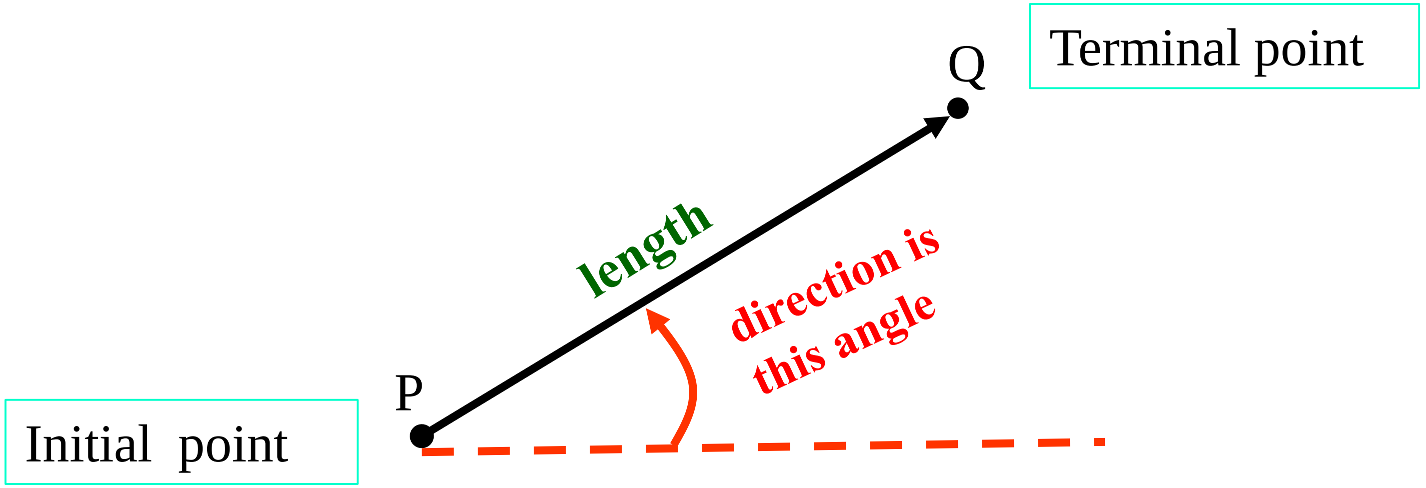
Vectors

- A vector in the plane is a directed line segment .

- The directed line segment $\overrightarrow{PQ}$ has initial point *P* and terminal point *Q*.

- length of the vector $\overrightarrow{PQ}$ is denoted by $\left|\overrightarrow{PQ}\right|$.

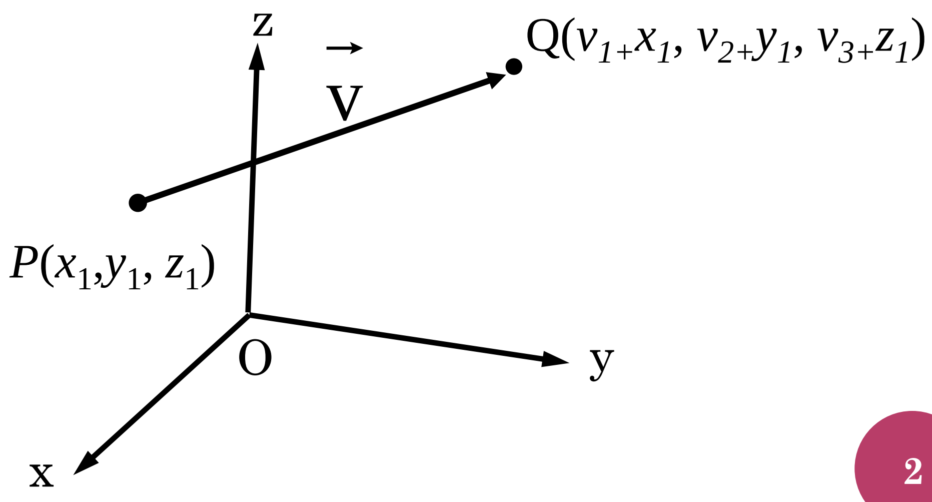


- a two-dimensional vector is an ordered pair $\vec{v} = \langle v_1, v_2 \rangle$ of real numbers, and a 3-dimensional vector is an ordered triple $\vec{v} = \langle v_1, v_2, v_3 \rangle$ of real numbers.

- v_1, v_2, v_3 are called the components of $\vec{v}$

- A vector $\vec{v} = \langle v_1, v_2, v_3 \rangle$ can be represented by an arrow from any point $P(x_1, y_1, z_1)$ to the point $Q(v_1 + x_1, v_2 + y_1, v_3 + z_1)$ in R^3.

$$\overrightarrow{PQ} = \vec{v} = \langle v_1, v_2, v_3 \rangle$$

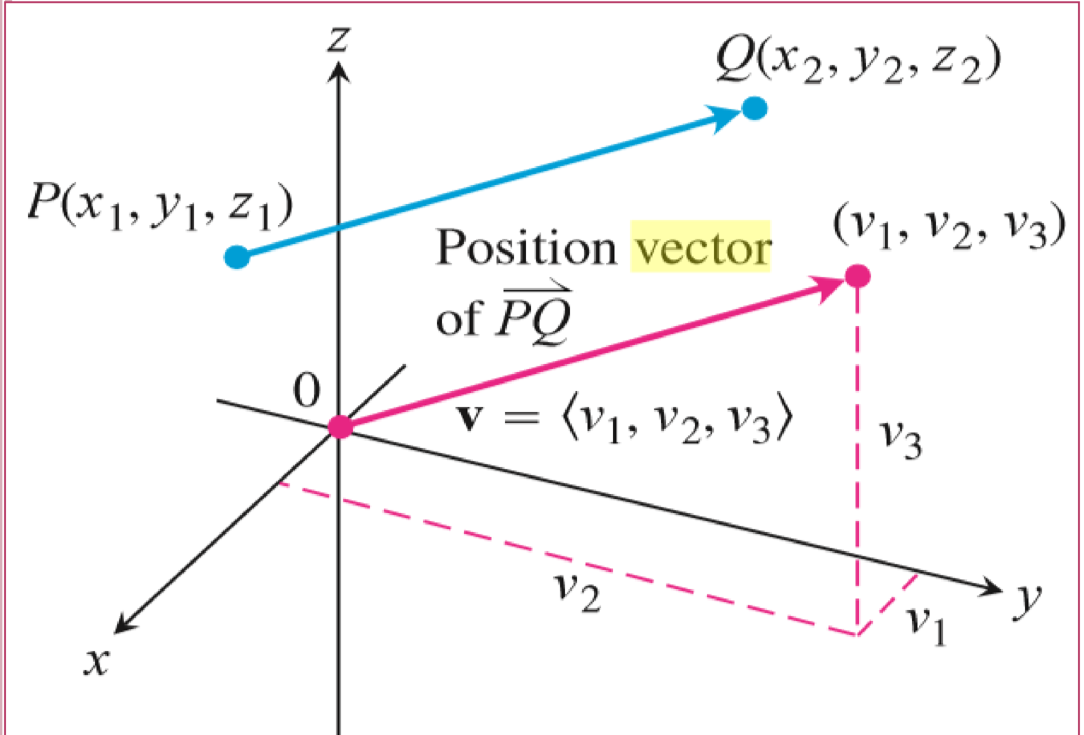


- If P is the origin O, $\vec{v}$ is called the position vector of the point O.

Standard Position Vector

❖ Given the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, the standard position vector $\vec{v}$ equal to $\overrightarrow{PQ}$ is $v = \langle v_1, v_2, v_3 \rangle$

$$= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

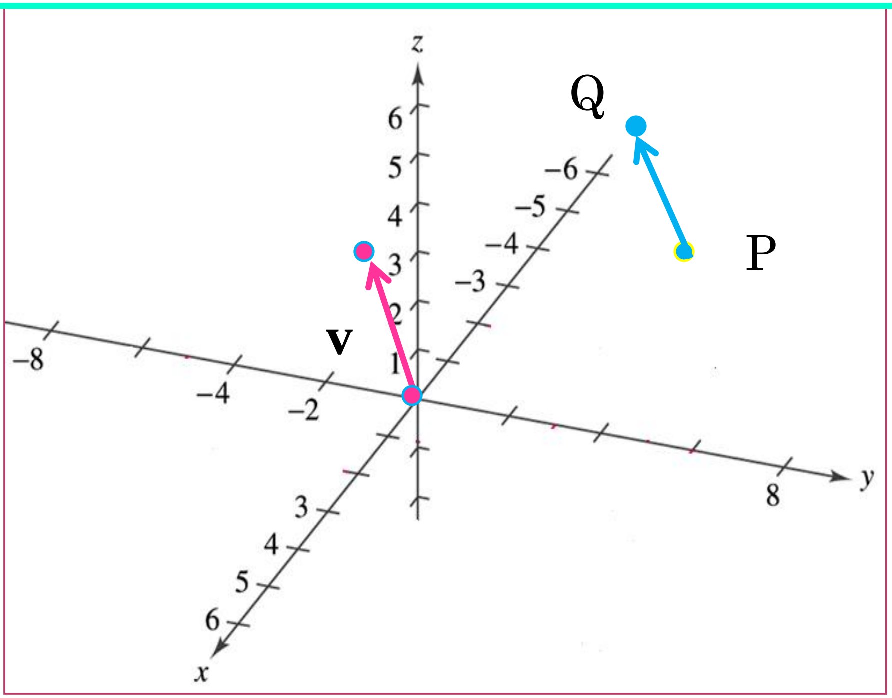


Example: Write the component form of the vector $\overrightarrow{PQ}$ where P(-3,4,1) and Q(-5,2,2).

Solution: The standard position vector v representing has components

$$V= \langle v_1, v_2, v_3 \rangle = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

$$= \langle -5-(-3), 2-4, 2-1 \rangle = \langle -2, -2, 1 \rangle$$



Length Of The Three-dimensional Vector

- The length of the vector v= $\overrightarrow{PQ}$ is the nonnegative number

$$|v| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- The length of the vector is sometimes called its magnitude or the norm of v and denoted by the symbol |v| or ||v|| .

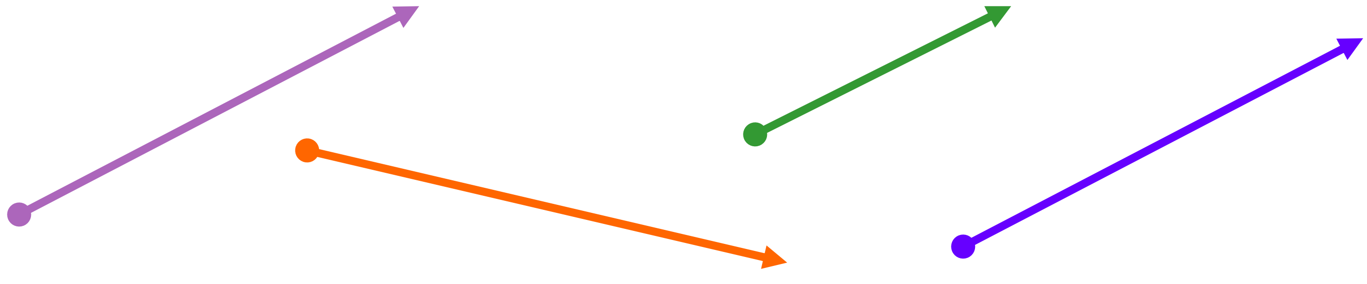
Example:

Find the length of the vector with initial point P(-3,4,1) and terminal point Q(-5,2,2).

Solution: the component form of $\overrightarrow{PQ}$ is v= $\langle -2,-2,1 \rangle$ so the length of v= $\overrightarrow{PQ}$ is

$$|v| = \sqrt{(-2)^2 + (-2)^2 + (1)^2} = \sqrt{9} = 3$$

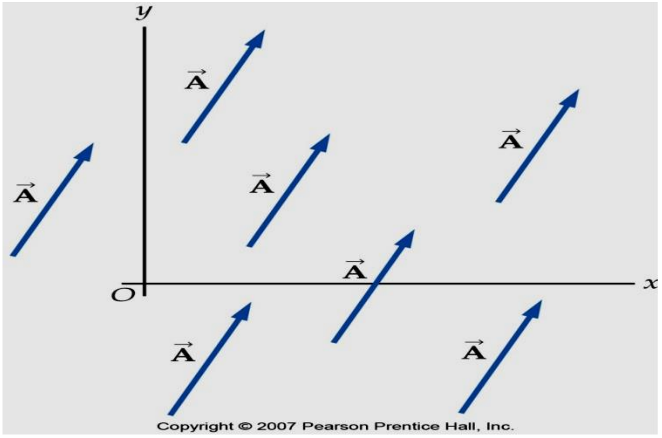
- Two vectors are equal if they have the same length and direction. SO, $\overrightarrow{PQ} \neq \overrightarrow{QP}$



algebraically if $v_1 = \langle x_1, y_1, z_1 \rangle$ and $v_2 = \langle x_2, y_2, z_2 \rangle$, then

$$v_1 = v_2 \text{ if and only if } x_1 = x_2, y_1 = y_2 \text{ and } z_1 = z_2$$

- **Note:** moving a vector does not change it. A vector is only defined by its magnitude and direction, not starting location.



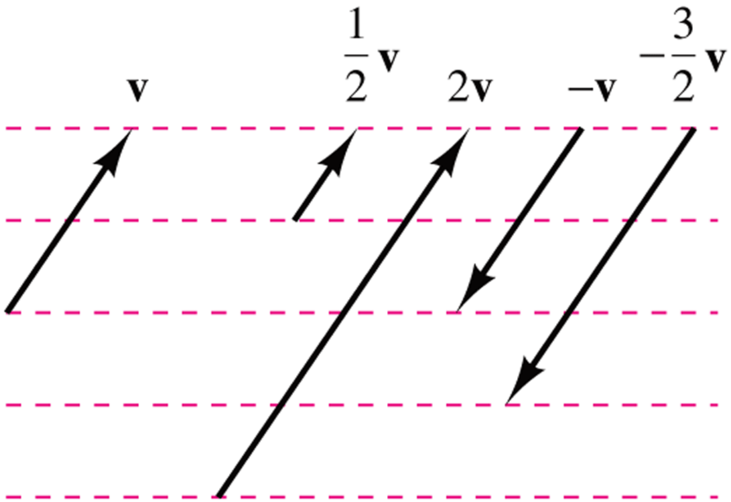
Vector Addition And Scalar Multiplication

Let u= $\langle u_1, u_2, u_3 \rangle$ and v= $\langle v_1, v_2, v_3 \rangle$ be vectors with k a scalar.

Addition: u + v = $\langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$

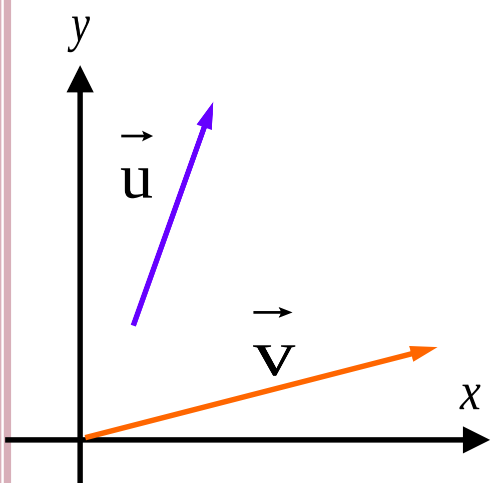
Difference: u-v= $\langle u_1 - v_1, u_2 - v_2, u_3 - v_3 \rangle$

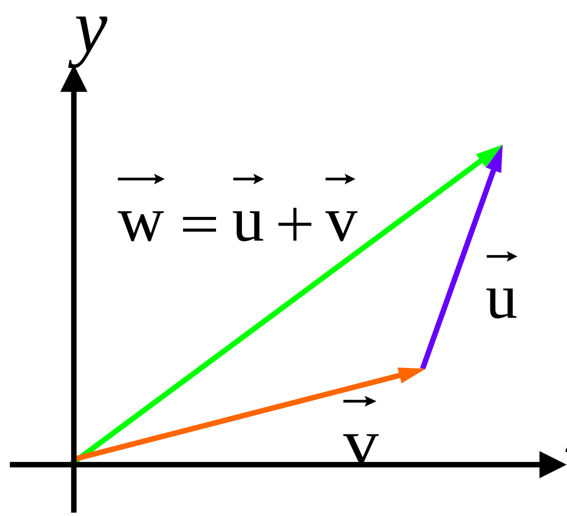
Scalar multiplication: ku= $\langle ku_1, ku_2, ku_3 \rangle$

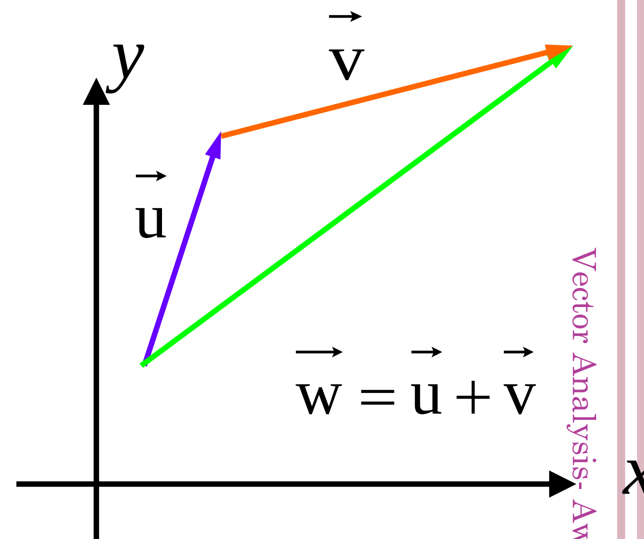


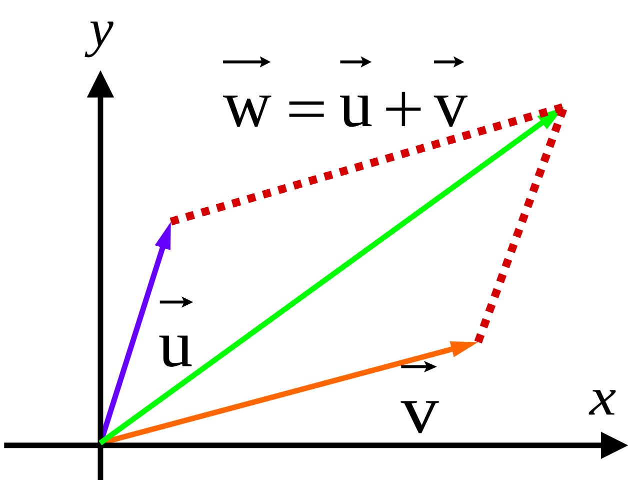
There are two basic ways to add vectors:

1-Tip to tail method 2-Parallelogram method

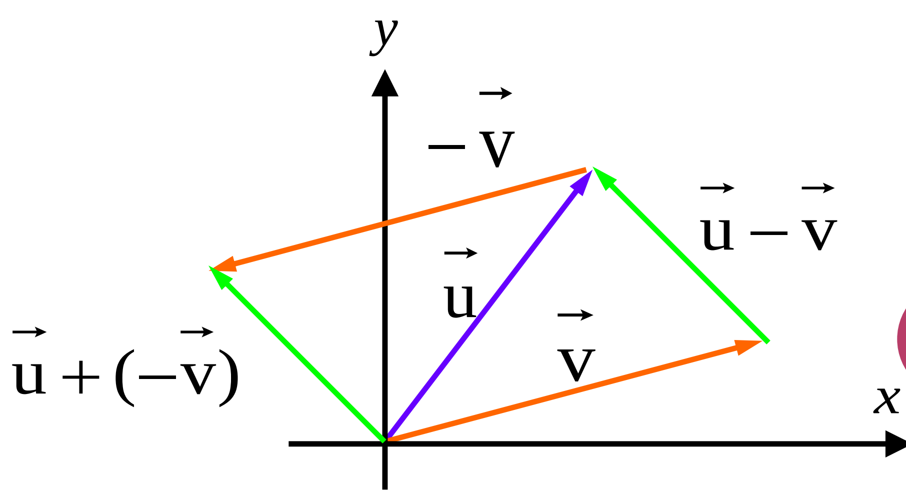








Parallelogram method



Example: Let $u=\langle -1,3,1\rangle$ and $v=\langle 4,7,0\rangle$. find

a. 2u+3v **b.** u-v **c.** $\left|\frac{1}{2}u\right|$

Solution:

a. $2u+3v = 2\langle -1,3,1\rangle + 3\langle 4,7,0\rangle = \langle -2,6,2\rangle + \langle 12,21,0\rangle = \langle 10,27,2\rangle$

b. $u-v = \langle -1,3,1\rangle - \langle 4,7,0\rangle = \langle -1-4, 3-7, 1-0\rangle = \langle -5,-4,1\rangle$

c. $\left|\frac{1}{2}u\right| = \left|\left\langle \frac{-1}{2}, \frac{3}{2}, \frac{1}{2}\right\rangle\right| = \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{2}\sqrt{11}$

Proposition (Properties of Vector Operations)

Let u,v,w be vectors and a,b be scalars.

1. u+v = v+u
2. (u+v)+w = u+(v+w)
3. u+0=u
4. u+(-u)=0
5. 0u=0
6. 1u=u
7. $a(b\mathrm{u})=(ab)\mathrm{u}$
8. $a(\mathrm{u}+\mathrm{v})=a\mathrm{u}+a\mathrm{v}$
9. $(a+b)\mathrm{u}=a\mathrm{u}+b\mathrm{u}$
10. $a\mathrm{v}=\mathrm{v}a$
11. $|a\mathrm{v}|=|a||\mathrm{v}|$

Linear Combinations

A vector r, is said to be a linear combination of the vectors $a, b, c, ..$ect. If there exist scalars $x, y, z, \ldots$ect. Such that

$$r = xa + yb + zc + \cdots.$$

Example:

The vectors

$$2a + b - 4c, \qquad a + 2b - 3c$$

are linear combinations of the vectors a, b, c.

Unit Vectors

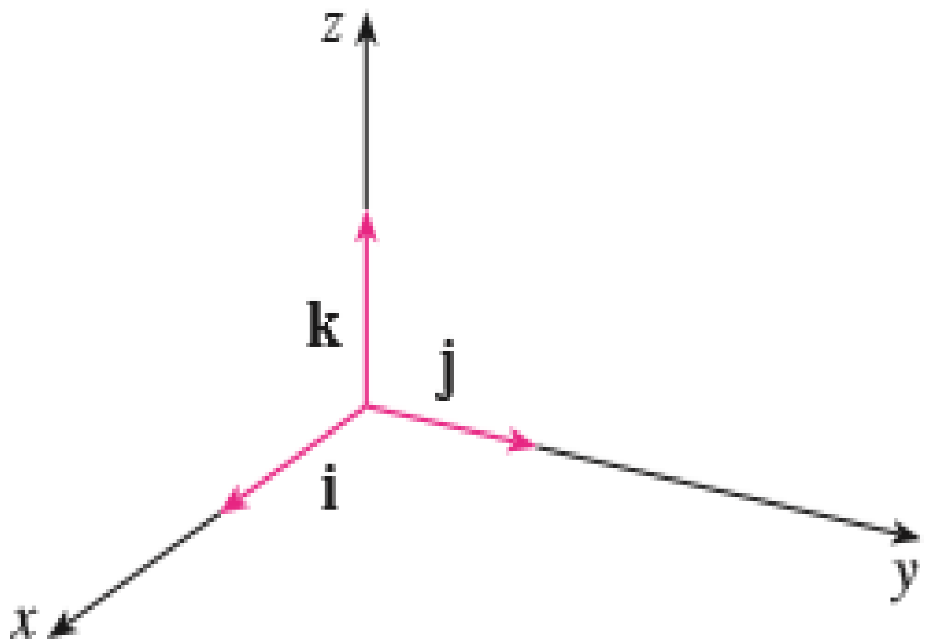
A vector v of length 1 is called a unit vector.

The standard unit vectors (**standard basis vectors**) are

$$\mathbf{i} = \langle 1, 0, 0 \rangle,$$

$$\mathbf{j} = \langle 0, 1, 0 \rangle,$$

$$\mathbf{k} = \langle 0, 0, 1 \rangle.$$



- Any vector v= $\langle v_1, v_2, v_3 \rangle$ can be written as a linear combination of the standard unit vectors as follows:

$$\mathbf{v} = \langle v_1, v_2, v_3 \rangle = \langle v_1, 0, 0 \rangle + \langle 0, v_2, 0 \rangle + \langle 0, 0, v_3 \rangle$$
$$= v_1 \langle 1, 0, 0 \rangle + v_2 \langle 0, 1, 0 \rangle + v_3 \langle 0, 0, 1 \rangle$$
$$= v_1 \mathbf{i} + v_2 \mathbf{j} + v_3 \mathbf{k}.$$

- We call the scalar (or number) v_1 the i-component of the vector v, v_2 the j-component, and v_3 the k-component.

- In component form, the vector from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is

$$\overrightarrow{P_1 P_2} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$$