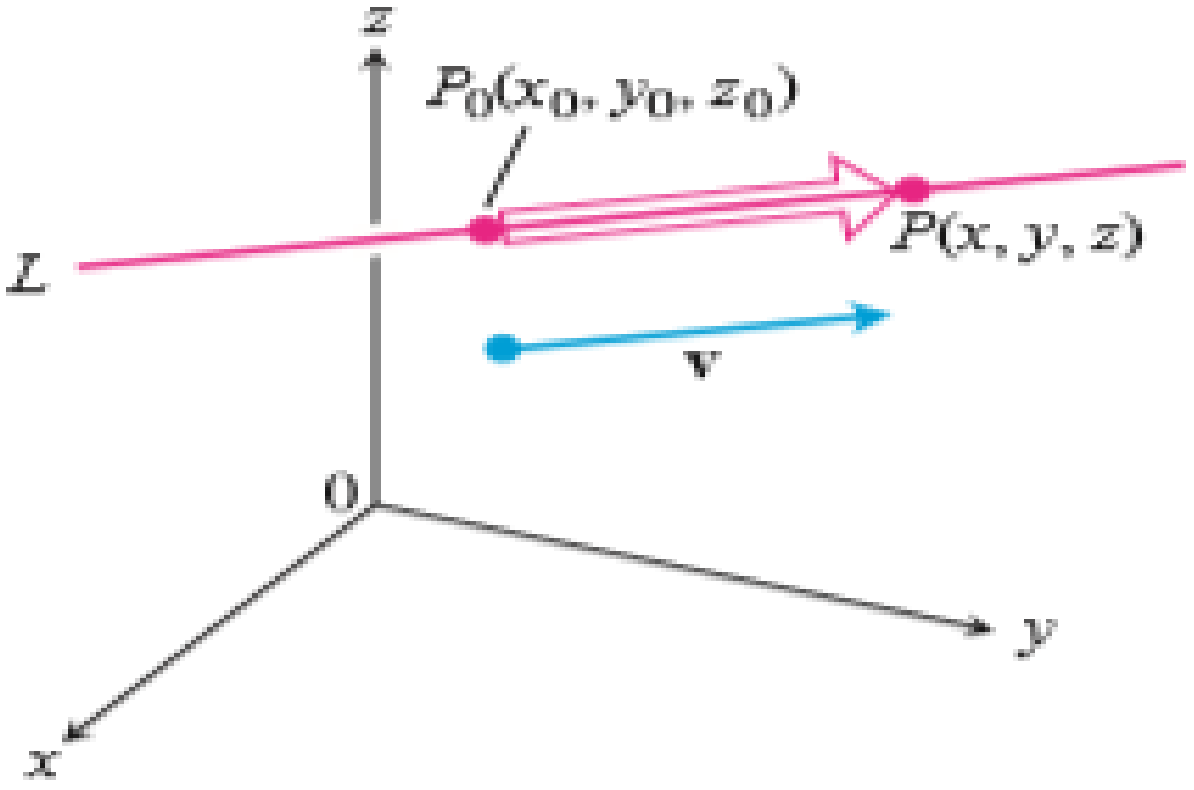
Lines and Planes in Space(R^3)

- In the plane, a line is determined by a point and a number giving the slope of the line.
- In space a line is determined by a point and a vector giving the direction of the line.

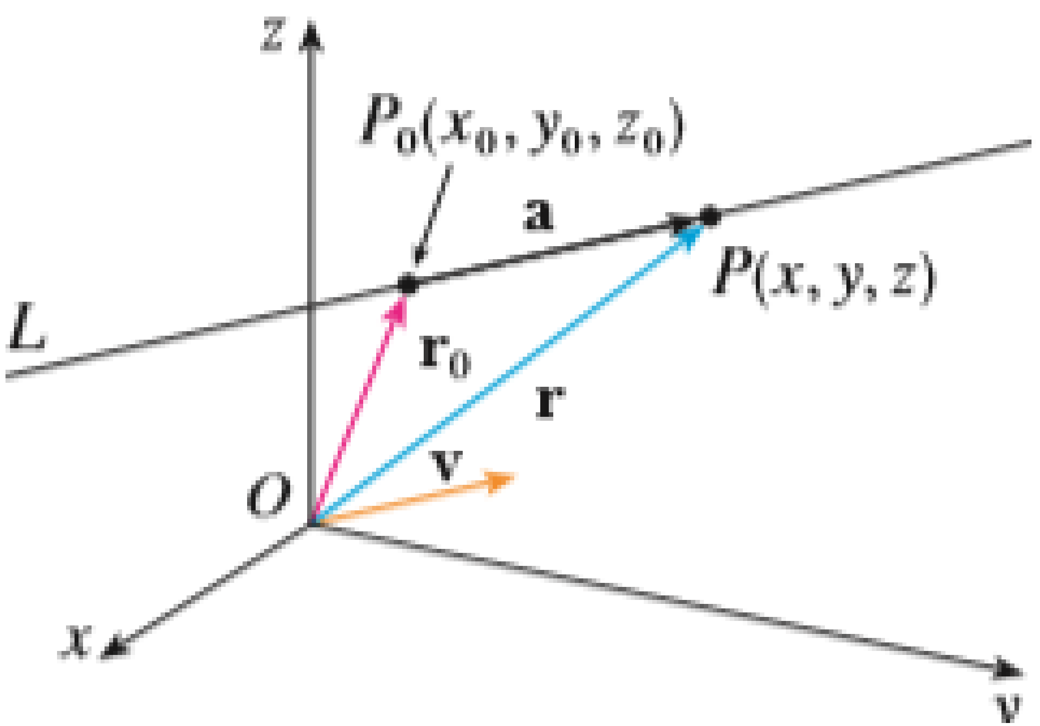


Vector Equation for a Line

A vector equation for the line L through $P_0(x_0, y_0, z_0)$ parallel to v is

$$r(t) = r_0 + tv, \qquad -\infty < t < \infty$$

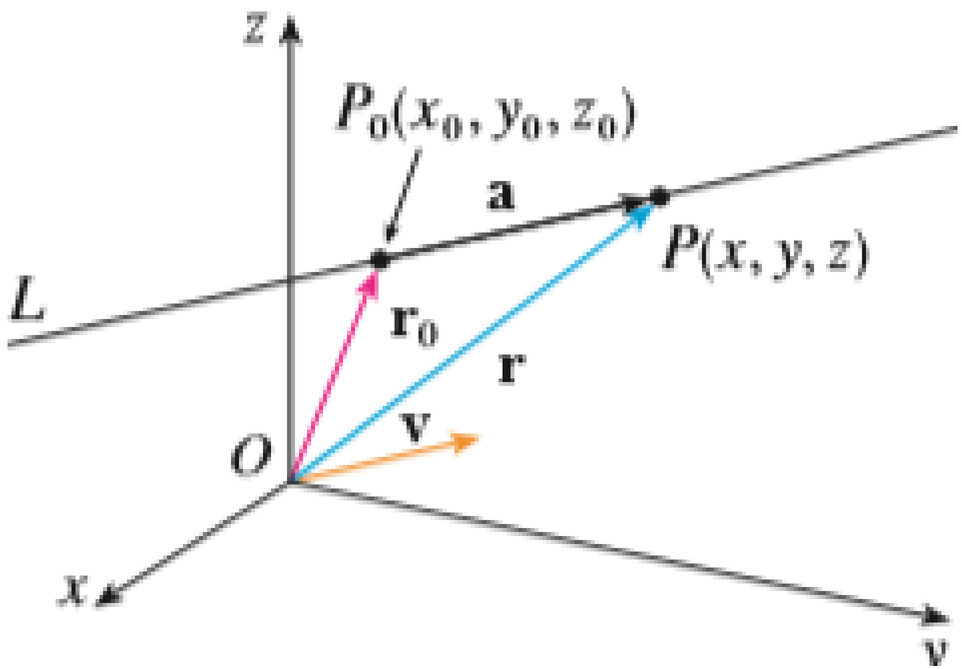
where r is the position vector of a point P(x, y, z) on L and r_0 is the position vector of $P_0(x_0, y_0, z_0)$.



Parametric Equations for a Line

The standard parameterization of the line L through $P_0(x_0, y_0, z_0)$ parallel to $\mathrm{v} = v_1 i + v_2 j + v_3 k$ is

$$x = x_0 + tv_1\ ,\quad \mathrm{y} = \mathrm{y}_0 + tv_2\ ,\quad \mathrm{z} = \mathrm{z}_0 + tv_3 \qquad -\infty < \mathrm{t} < \infty$$



symmetric equations for a Line

Another way of describing a line is to eliminate the parameter from . If none of v_1, v_2 or v_3 is 0, then

$$\frac{x-x_0}{v_1}=\frac{y-y_0}{v_2}=\frac{z-z_0}{v_3}$$

Is said to be symmetric equation for a line.

- The numbers v_1, v_2, v_3 are called the direction numbers, or direction cosines of the straight line.

- If v_1, v_2 or v_3 is zero, we may still write the symmetric equation of the line. For example, if $v_1 = 0$,we shall the symmetric equations as

$$x = x_0,\ \frac{y-y_0}{v_2}=\frac{z-z_0}{v_3}$$

which is a line lying on the plane x0.

Examples:

1- Find parametric equations for the line through (-2,0,4) parallel to v=2i+4j-2k.

2- Find parametric equations for the line through p(-3,2,-3) and Q(1,-1,4).

3- (a) Find a vector equation and parametric equations for the line that passes through the point (5,1,3) and is parallel to the vector v=i+4j-2k.

(b) Find two other points on the line.

4- Show that the lines $L1: x = 1 + t,\ y = -2 + 3t,\ z = 4 - t,$

$$L2: x = 2s,\ y = 3 + s,\ z = -3 + 4s,$$

are skew, i.e. they do not intersect. Hence they do not lie in the same plane.

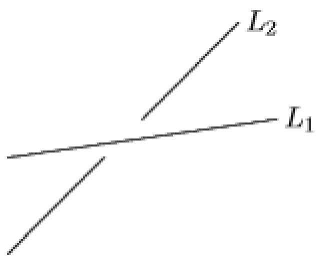
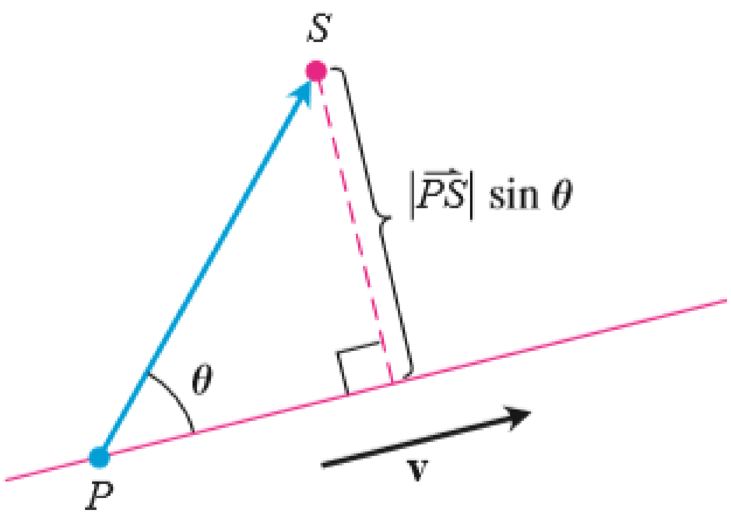


Figure 17 Skew Lines

The Distance from a Point to a Line in Space

Distance from a point S to a line through P parallel to v

$$d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|}$$



Example:

Find the distance from the point S(1, 1, 5) to the line

L: x = 1 + t, y = 3 - t, z = 2t.

Equation for a Plane

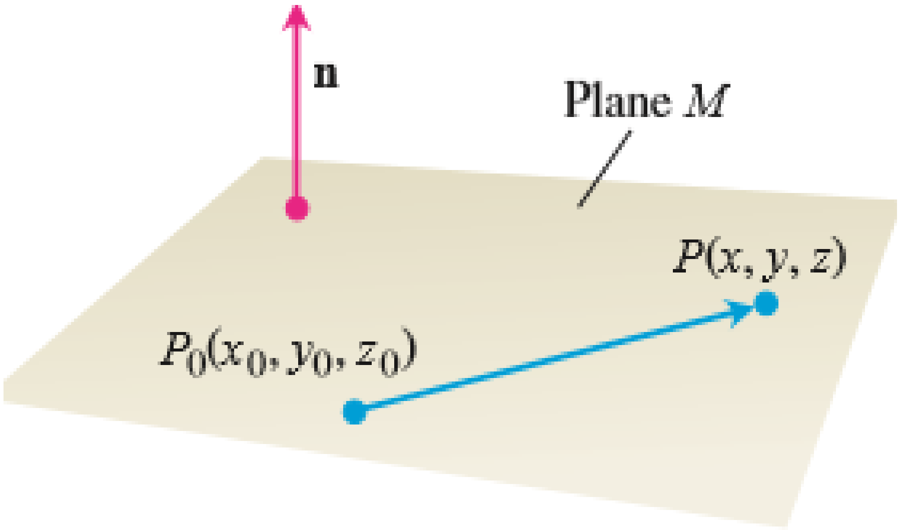
The plane through $P_0(x_0, y_0, z_0)$ normal to n=Ai+Bj+Ck has

Vector equation: $n.\overrightarrow{P_0P} = 0$

Component equation: $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

Component equation simplified:

$$Ax + By + Cz = D \quad \text{where} \quad D = Ax_0 + By_0 + Cz_0$$



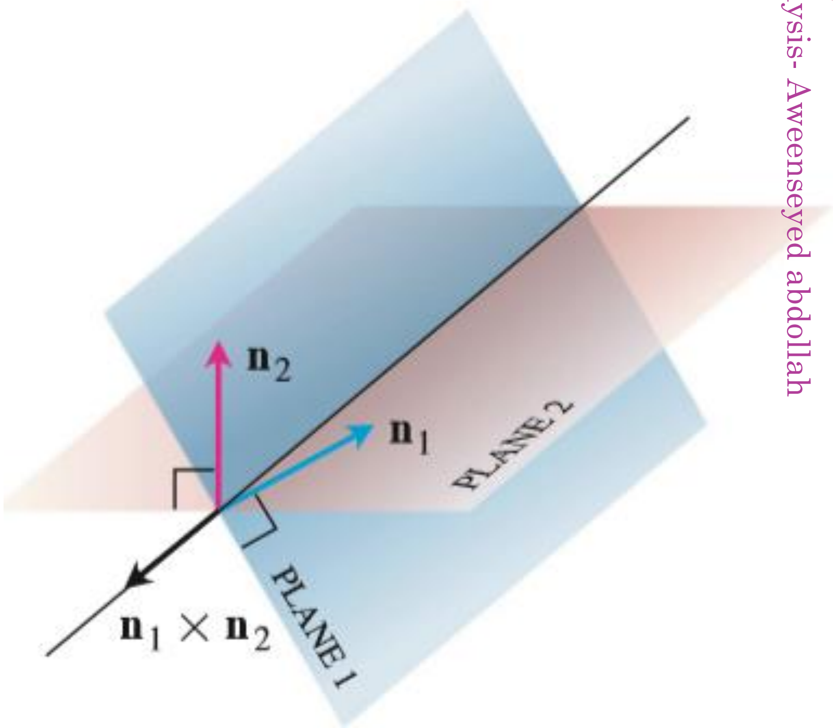
Examples:

1-Find an equation for the plane through $p_0 = (-3,0,7)$ perpendicular to n=5i+2j-k.

2-Find an equation for the plane through A(0, 0, 1), B(2, 0, 0), and C(0, 3, 0).

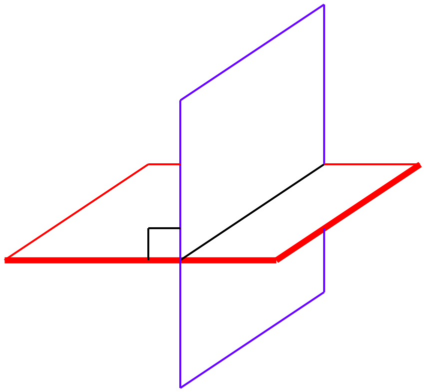
Lines of Intersection

Just as lines are parallel if and only if they have the same direction, two planes are parallel if and only if their normals are parallel, or $n_1 = kn_2$ for some scalar k. Two planes that are not parallel intersect in a line.

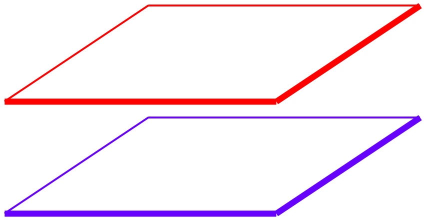


Remark:

Let $n_1 = A_1 i + B_1 j + C_1 k$ and $n_2 = A_2 i + B_2 j + C_2 k$ be two normal vector for two deferent plane then



- Two plane are perpendicular if $n_1 . n_2 = 0$



- Two plane are parallel if $n_1 \times n_2 = 0$
- The angle between two plane is

$$\theta = \cos^{-1}\left(\frac{n_1 . n_2}{|n_1| . |n_2|} \right)$$

Examples:

1-Find a vector parallel to the line of intersection of the planes $2x+y-2z=5$ and $3x-6y-2z=15$.

2- Find parametric equations for the line in which the planes $3x-6y-2z=15$ and $2x+y-2z=5$ intersect.

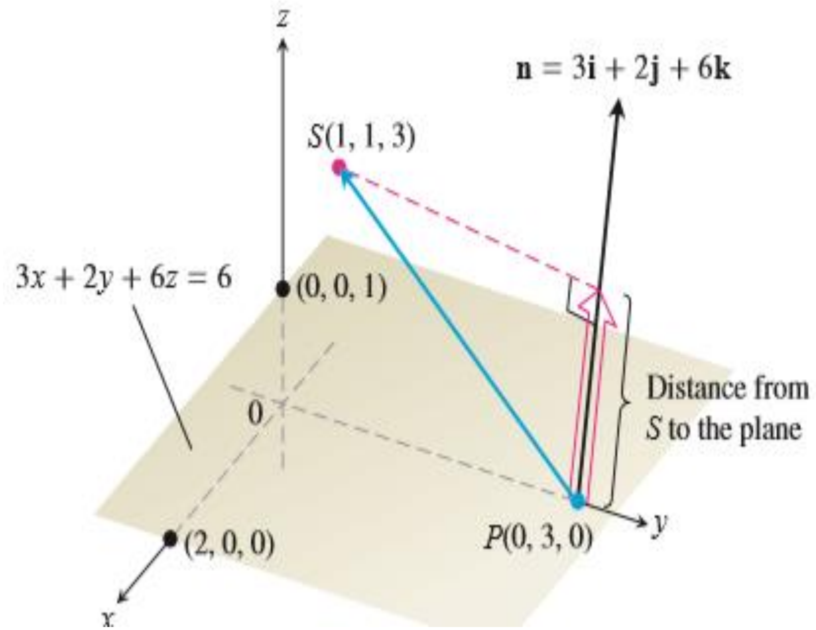
3-Find the point where the line $x=(8/3)+2t, \quad y=-2t, \quad z=1+t$ intersects the plane $3x+2y+6z=6$.

Proposition (The Distance from a Point to a Plane)

If P is a point on a plane with normal **n**, then the distance from any point S to the plane is the length of the vector projection of $\overrightarrow{PS}$ onto **n**. That is, the distance from S to the plane is

$$d = \left| \overrightarrow{PS} . \frac{n}{|n|} \right|$$

where n=Ai+Bj+Ck is normal to the plane.



Example:

Find the distance from S(1, 1, 3) to the plane 3x+2y+6z=6.

Vector Analysis- Aweenseyed abdollah

Homework :

1- a) Find the angle $\theta,(0 \leq \theta \leq 90^\circ)$ between the planes $x + y + z = 1$ and $x-2y + 3z = 1$.

b) Find the symmetric equations for the line of intersection of the planes in (a).

2- Find the distance between the parallel planes $10x+2y-2z = 5$ and $5x+y-z = 1$.

3- Find the distance between the skew lines:

$$L1 : x = 1 + t,\ y = -2 + 3t, z = 4-t$$

$$L2 : x = 2s, y = 3 + s,\ z = -3 + 4s$$

4- Find the equation of the straight line passing through the point $P0(1,5,-1)$ and perpendicular to the lines

$$L1 : x = 5 + t,\ y = -1-t,\ z = 2t \text{ and}$$

$$L2 : x = 11t,\ y = 7t,\ z = -2t.$$