

1. Let  $C$  be any curve in  $\mathbb{R}^3$  from  $(0, 0, 0)$  to  $(1, 1, 1)$ . Let  $\mathbf{W}$  be the vector field  $\langle y^2 z^2, 2xyz^2, 2xy^2 z \rangle$ . Calculate  $\int_C \mathbf{W} \cdot d\mathbf{s}$ .
2. Let  $\sigma$  be the region parameterized by the following:

$$\phi(u, v) = (uv^2, u^3v), \quad 1 \leq u \leq 2, \quad 1 \leq v \leq 2$$

Calculate

$$\int_{\partial\sigma} (1, -\ln x) \cdot d\mathbf{s}$$

3. Let  $C_1$  and  $C_2$  be curves given by the following parameterizations (with the induced orientations):

$$C_1 : \phi(t) = (t, 0, 0), \quad 2\pi \leq t \leq 4\pi$$

$$C_2 : \psi(t) = (t \cos t, t \sin t, 0), \quad 2\pi \leq t \leq 4\pi$$

Show that for any vector field  $\mathbf{W}$  such that  $\nabla \times \mathbf{W} = \langle 0, 0, 0 \rangle$  the following is true:

$$\int_{C_1} \mathbf{W} \cdot d\mathbf{s} = \int_{C_2} \mathbf{W} \cdot d\mathbf{s}$$

1. Find the curl of the following vector field:

$$\left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2}, 0 \right\rangle$$

2. Find the divergence of the following vector field:

$$\langle x^2 + y^2, y^2 - x^2, 0 \rangle$$

3. Find the gradient of

$$f(x, y, z) = x^2 \sin(y - z)$$

4. Find two vector fields whose curls are  $\langle 0, 0, \frac{y}{x} \rangle$ .

1. Evaluate the following integrals:

a.  $\int_1^2 \int_2^3 \cos(2x + y) \, dx \, dy$

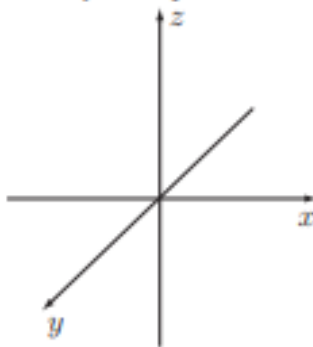
b.  $\int_0^1 \int_x^1 \sqrt{1 + y^2} \, dy \, dx$

2. Set up an integral for the volume which lies between the cone  $\sqrt{1 - x^2 - y^2}$  and the  $xy$ -plane.

Let  $f(x, y) = x^2y + x^3y^2$ .

1. Find  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$ .
2. If  $\phi(t) = (t^2, t - 1)$ , then what is  $f(\phi(t))$ ?
3. Suppose you don't know what  $\psi(t) = (x(t), y(t))$  is, but you know  $\psi(2) = (1, 1)$ ,  $\frac{dx}{dt}(2) = 3$ , and  $\frac{dy}{dt}(2) = 1$ . Find the derivative of  $f(\psi(t))$  when  $t = 2$ .
4. Suppose  $x$  and  $y$  are functions of  $u$  and  $v$ ,  $x(u, v) = u^2 + v$ , and  $y(1, 1) = 1$ . What would  $\frac{\partial y}{\partial u}$  have to be when  $(u, v) = (1, 1)$ , if  $\frac{\partial f}{\partial u} = 12$ ?

1. Determine if the coordinate system pictured is left or right handed.



2. Let  $f(x, y) = \frac{y}{x^2+1}$ .
  - a. Sketch the intersections of the graph of  $f(x, y)$  with the  $xy$ -plane, the  $xz$ -plane, and the  $yz$ -plane.
  - b. Sketch the level curves for  $f(x, y)$ .
  - c. Sketch the graph of  $f(x, y)$ .

1. Show that the function

$$f(x, y) = \frac{x \sin y}{x^2 + y^2}$$

does not have a limit as  $(x, y) \rightarrow (0, 0)$ .

2. Is the function

$$f(x, y) = \begin{cases} \frac{x+y}{x+y} & (x, y) \neq (0, 0) \\ 1 & (x, y) = (0, 0) \end{cases}$$

continuous at  $(0, 0)$ ?

3. Find the domain of the function

$$f(x, y) = \ln \frac{1}{x - y^2}.$$

1. Let  $f(x, y)$  be the following function:

$$f(x, y) = xy + x - 2y + 4$$

- a. Sketch the intersections of  $f(x, y)$  with the  $xz$ - and  $yz$ -planes.
- b. Find the critical point(s) and compute the value of

$$\begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix}$$

Can you say if the graph has a max, min, or saddle at the critical point(s)?

- c. Find the slope of the tangent line to the graph of  $f(x, y)$ , in the direction of  $\langle 1, 2 \rangle$ , at the point  $(0, 1)$ .
- d. Find the volume under the graph of  $f(x, y)$ , and above the rectangle in the  $xy$ -plane with vertices at  $(0, 0)$ ,  $(1, 0)$ ,  $(0, 2)$ , and  $(1, 2)$ .