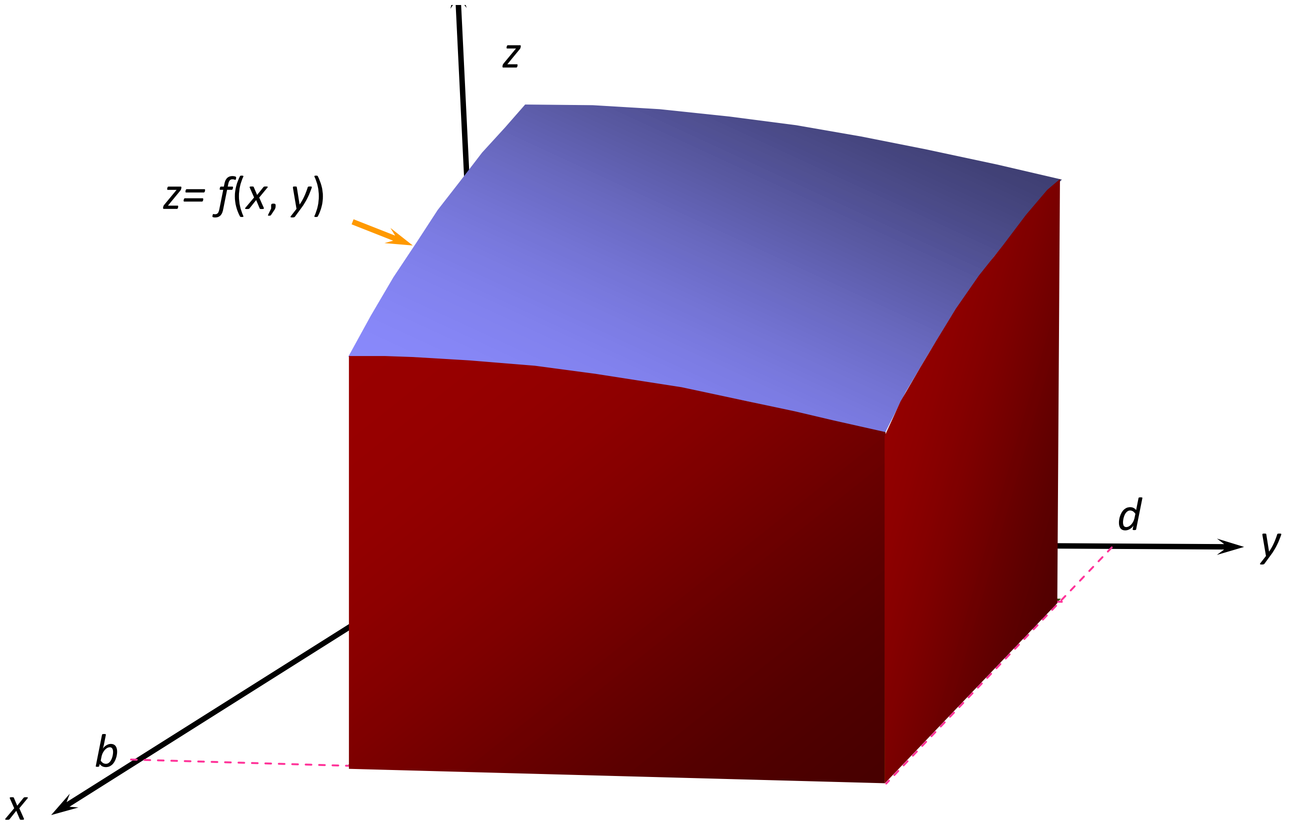
Double Integral

Suppose that *f(x,y)* defined on a closed rectangle

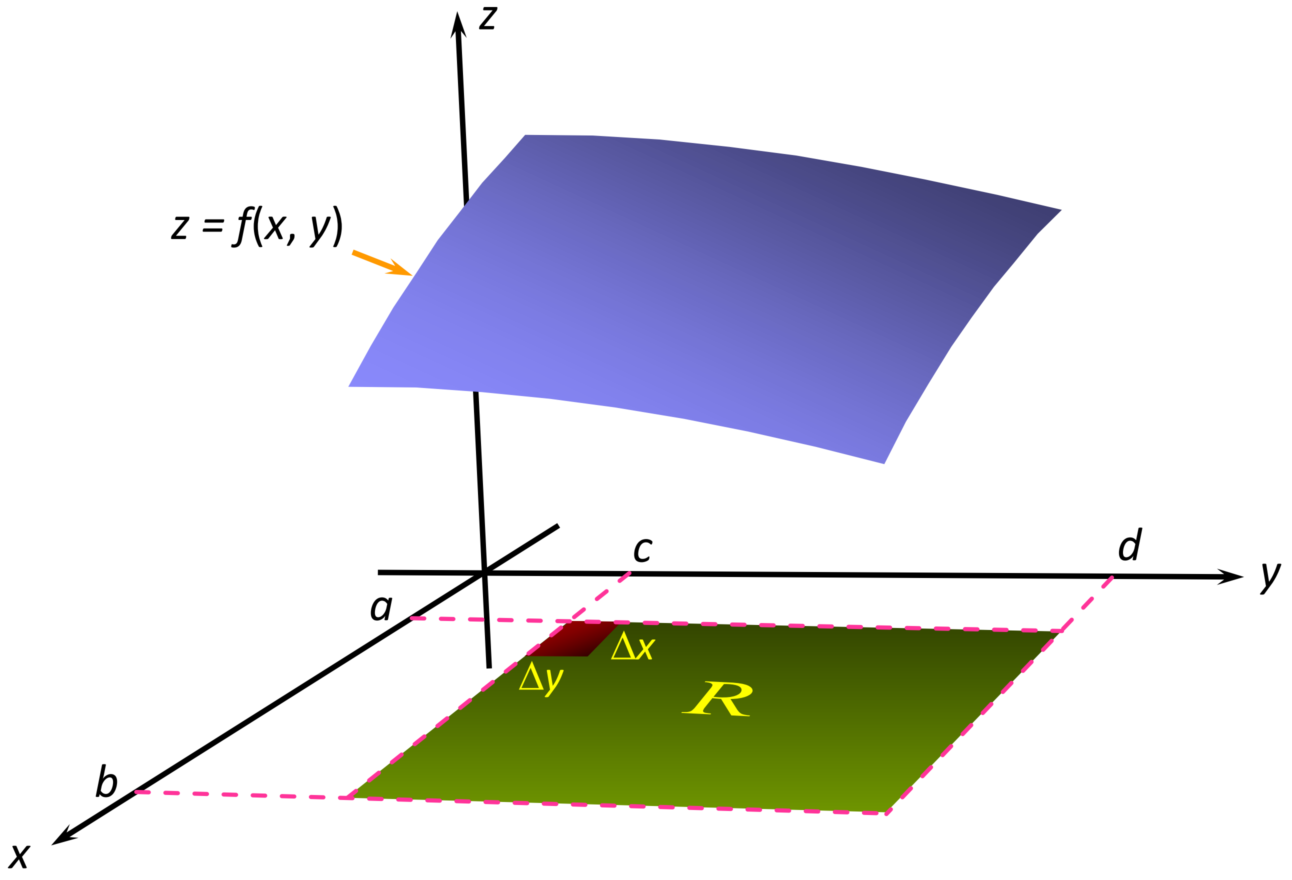
$$R = [a,b] \times [c,d] = \{(x,y) \in \mathbb{R}^2 \mid a \leqslant x \leqslant b,\ c \leqslant y \leqslant d\}$$

and we first suppose that $f(x,y) \geqslant 0$. The graph of *f* is a surface with equation *z=f(x,y)*. Let S be the solid that lies above R and under the graph of *f*, that is $S = \{(x,y,z) \in \mathbb{R}^3 \mid 0 \leqslant z \leqslant f(x,y),\ (x,y) \in R\}$



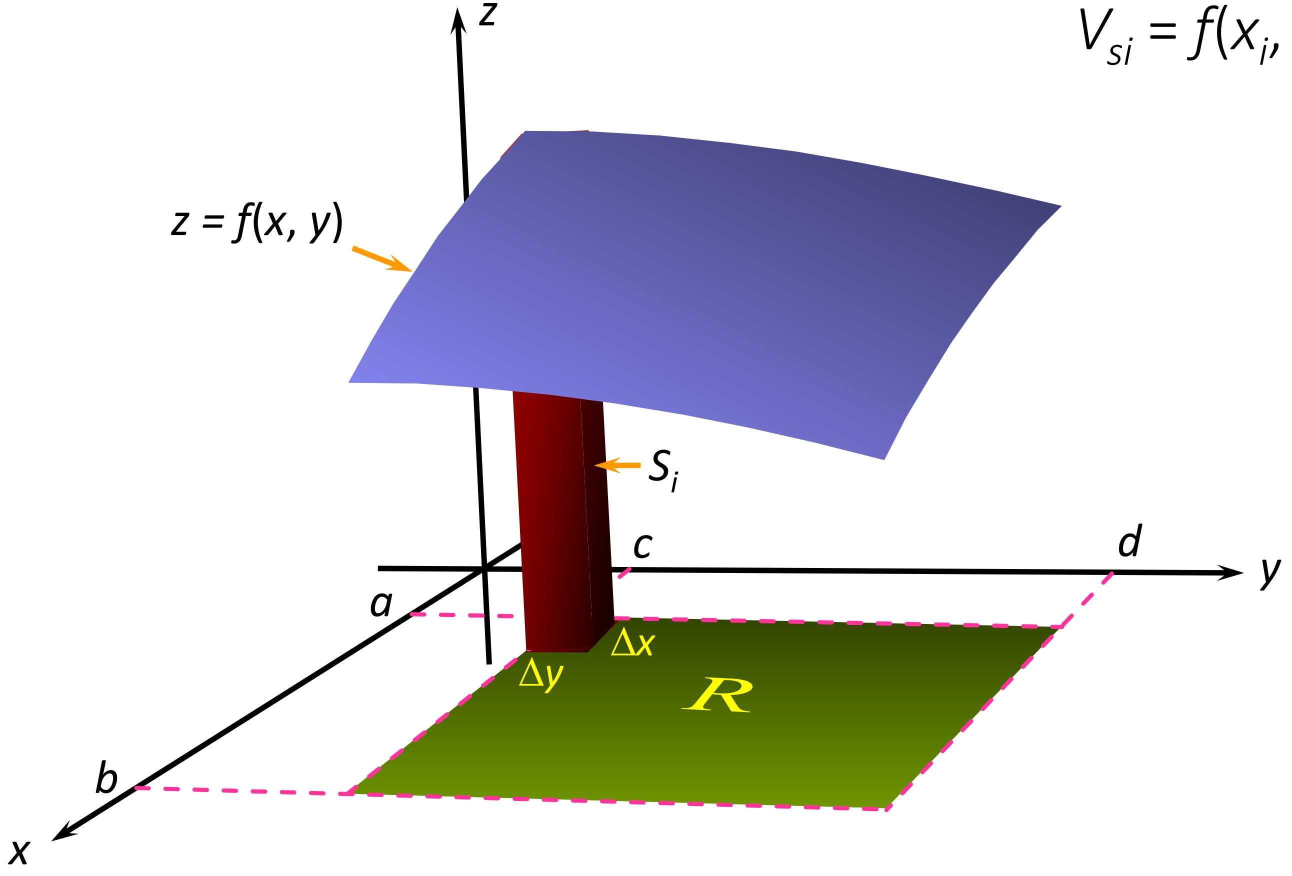
- Our goal is to find the volume of S. To find the volume of the solid under the surface, we can perform a Riemann sum of the volume S_i of parallelepipeds with base

  $R_i = \Delta x \times \Delta y$ and height $f(x_i, y_i)$:



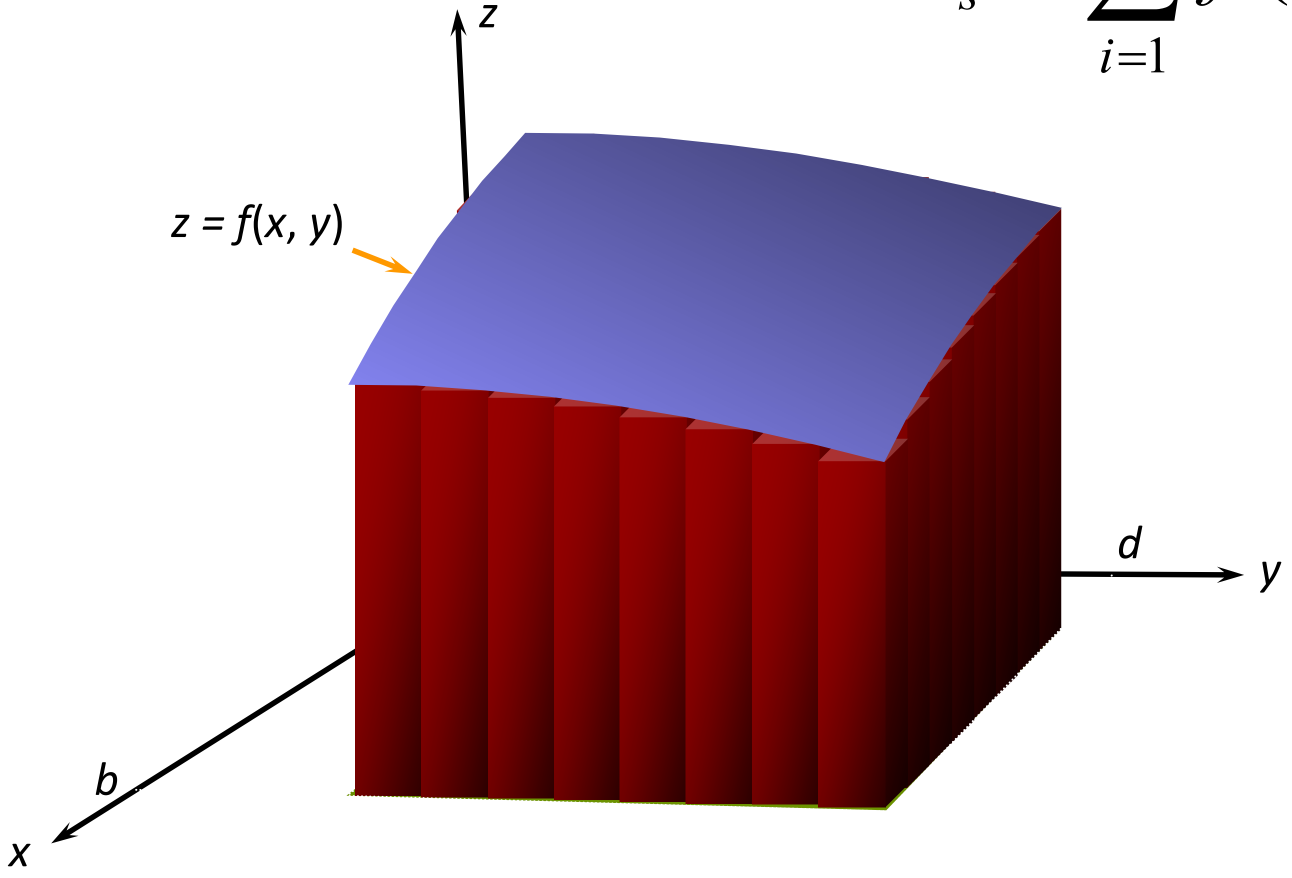
A Geometric Interpretation of the Double Integral

$V_{si} = f(x_i, y_i).\ R_i$



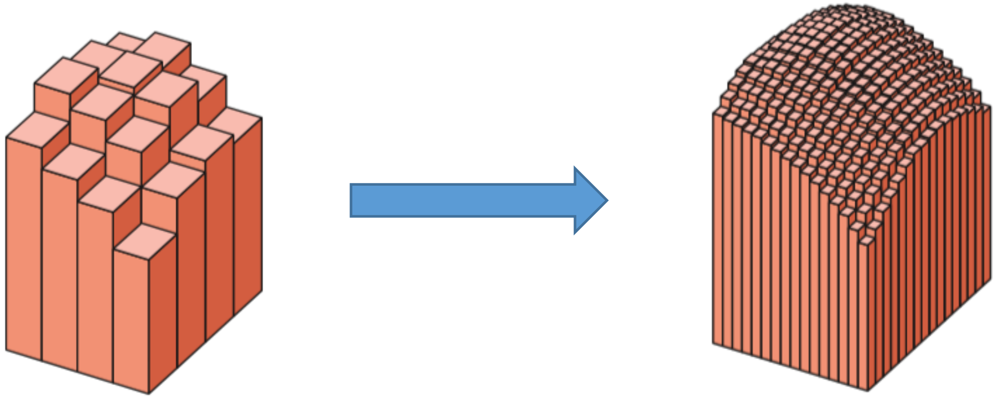
A Geometric Interpretation of the Double Integral

$$V_s \cong \sum_{i=1}^{n} f(x_i, y_i) R_i$$



- The limit of the Riemann sum obtained when Δx and Δy *go to zero* is the value of the double integral of *f*(*x*, *y*) over the region *R* and is denoted by

$$\iint_R f(x,y)\,dA = \lim_{\substack{\Delta x_i \to 0 \\ \Delta y_i \to 0}} \sum_{i=1}^{n} f(x_i, y_i) R_i$$



- The double integral represent the volume above the region R and the under the surface f(x,y).

- If f(x,y)=1 then the double integral represents the area means that $A = \iint_R dA$