Note if $v \neq 0$, then

1. The unit vector that has the same direction as v is $u = \frac{v}{|v|}$

2. The equation $v = |v| \frac{v}{|v|}$ expresses v in terms of its length and direction.

Examples:

1- Find a unit vector u in the direction of the vector from $P_1(1,0,1)$ to $P_2(3,2,0)$.

2- Find a vector of magnitude 7 in the direction of v=12i-5k

The Dot Product

The dot product or scalar product of vectors $\mathrm{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathrm{v} = \langle v_1, v_2, v_3 \rangle$ is the number $\mathrm{u.v} = u_1.v_1 + u_2.v_2 + u_3.v_3$

Example: Let a =<1, 2, 3> and b =<-1, 0, -1>. Find a·b.

Proposition (Properties of the Dot Product)

If u,v, and w are any vectors and c is a scalar, then

1. u.v= v.u
2. (cu).v= u.(cv)=c(u.v)
3. u.(v+w)= u.v+u.w
4. 0.u=0
5. $\mathrm{u.u} = |\,\mathrm{u}\,|^2$

Theorem (Angle Between Two Vectors)

If θ is the angle between two nonzero vectors

$\mathrm{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathrm{v} = \langle v_1, v_2, v_3 \rangle$ then

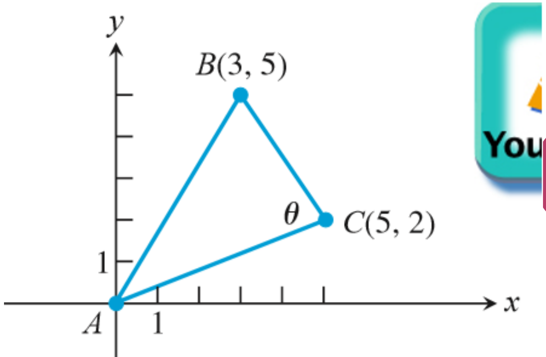
$$\mathrm{u}.\mathrm{v} = |u\|v|\cos\theta \qquad 0 \le \theta \le \pi$$

Examples:

1-Find the angle between u=i-2j-2k and v=6i+3j+2k

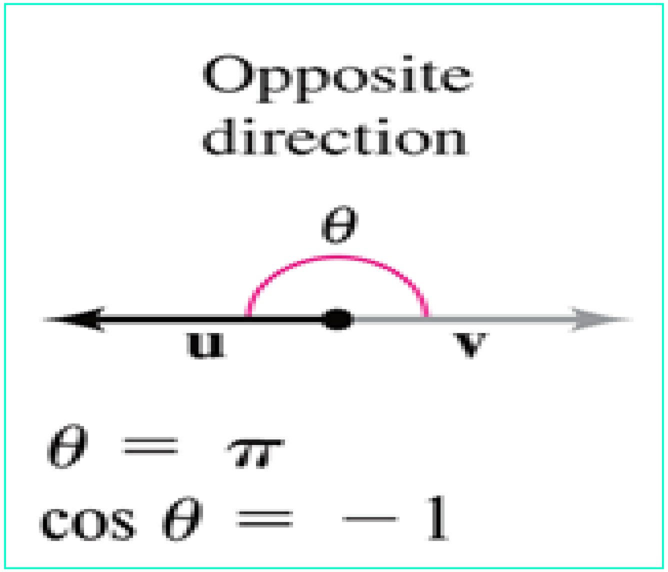
2-Find the angle θ in the triangle ABC determined by the vertices A(0,0), B(3,5), C(5,2).

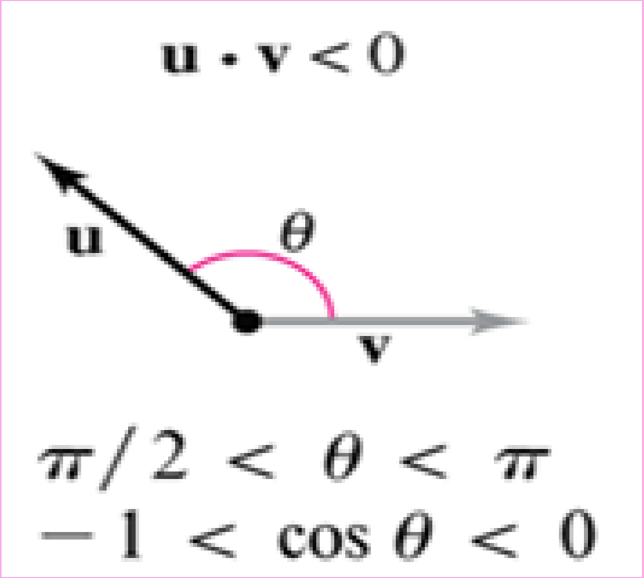
3-Prove that $ma.nb = mn(a.b)$ where a, b are any vectors and m, n any scalars.

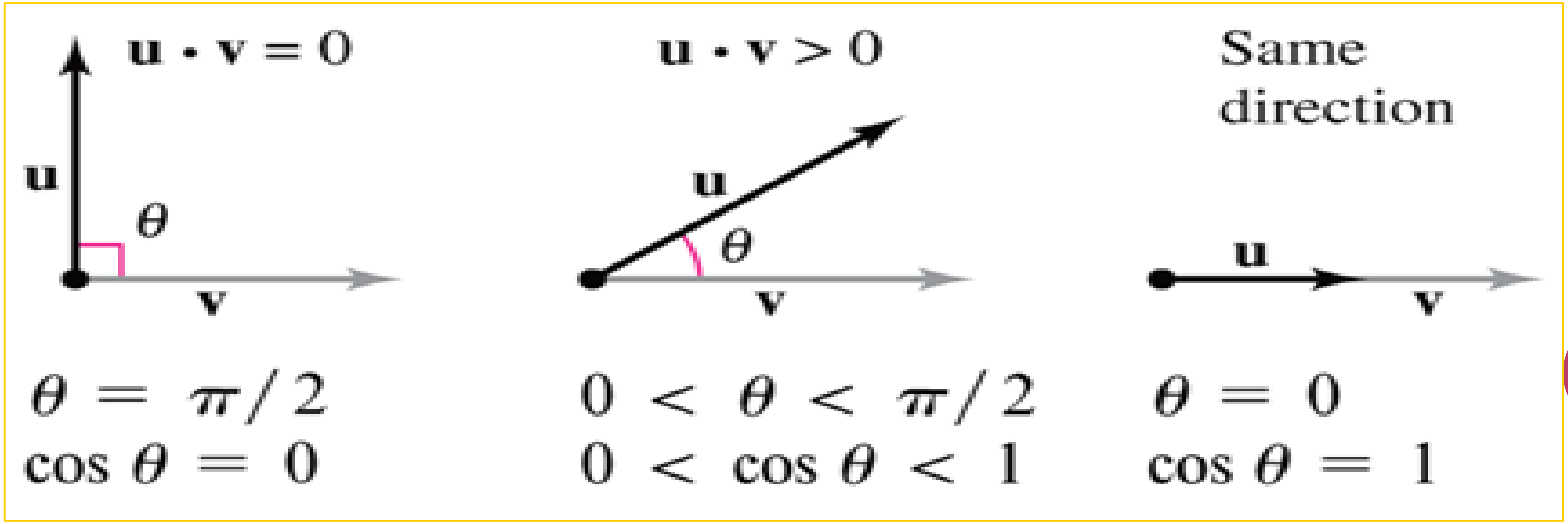


Sign of the scalar product

$$u.v = |u||v|\cos\theta$$







Orthogonal Vectors

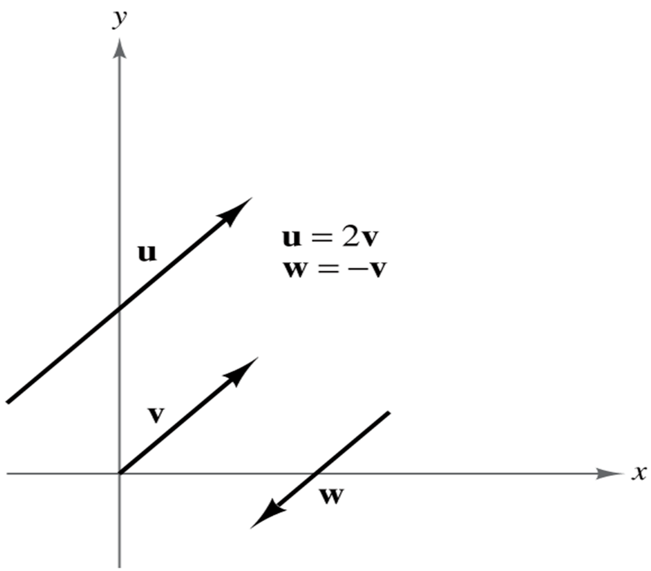
- Two vectors u and v are said to be orthogonal or perpendicular if the angle between them is 90◦. In other words,

$$\text{u and v are orthogonal} \Longleftrightarrow u \cdot v = 0.$$

- The zero vector is considered to be perpendicular to all vectors.

Parallel Vectors

Two nonzero vectors u and v are parallel if there is some scalar c such that u=cv.



Examples:

1. $2i + 2j - k$ is orthogonal to $5i - 4j + 2k$ because

$$(2i + 2j - k)(5i - 4j + 2k) = (2)(5) + (2)(-4) + (-1)(2) = 0.$$

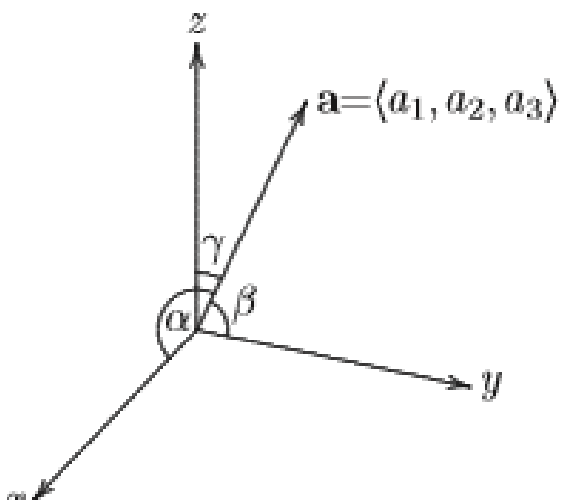
1. Prove that the angle in a semi-circle is a right angle.

Homework:

1-Find the angle between u=i-2j+2k and v= -3i+6j-6k

2. Let $a = (a_1, a_2, a_3) \neq 0$ and α, β, γ in $[0, \pi]$ are angles that a makes with the x,y,z axes respectively. Show that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

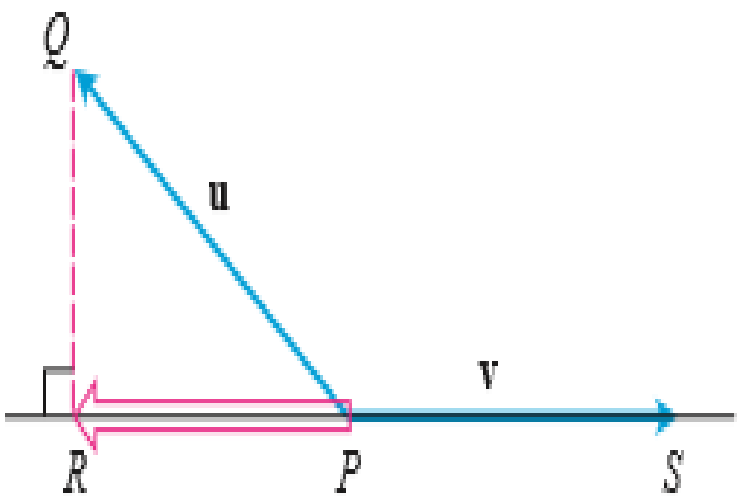


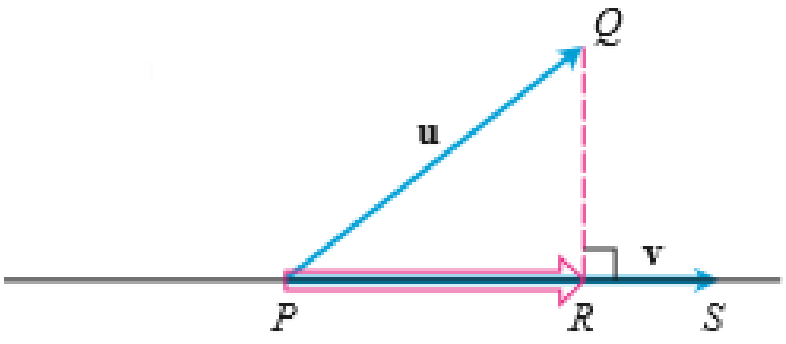
Vector Analysis- Aweenseyed abdollah

Vector projection

The vector projection of $\mathrm{u} = \overrightarrow{PQ}$ onto a nonzero vector $\mathrm{v} = \overrightarrow{PS}$ is the vector $\overrightarrow{PR}$ determined by dropping a perpendicular from Q to the line PS. The notation for this vector is

$$\mathrm{proj}_{\mathrm{v}}\mathrm{u} \quad (\text{“the vector projection of u onto v”}).$$



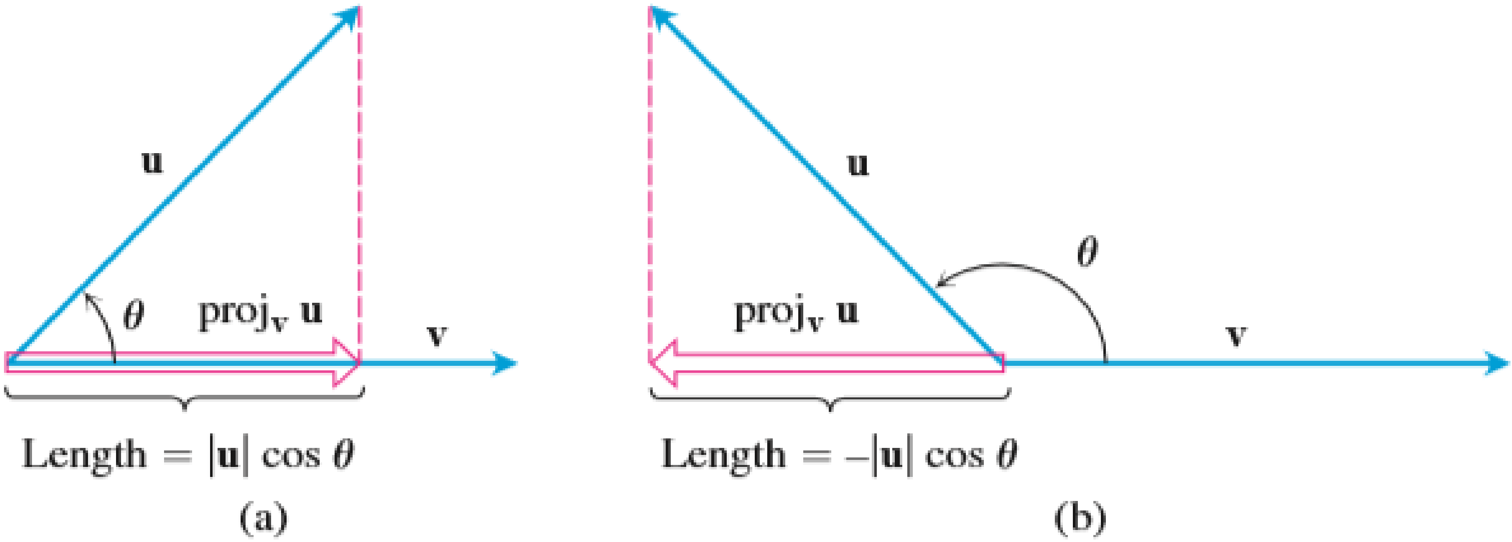


Vector projection

- Vector projection of u onto v is: $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$

- Scalar projection of u onto v is: $|\mathbf{u}|\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|} = \mathbf{u}\cdot\dfrac{\mathbf{v}}{|\mathbf{v}|}$

Note that the scalar projection is negative if θ > 90◦.



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Example:

Find the scalar and vector projection of u=6i+3j+2k onto v=i-2j-2k.

Cross Product

If $\mathrm{u} = u_1 i + u_2 j + u_3 k$ and $\mathrm{v} = v_1 i + v_2 j + v_3 k$ then the cross product or vector product of u and v is

$$u \times v = \begin{vmatrix} \mathrm{i} & j & k \\ \mathrm{u}_1 & \mathrm{u}_2 & \mathrm{u}_3 \\ \mathrm{v}_1 & \mathrm{v}_2 & \mathrm{v}_3 \end{vmatrix}$$

$$= \begin{vmatrix} \mathrm{u}_2 & \mathrm{u}_3 \\ \mathrm{v}_2 & \mathrm{v}_3 \end{vmatrix} \mathrm{i} - \begin{vmatrix} \mathrm{u}_1 & \mathrm{u}_3 \\ \mathrm{v}_1 & \mathrm{v}_3 \end{vmatrix} \mathrm{j} + \begin{vmatrix} \mathrm{u}_1 & \mathrm{u}_2 \\ \mathrm{v}_1 & \mathrm{v}_2 \end{vmatrix} \mathrm{k}.$$

$$= (\mathrm{u}_2\mathrm{v}_3 - \mathrm{v}_2\mathrm{u}_3)\mathrm{i} - (\mathrm{u}_1\mathrm{v}_3 - \mathrm{v}_1\mathrm{u}_3)\mathrm{j} + (\mathrm{u}_1\mathrm{v}_2 - \mathrm{v}_1\mathrm{u}_2)k$$

- i×j = k, j×k = i, k×i = j and i×i = j×j = k×k = 0.

Example: Let $u = \langle 1,3,4 \rangle$ and $v = \langle 2,7,-5 \rangle$ .Find u×v.

Theorem :

Let $u = \langle u_1, u_2, u_3 \rangle$, $v = \langle v_1, v_2, v_3 \rangle$ and $w = \langle w_1, w_2, w_3 \rangle$, then

$$u.(v \times w) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Corollary: u×v is perpendicular to both u and v.

Theorem:

If θ is the angle between u and v (so $0 \leq \theta \leq \pi$), then

$$|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin\theta$$