**Chapter two**

2-1: interval estimation :-

We study interval estimate in the form of confidence intervals (C.I.),and the C.I. )would be as $\hat{θ}\_{l}<θ<\hat{θ}\_{u}$,there the true value of $θ$lies in the given interval with a probability (1-$∝$)

$∝$= level of significant

 = size of critical region

 =probability of type 1 error

(1-$∝$)=is cofficent of C.I.

2-1-1: confidence interval for mean of normal population :-



Now observe that using the curve of standard Normal distribution as give , we have

$p(-z\_{^{∝}/\_{2}}<z<z\_{^{∝}/\_{2}}) $=(1-$∝$)%

Where

$$z=\frac{\overbar{x}-θ}{^{σ}/\_{\sqrt{n}}}$$

We have two case

a)- the variance is known:-

$p(-z\_{^{∝}/\_{2}}<z<z\_{^{∝}/\_{2}}) $=(1-$∝$)%

$p(-z\_{^{∝}/\_{2}}<\frac{\overbar{x}-θ}{^{σ}/\_{\sqrt{n}}}<z\_{^{∝}/\_{2}}) $=(1-$∝$)%

$p(\overbar{x}-^{σ}/\_{\sqrt{n}} \*z\_{^{∝}/\_{2}}<θ<\overbar{x}+^{σ}/\_{\sqrt{n}} \*z\_{^{∝}/\_{2}}) $=(1-$∝$)%

Example // a r.s. of size n=20 ,with $σ=15$ has a mean =64.3 construct a 95% Confidence interval for the population means?

**B)- IF** $σ=Unkown$

**1)- if n>30**  the C.I. IS same as about except that we replace $σ$ with (S) and the required C.I. with coefficient (1-$∝$)% is :-

$p(\overbar{x}-^{s}/\_{\sqrt{n}} \*z\_{^{∝}/\_{2}}<θ<\overbar{x}+^{s}/\_{\sqrt{n}} \*z\_{^{∝}/\_{2}}) $=(1-$∝$)%

**2)- if n < 30**

The C.I. would be as :-

$p(-t\_{(\frac{∝}{2},n-1)}<t<t\_{(\frac{∝}{2},n-1)})$=(1-$∝$)%

T=$\frac{\overbar{x}-θ}{s/\sqrt{n}}$

$p(-t\_{\left(\frac{∝}{2},n-1\right)}<\frac{\overbar{x}-θ}{\frac{s}{\sqrt{n}}} <t\_{(\frac{∝}{2},n-1)})$=(1-$∝$)%

$p(\overbar{x}-\frac{s}{\sqrt{n}} \* t\_{\left(\frac{∝}{2},n-1\right)}<θ<\overbar{x}+\frac{s}{\sqrt{n}} \*t\_{(\frac{∝}{2},n-1)})$=(1-$∝$)%

Example// construct a 95% C.I. for the mean life of light bulbs given that a r.s. of size n=7 ,and with a s.d. =20 hours the average live time = 420 , t=(0.025,6)=

**2-1-3: confidence interval for the variance:-**

If $x\_{1},x\_{2},x\_{3},…,x\_{n}$is a random sample taken a normal population with mean(m) and variance $σ^{2}$ and if the sample variance is denoted by $s^{2}$ , the random variable

$$x^{2}=\frac{(n-1)s^{2}}{σ^{2}}$$

Has a chi-squared dist. With ( n-1) degrees of freedom this knowledge enables us to construct a confidence interval as follows

$p(-x^{2}\_{(\frac{∝}{2},n-1)}<x^{2}<x^{2}\_{(1-\frac{∝}{2},n-1)})$=(1-$∝$)%

$p(-x^{2}\_{(\frac{∝}{2},n-1)}<\frac{(n-1)s^{2}}{σ^{2}}<x^{2}\_{(1-\frac{∝}{2},n-1)})$=(1-$∝$)%

$p(\frac{(n-1)s^{2}}{x^{2}\_{(\frac{∝}{2},n-1)}}<σ^{2}<\frac{(n-1)s^{2}}{x^{2}\_{(1- \frac{∝}{2},n-1)}})$=(1-$∝$)%

Example// a random sample of