Question Bank

Linear Algebra

1. Define a column vector **U** in $R^n$ ?
2. Find the value of x,y that makes (x,3) and (2,x+y) equals?
3. To which vector space $R^n$ does each vector belong? Use (3,-2,5,3i) , (3,2), ($\pi$, 2,-5 $\pi$)?
4. Prove that **K**.**0**=**0**?
5. Define an **mxn** linear system and then solve the following system of linear equations:

$$x_1 + 2x_2 + x_3 = 3$$
$$3x_1 - x_2 - 3x_3 = -1$$
$$2x_1 + 3x_2 + x_3 = 4$$

   Hint:[Using augmented matrix form] ?

6. Label the following statements as true or false, and correct it if it's false?

   a. The zero vector space has no basis.

   b. Every vector space has a finite basis.

   c. A vector space can't have more than one basis.

   d. The dimension of $M_{mxn}(F)$ is m+n .

   e. If S generates the vector space V , then every vector in V can be written as a linear combination of vectors in S in only one way.

7. Define the concept of dimension of a vector space, and then show that ?
8. Let $v_1 = (1,1,1,1), v_2 = (0,1,1,1), v_3 = (0,0,1,1)$ *and* $v_4 = (0,0,0,1)$ form a basis for $R^4$ .Find the unique representation of an arbitrary vector (a,b,c,d) in $R^4$ as a linear combination of $v_1, v_2, v_3$ *and* $v_4$ . ?
9. Find dimension of a subspace M+N in $R^3$ if M={(x, y, z): x-2y+z=0},

   N= {(x, y, z): 2x+5y=0}.

Show that Both L and T be two linear transformations on $R^3$ **where each one** defined by

L(x, y, z)=(0, y, z), T(x, y, z)=(x, 0, 0) ?

10. Let N, M be two subspaces which N= {a+ bx+ $cx^2$ ;a=b=c}, M= {a+bx+$cx^2$ ;b=0},then find M$\oplus$N?

11. **Let** U= {(x, y, z) ,x-y=z}, is U a subspace or not of $R^3$ ?

12. Find a linear transformation , L: $R^3 \rightarrow R^2$ such that S={(1,-1,0),(2,0,1)} is a basis for it's kernel.

13. Let **V** be the vector space of all functions from the real field **R** into **R** .Show that **W** is a subspace of **V**, where **W**= {f: f (7) =f (1)}?

14. **Is** the real field **R** a vector space? **a-**over **Q** **b-** over Z **c-**over **C** ?

15. Let **U** and **W** be the subspaces of $\mathbf{R^3}$ defined by **U**={(a,b,c):a=b=c} and **W**={(0,b,c):b,c $\in$ **R**}. Show that $\mathbf{R^3 = U \oplus W}$**?**