)=exp[ln f(x;)]

 = exp[q() + p() k() + s(x)]

Example//let be arssn from Ber() show that if the dist. Of x can be written in exponential form ?( belongs to the expon. Family )

Answer//

1-3:Rao-Black well Theorem

Let x has a p.d.f. f(x;) and u be unbiased estimated to and T be sufficient estimator ,then :

1. E[E(u/T)]= =E(u) , Where w=E(u/T)
2. V(u) v[E(U/T)]

Answer//