VECTOR ANALYSIS

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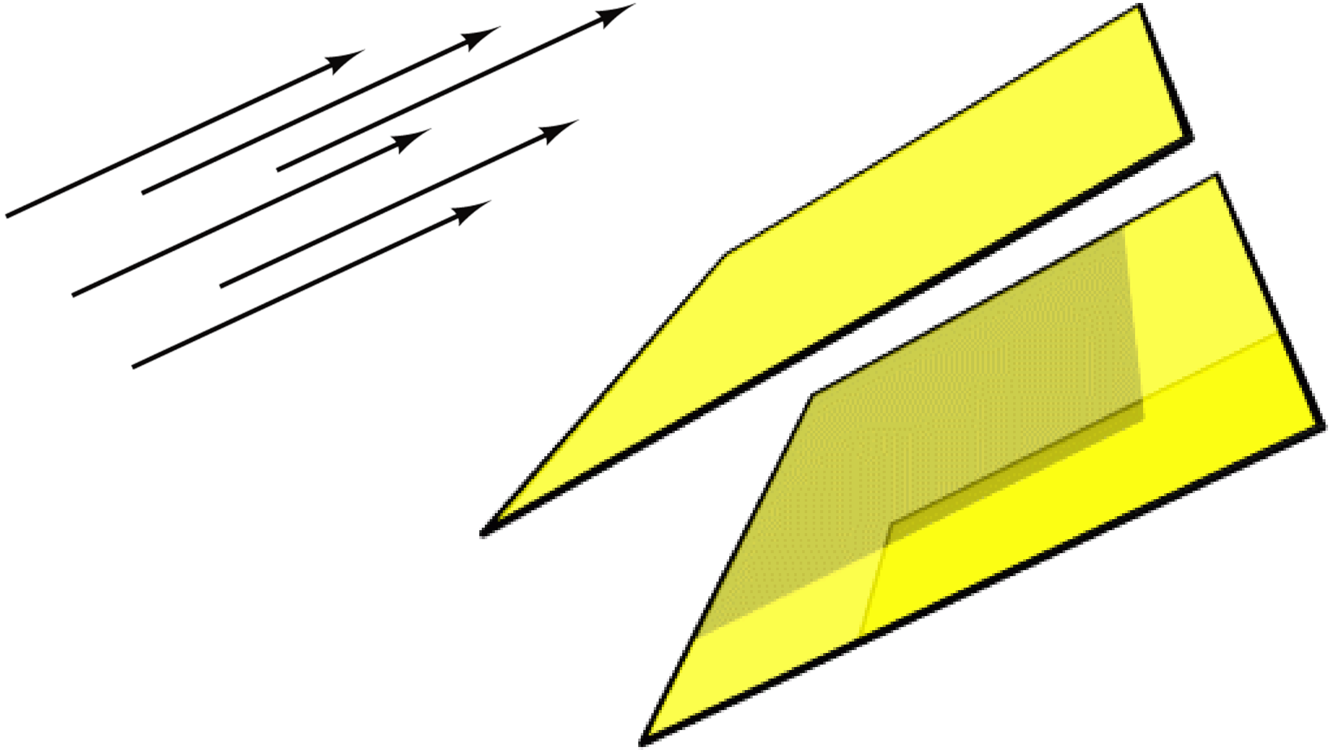
Chapter 1: Vectors and the Geometry of Space

Chapter 2: Functions of Several Variables

Chapter one

Vectors And The Geometry Of Space



The Euclidean 2-space

- The Euclidean 2-space denoted by R^2 is the set

$$\{(x,y) \mid x,y \in R\}.$$

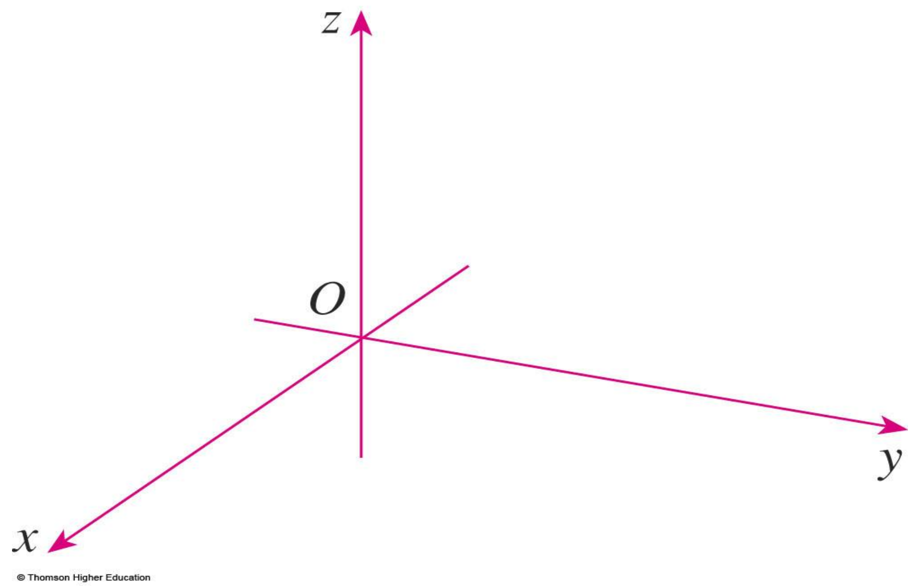
The Euclidean 3-space

- The Euclidean 3-space denoted by R^3 is the set

$$\{(x,y,z) \mid x,y,z \in R\}.$$

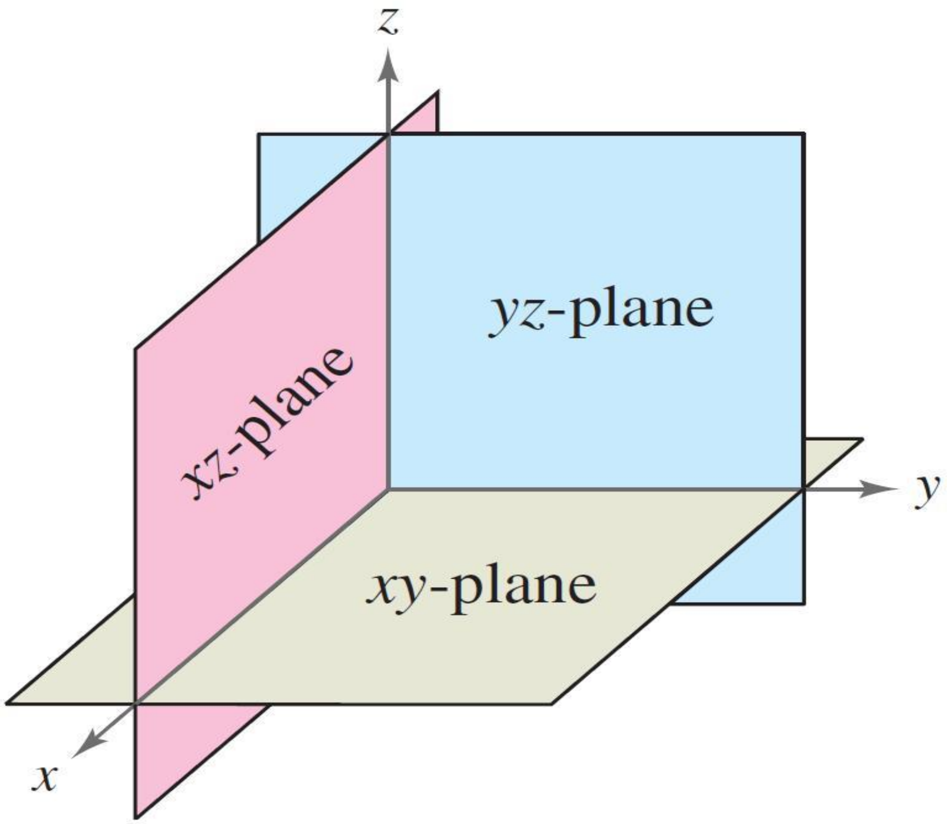
Three-dimensional Coordinate Systems

- To locate a point in space, three numbers are required.
- We represent any point in space by an ordered triple (x, y, z) of real numbers.
- In order to represent points in space, we use three mutually perpendicular coordinate axes.



Coordinate Planes

- The three coordinate axes determine the three coordinate planes.



- The *xy*-plane (z=0 plane) contains the *x*- and *y*-axes.
- The *yz*-plane (x=0 plane) contains the *y*- and *z*-axes.
- The *xz*-plane (y=0 plane) contains the *x*- and *z*-axes.

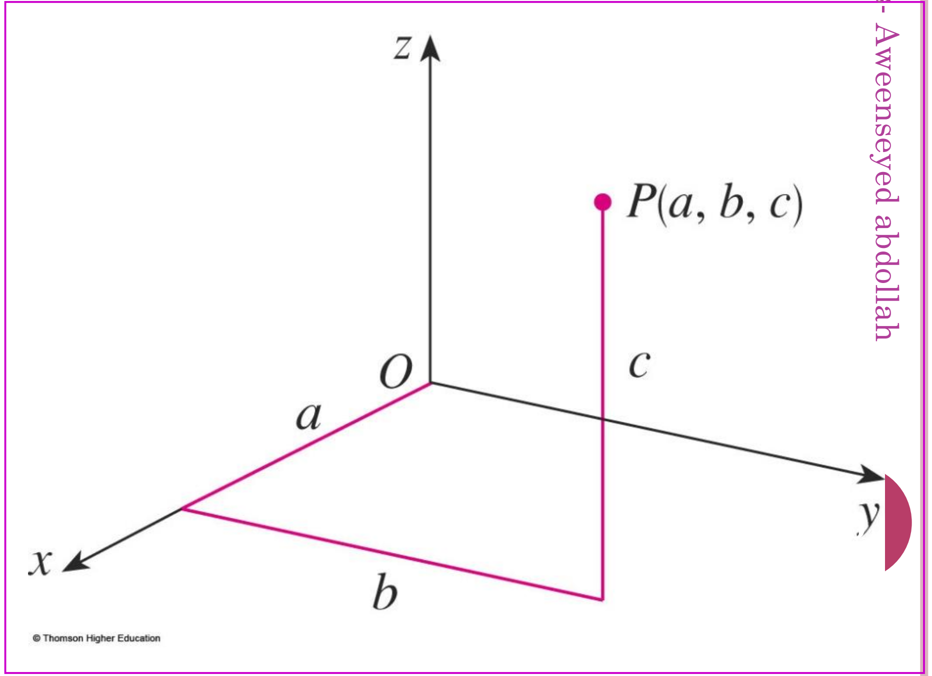
Plotting Points in Space

- To locate the point P(a, b, c), we can start at the origin O and proceed as follows:

  - First, move a units along the x-axis.

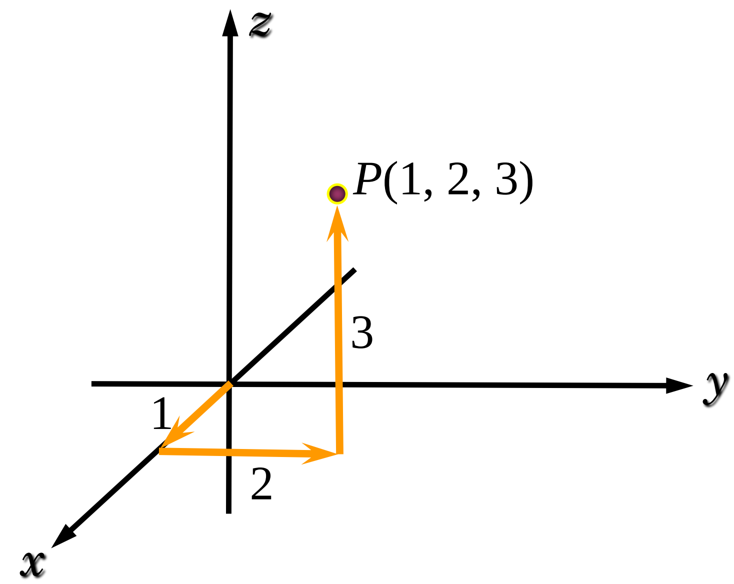
  - Then, move b units parallel to the y-axis.

  - Finally, move c units parallel to the z-axis

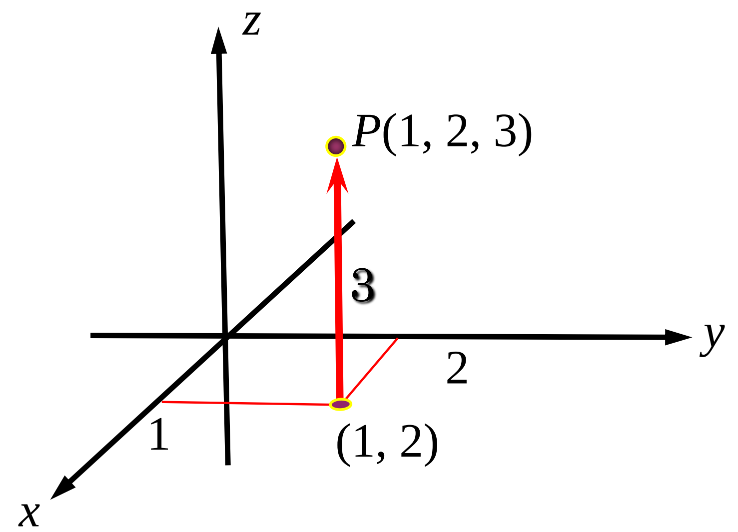


Example:

- Consider the task of locating $P(1, 2, 3)$ in 3-space:



- One method to achieve this is to start at the origin and measure out from there, axis by axis:



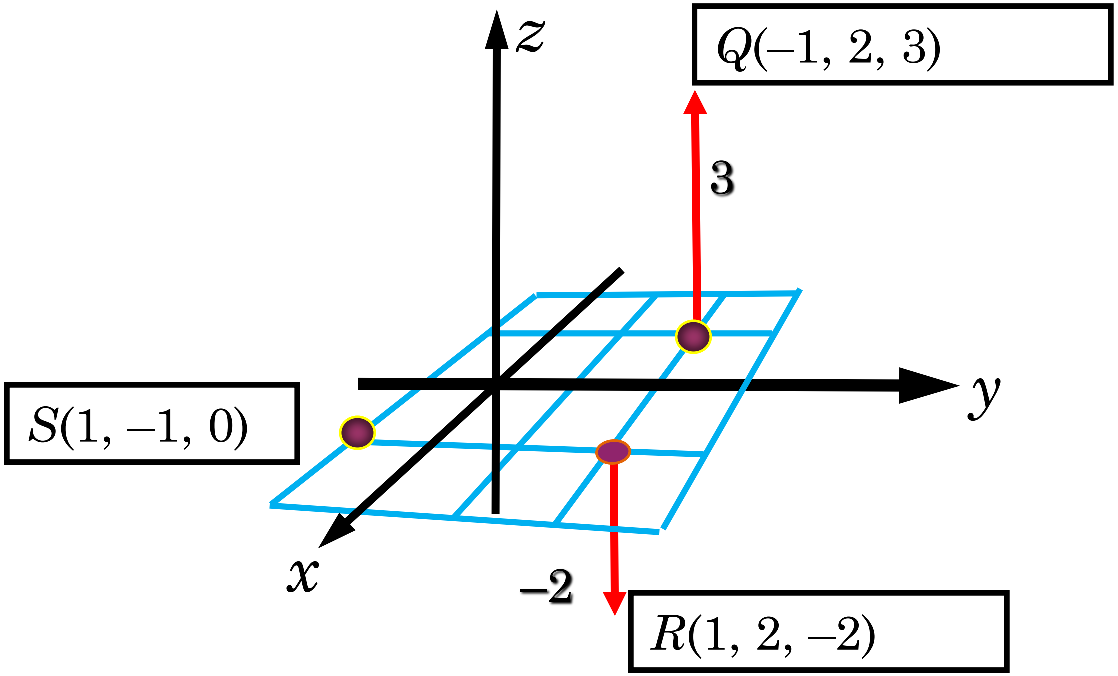
- Another common method is to find the *xy* coordinate and from there elevate to the level of the *z* value:

Example:

Locate the following points in 3-space:

$Q(-1, 2, 3)$, $R(1, 2, -2)$, and $S(1, -1, 0)$.

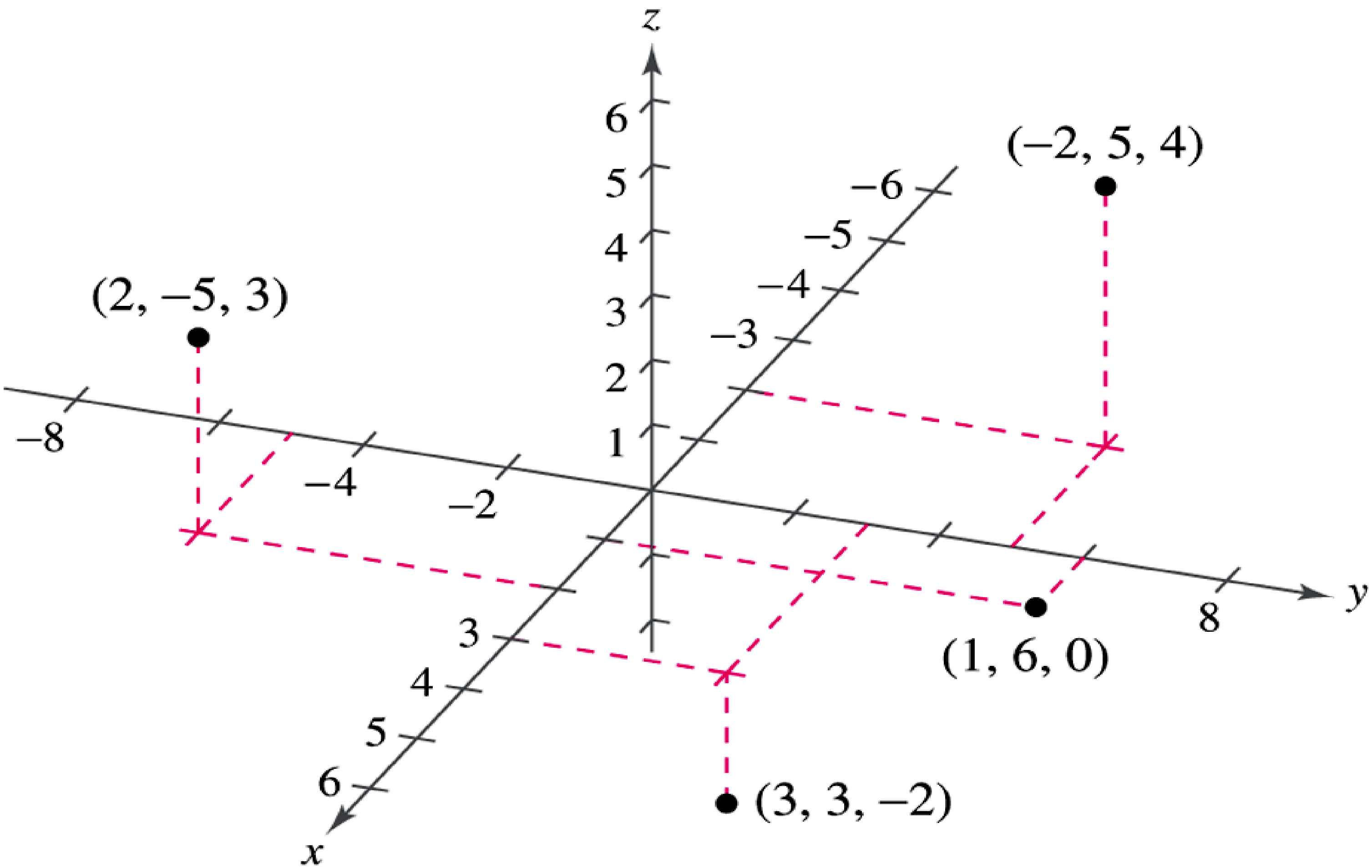
<u>Solution</u>



Example:

Locate the following points in 3-space:

(–2,5,4) ,(1,6,0),(3,3,-2)and(2,-5,3).



Distance Formula In Three Dimensions

The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is:

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example:

The distance from the point $P(2, -1, 7)$ to the point $Q(1, -3, 5)$ is:

Solution:

$$|PQ| = \sqrt{(1-2)^2 + (-3+1)^2 + (5-7)^2}$$

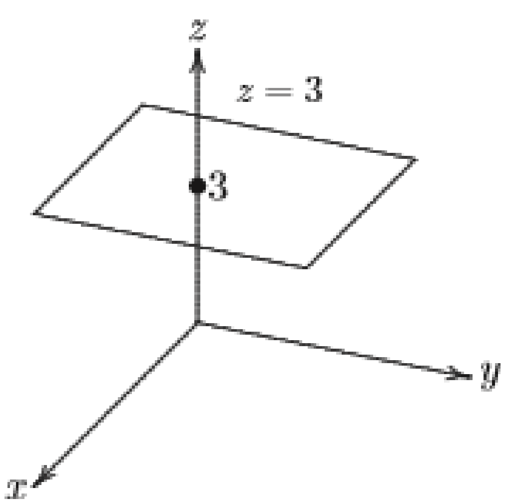
$$= \sqrt{1+4+4}$$

$$= 3$$

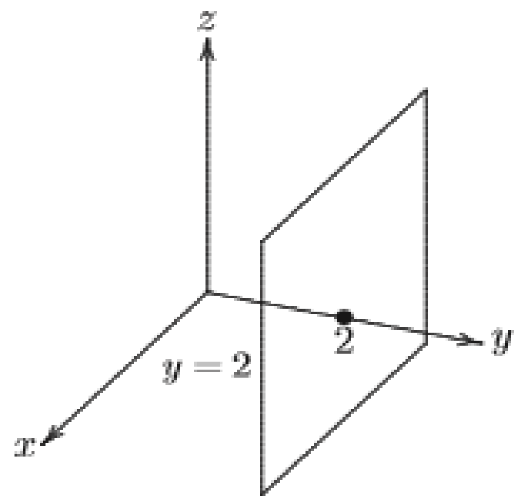
2-D Vs. 3-D Analytic Geometry

- In 2-D analytic geometry, the graph of an equation involving *x* and *y* is a curve in R^2.
- In 3-D analytic geometry, an equation in *x*, *y*, and *z* represents a surface in R^3.

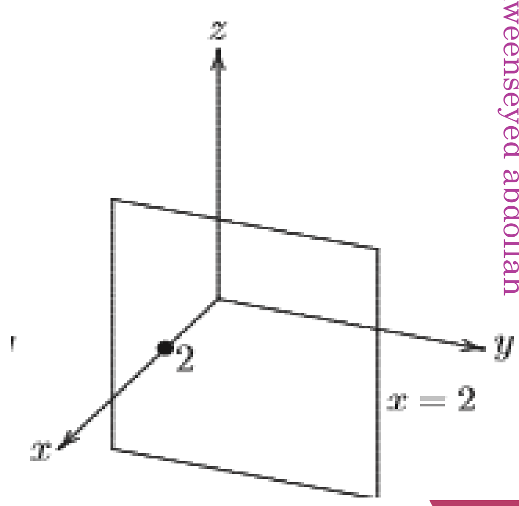
Example 1: What surfaces in R^3 are represented by the following equations? a) z=3 b)y=2 c)x=2



a) z=3, a plane in R^3 parallel to the *xy*-plane



b) y=2, a plane in R^3 parallel to the *xz*-plane

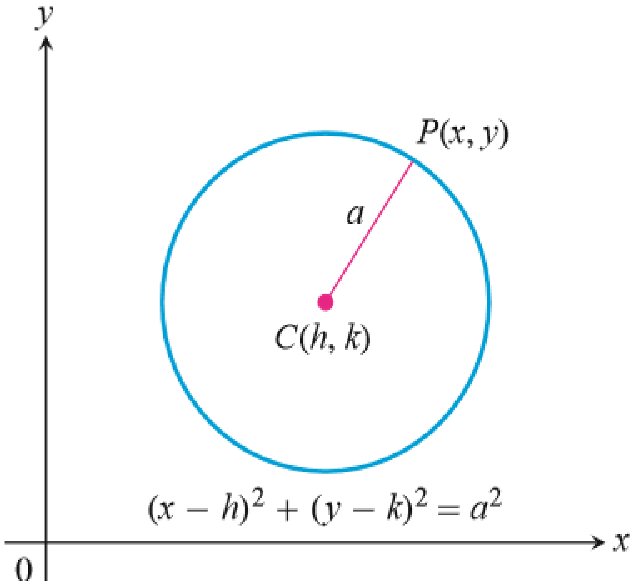


c) x=2, a plane in R^3 parallel to the *yz*-plane

Example 2:

An equation of a circle with centre c(h,k) and radius a is

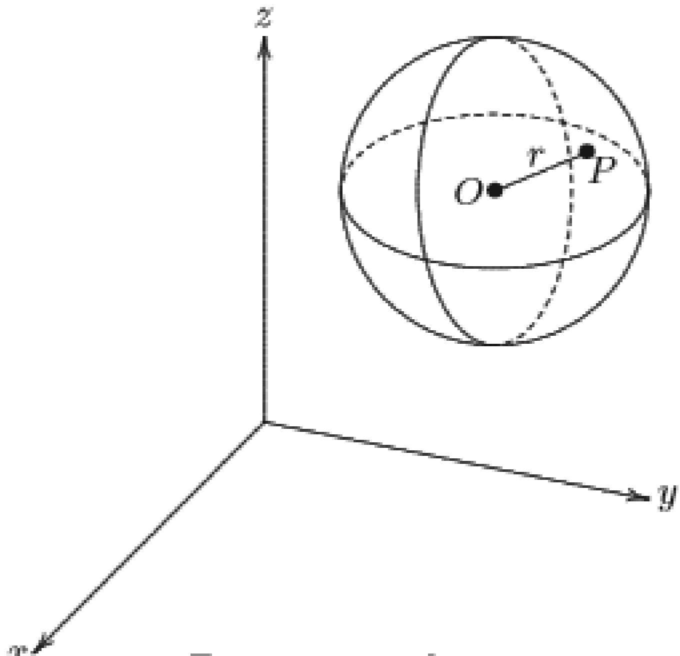
$$(x-h)^2+(y-k)^2=a^2$$



Example 3:

An equation of a sphere with centre O(a,b,c) and radius r is

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$



Exercise:

Show that $x^2+y^2+z^2+4x-6y+2z+6=0$ is the equation of a sphere. Describe its intersection with the plane z = 1.