Salahaddin University-Erbil

College of Engineering

Electrical Engineering Department

Network Analysis II

Chapter One

Magnetically Coupled Circuits

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Chapter One

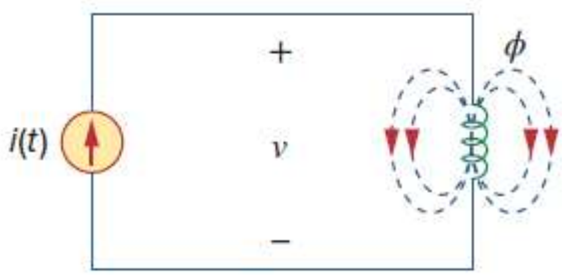
Magnetically Coupled Circuits

1.1 Introduction

- Magnetically coupled circuit means that two loops with or without contacts between them, affect each other through the magnetic field generated by one of them.
- The transformer is an electrical device designed on the basis of the concept of magnetic coupling.
- It uses magnetically coupled coils to transfer energy from one circuit to another.
- Transformers are key circuit elements. They are used in power systems for stepping up or stepping down AC voltages or currents. They are used in electronic circuits such as radio and television receivers for such purposes as impedance matching, isolating one part of a circuit from another.

1.2 Self Inductance

When current i flows through the coil, a magnetic flux ϕ is produced around it. According to **Faraday's law**, the voltage v induced in the coil is proportional to the number of turns N and the time rate of change of the magnetic flux ϕ, that is,



Magnetic flux produced by a single coil with *N* turns.

$$v = N\frac{d\phi}{dt}$$

But the flux ϕ is produced by current i so that any change in ϕ is caused by a change in the current.

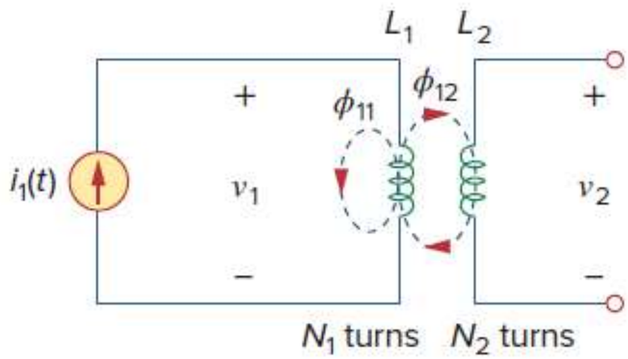
$$v = N\frac{d\phi}{di}\frac{di}{dt}$$

$$v = L\frac{di}{dt}$$

1.3 Mutual Inductance

- Mutual Inductance is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (**H**).
- When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter. This phenomenon is known as ***mutual inductance***.

Two coils with self-inductances L_1 and L_2 that are in close proximity with each other as shown in figure below. The magnetic flux ϕ_1 emanating from coil 1 has two components: One component ϕ_{11} links only coil **1**, and another component ϕ_{12} links both coils.



Mutual inductance M_{21} of coil 2 with respect to coil 1

$$\phi_1 = \phi_{11} + \phi_{12}$$

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

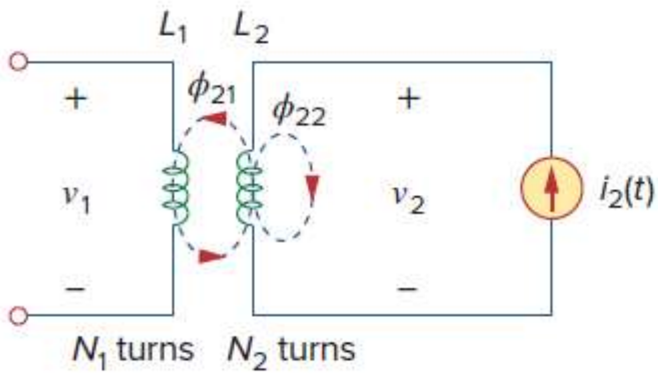
$$v_1 = N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

$$M_{21} = N_2 \frac{d\phi_{12}}{di_1}$$

$$\boxed{v_2 = M_{21} \frac{di_1}{dt}}$$

In figure below, the magnetic flux ϕ_2 emanating from coil **2** comprises flux ϕ_{22} that links only coil **2** and flux ϕ_{21} that links both coils.



Mutual inductance M_{12} of coil 1 with respect to coil 2

$$\phi_2 = \phi_{21} + \phi_{22}$$

$$v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi_2}{di_2}\frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

$$v_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2}\frac{di_2}{dt} = M_{12}\frac{di_2}{dt}$$

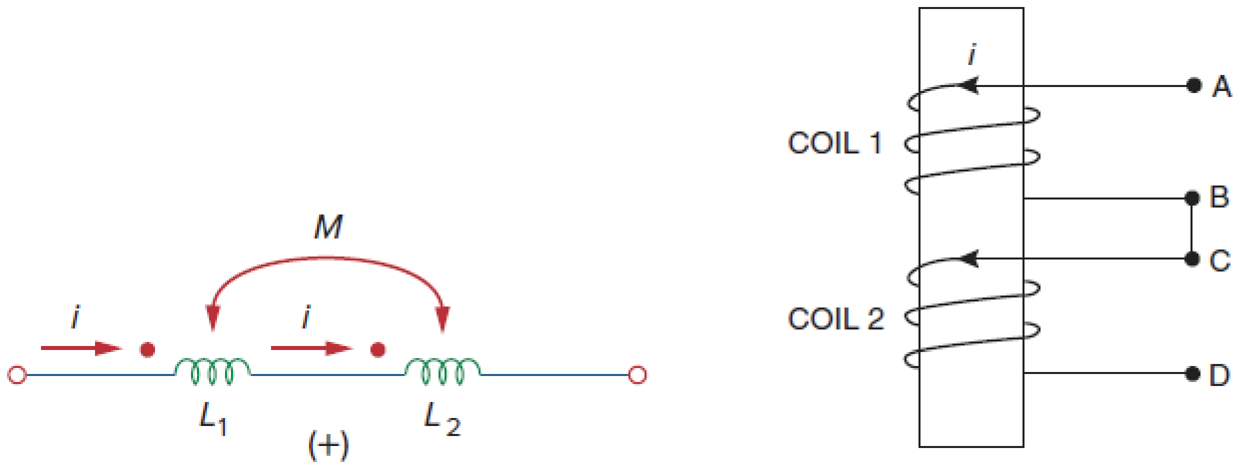
$$M_{12} = N_1 \frac{d\phi_{21}}{di_2}$$

$$\boxed{v_1 = M_{12}\frac{di_2}{dt}}$$

$$M_{12} = M_{21} = M$$

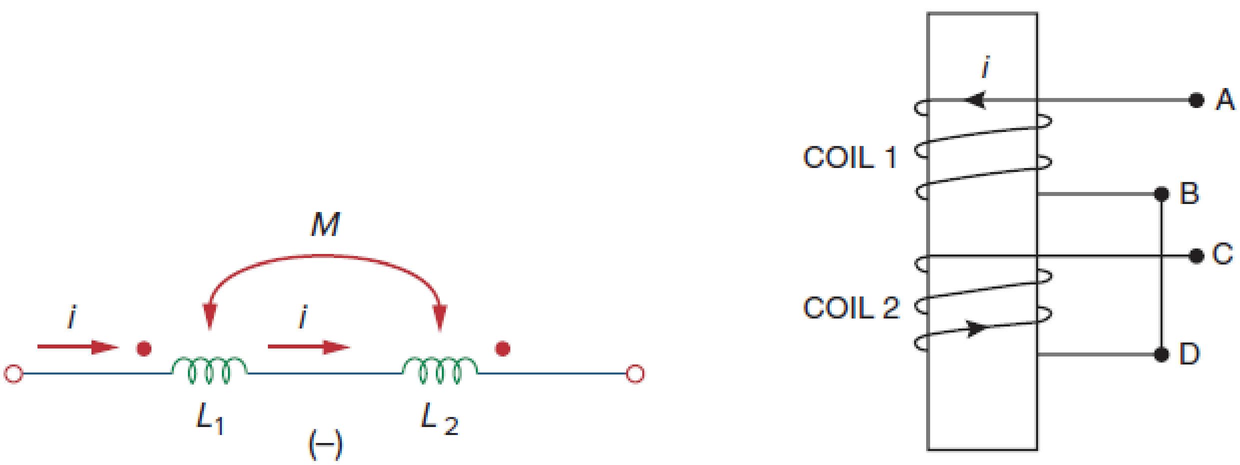
1.4 Types of Connected Coils

The figure below shows the **dot convention** for coupled coils in series and the total inductance is



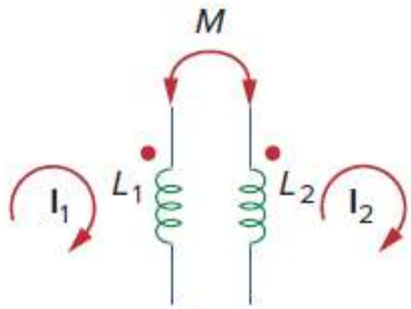
Dot convention for coils in series: series aiding connection

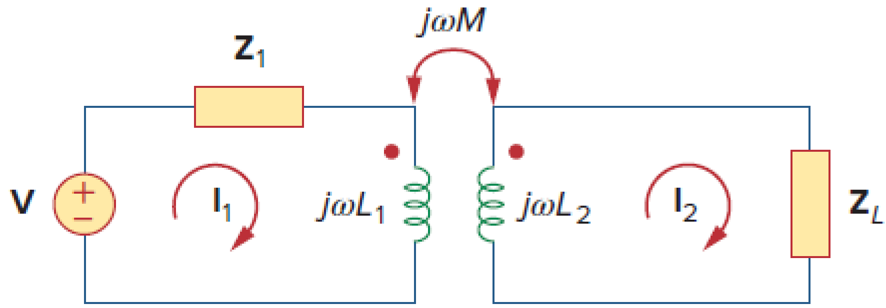
$$L = L_1 + L_2 + 2M \qquad \text{(Series-aiding connection)}$$

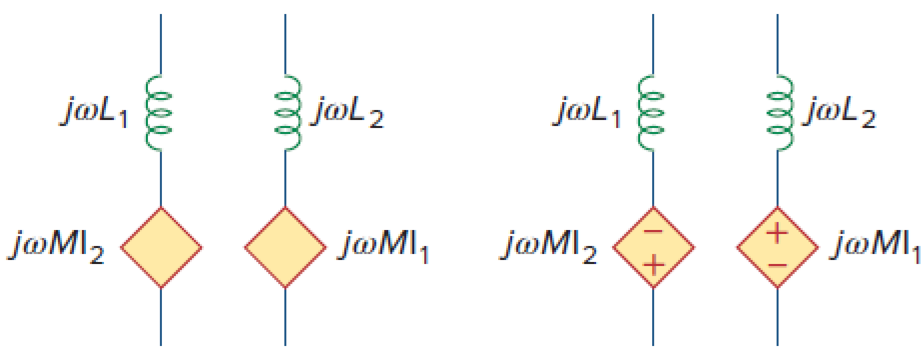


Dot convention for coils in series: series opposing connection

$$L = L_1 + L_2 - 2M \qquad \text{(Series-opposing connection)}$$







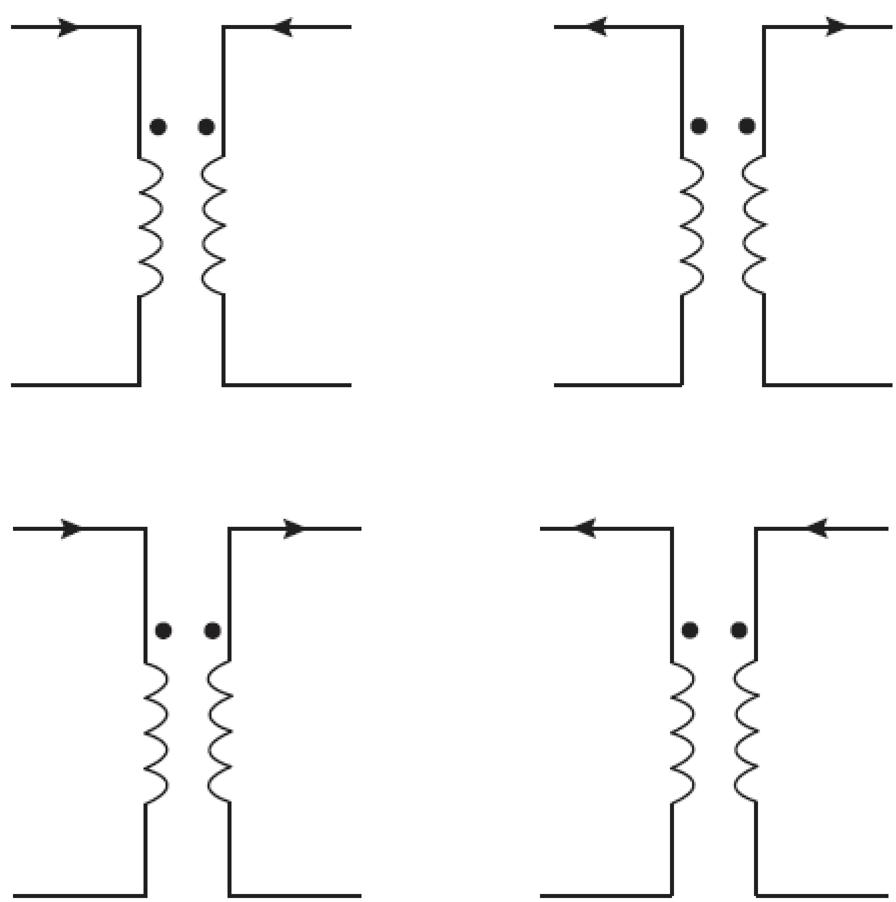
$$\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$$

$$0 = -j\omega M\mathbf{I}_1 + (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2$$

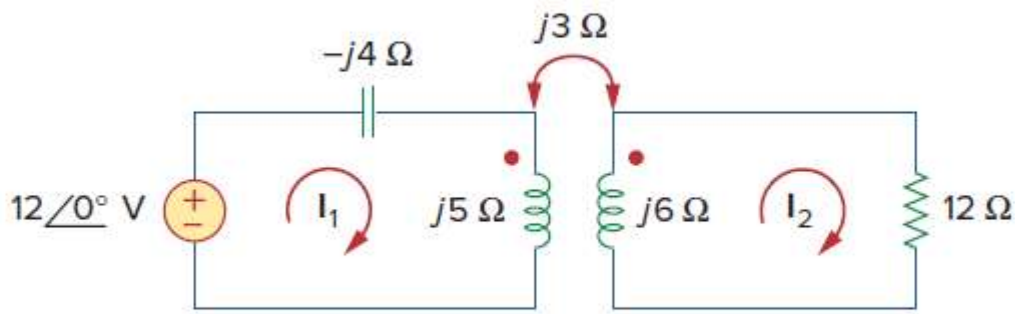
The **dot rule** determines the sign of the voltage of mutual inductance in the Kirchhoff's law equations, and states:

- *When **both** currents **enter**, or **both** currents **leave**, a pair of coupled coils at the dotted terminals, the signs of the '**M' terms** will be **the same as the signs** of the **'L' terms**.*

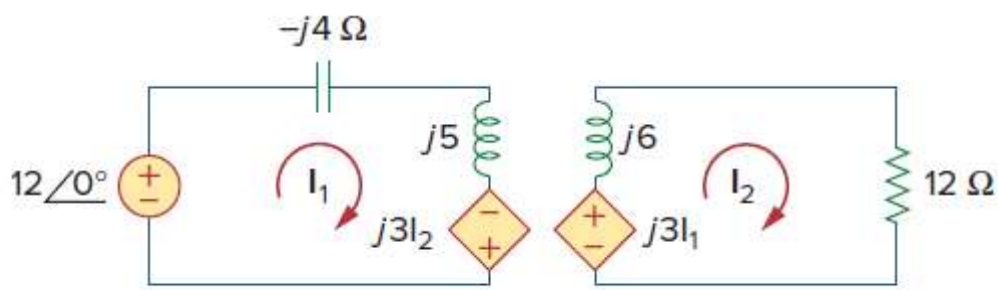
- *When **one** current **enters** at a dotted terminal **and** one **leaves** by a dotted terminal, the signs of the **'M' terms** are **opposite to the signs** of the **'L' terms**.*



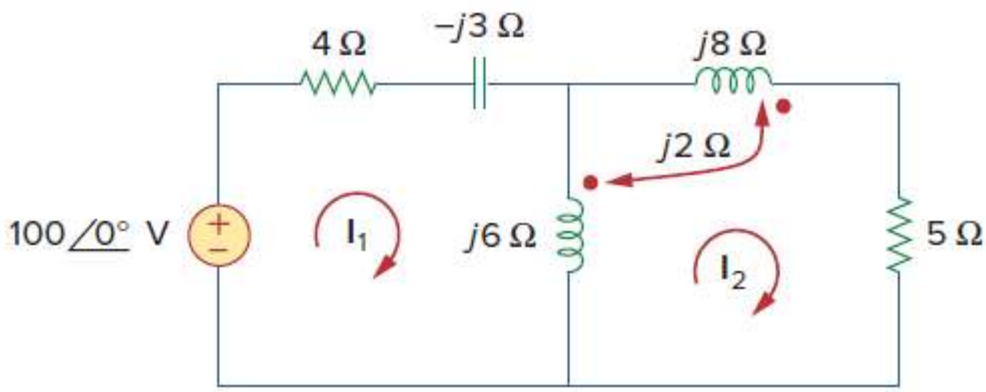
Example 1.1: Calculate the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit shown in figure below.



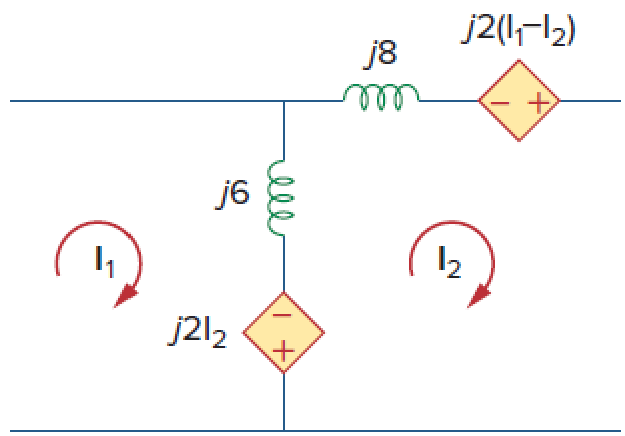
Solution:



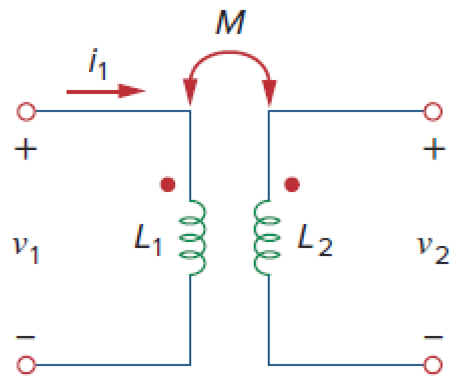
Example 1.2: Calculate the mesh currents in the circuit shown in figure below.



Solution:



1.5 Energy in a Coupled Circuit



Energy stored in magnetically coupled coils as described in the following steps.

$$w = \frac{1}{2} L i^2$$

$$p_1(t) = v_1 i_1 = i_1 L_1 \frac{di_1}{dt}$$

$$w_1 = \int p_1 dt = L_1 \int_0^{I_1} i_1 \, di_1 = \frac{1}{2} L_1 I_1^2$$

$$p_2(t) = i_1 M_{12} \frac{di_2}{dt} + i_2 v_2 = I_1 M_{12} \frac{di_2}{dt} + i_2 L_2 \frac{di_2}{dt}$$

$$w_2 = \int p_2 dt = M_{12} I_1 \int_0^{I_2} di_2 + L_2 \int_0^{I_2} i_2 di_2$$

$$= M_{12} I_1 I_2 + \frac{1}{2} L_2 I_2^2$$

$$w = w_1 + w_2 = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2$$

$$w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{21} I_1 I_2$$

$$M_{12} = M_{21} = M$$

$$w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2$$

$$w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 - M I_1 I_2$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

The energy stored in the circuit cannot be negative because the circuit is passive. This means that the below quantity must be greater than or equal to zero:

$$\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2 \geq 0$$

To complete the square, we both add and subtract the term $i_1 i_2 (\sqrt{L_1 L_2})$ on the right-hand side of the above equation and obtain

$$\frac{1}{2}(i_1\sqrt{L_1} - i_2\sqrt{L_2})^2 + i_1 i_2(\sqrt{L_1 L_2} - M) \geq 0$$

The squared term is never negative, at its least it is zero. Therefore, the second term on the right-hand side of the above equation must be greater than zero, that is

$$\sqrt{L_1 L_2} - M \geq 0$$

$$M \leq \sqrt{L_1 L_2}$$

Thus, the mutual inductance cannot be greater than the geometric mean of the self-inductances of the coils. The extent to which the mutual inductance ***M*** approaches the upper limit is specified by the ***coefficient of coupling k***, given by

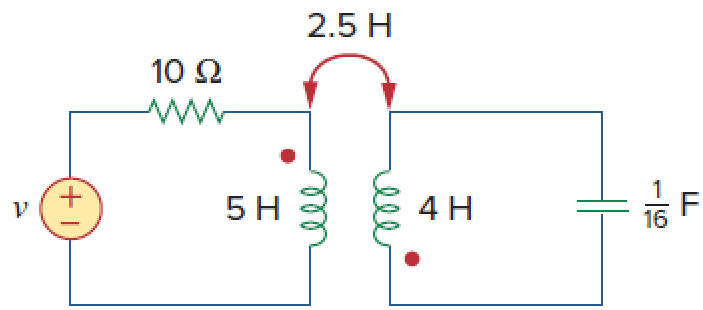
$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$M = k\sqrt{L_1 L_2}$$

The ***coupling coefficient k*** is a measure of the magnetic coupling between two coils

$$\mathbf{0 \leq k \leq 1}$$

Example 1.3: Consider the circuit shown in figure below. Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time $\boldsymbol{t = 1}$**s** if $\boldsymbol{v = 60 \cos(4t + 30°)}$ **V**.



Solution:

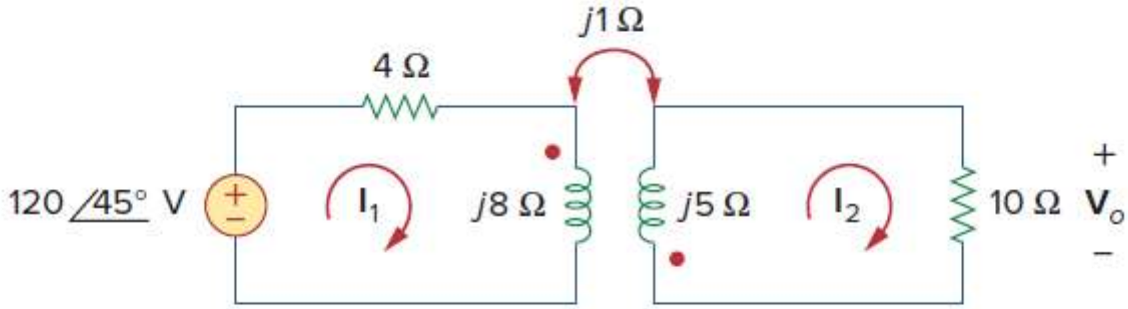
Example 1.4: Two coils are connected in series and their effective inductance is found to be **15 mH**. When the connection to one coil is reversed, the effective inductance is found to be **10 mH**. If the coefficient of coupling is **0.7**, determine:

(a) The self inductance of each coil

(b) The mutual inductance.

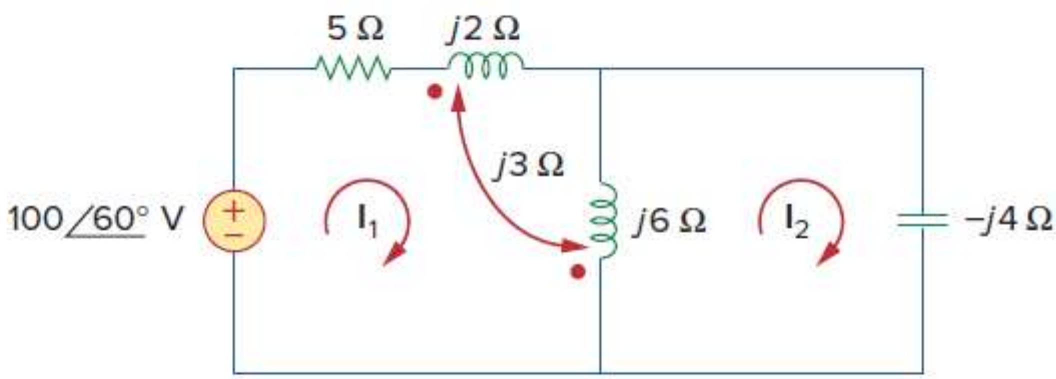
Homework-1:

A) Determine the voltage $\mathbf{V_O}$ in the circuit shown in figure below.



Answer: $12\angle -45^\circ$ V.

B) Determine the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit shown in figure below.



Answer: $I_1 = 17.889\angle 86.57^\circ$ A, $I_2 = 26.83\angle 86.57^\circ$ A.

C) Two coils connected in series have self inductance of **40 mH** and **10 mH**, respectively. The total inductance of the circuit is found to be **60 mH**. Determine:

(a) The mutual inductance between the two coils

(b) The coefficient of coupling.

Answer: coefficient of coupling, ***k* = 0.25**

D) Two mutually coupled coils, **X** and **Y**, are connected in series to a **240 V DC**. supply. Coil **X** has a resistance of **5Ω** and an inductance of **1H**. Coil **Y** has a resistance of **10Ω** and an inductance of **5 H**. At a certain instant after the circuit is connected, the current is 8 A and increasing at a rate of **15 A/s**. Determine:

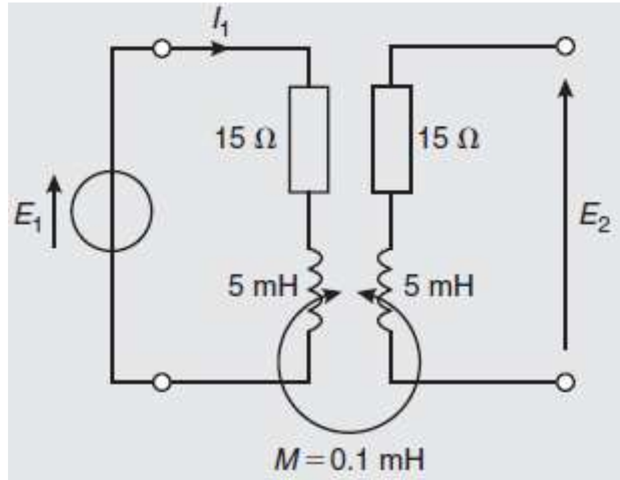
(a) The mutual inductance between the coils

(b) The coefficient of coupling.

Answer: coefficient of coupling, ***k* = 0.447**

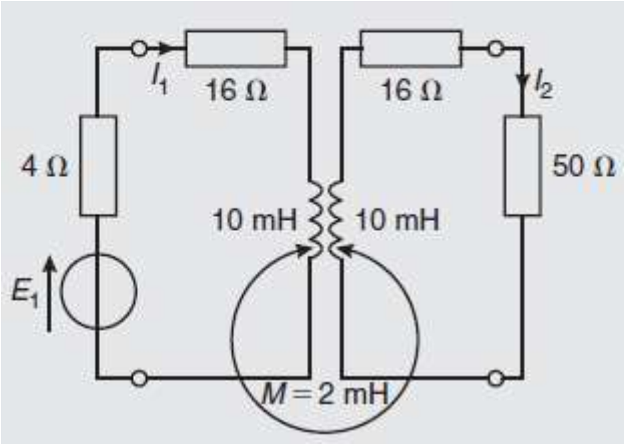
Example 1.5: For the circuit shown in figure below, determine the E_2 which appears across the open-circuited secondary winding, given that E_1 **= 8sin 2500*t* volts**.

Solution:



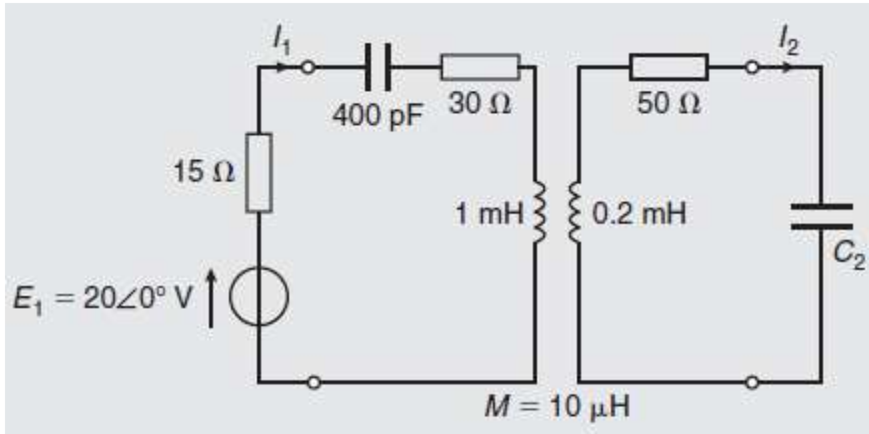
Example 1.6: For the circuit shown in figure below, determine the value of the secondary current I_2 if $E1 = 2\angle 0^\circ$ volts and the frequency is $10^{3}/\pi$ **Hz**.

Solution:



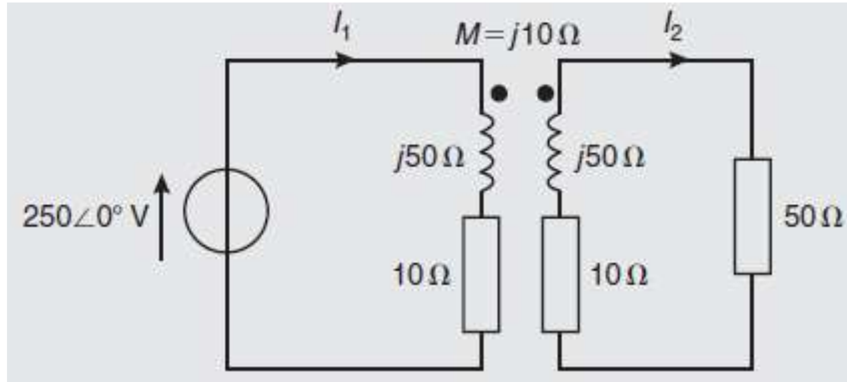
Example 1.7: For the circuit shown in figure below each winding is tuned to resonate at the same frequency. Determine (a) the resonant frequency, (b) the value of capacitor *C*2, (c) the primary current, (d) the voltage across capacitor C2 and (e) the coefficient of coupling.

Solution:



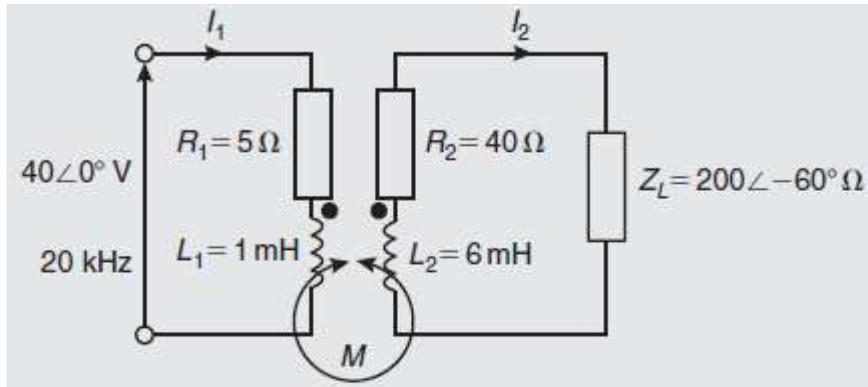
Example 1.8: For the coupled circuit shown in figure below, determine the values of currents $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$.

Solution:



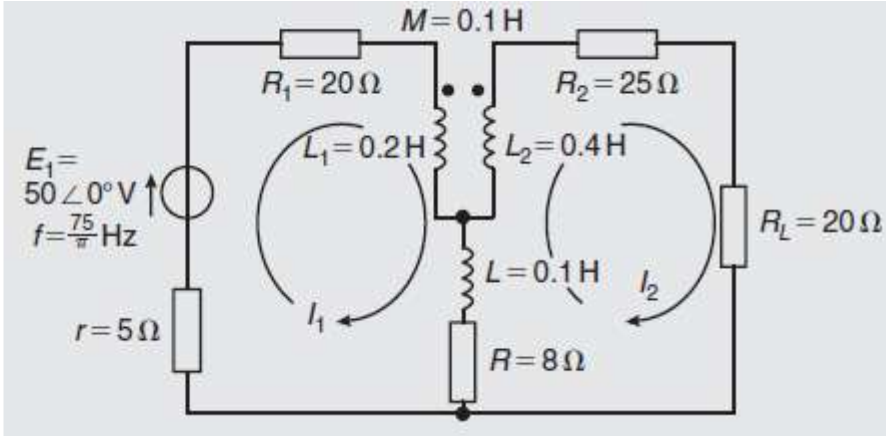
Example 1.9: The circuit diagram of an air-cored transformer winding is shown in figure below. The coefficient of coupling between primary and secondary windings is **0.70**. Determine for the circuit (a) the mutual inductance $\boldsymbol{M}$, (b) the primary current $\boldsymbol{I_1}$ and (c) the secondary current $\boldsymbol{I_2}$.

Solution:



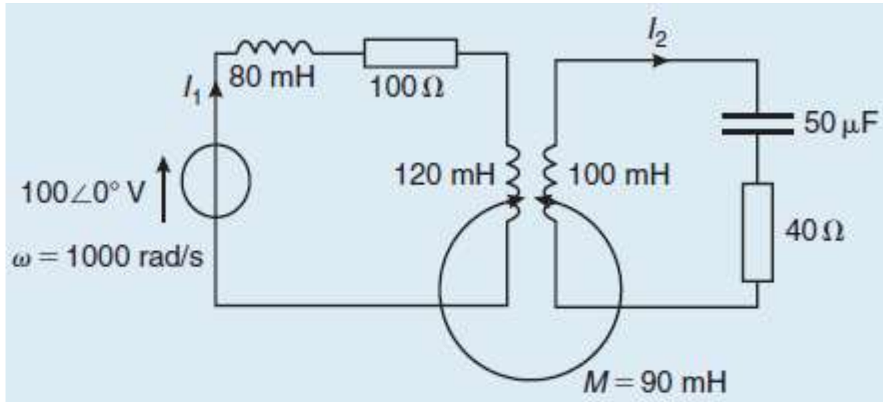
Example 1.10: For the coupled circuit shown in figure below. Determine the source and load currents for (a) the windings as shown (i.e. with the dots adjacent), and (b) with one winding reversed (i.e. with the dots at opposite ends).

Solution:

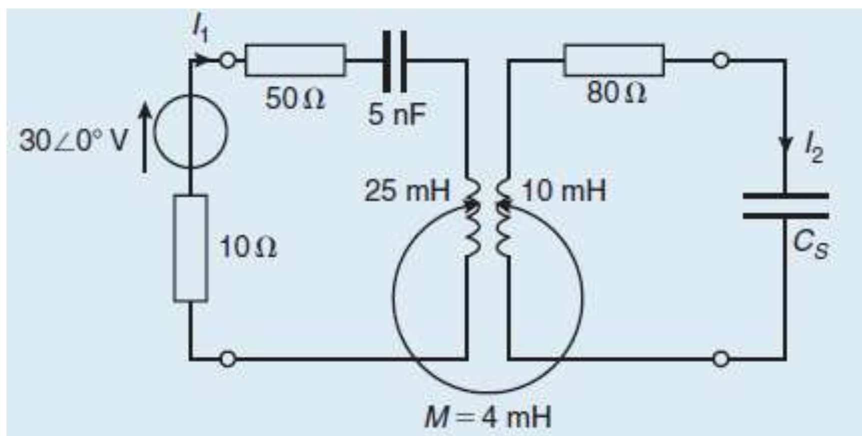


Homework-2:

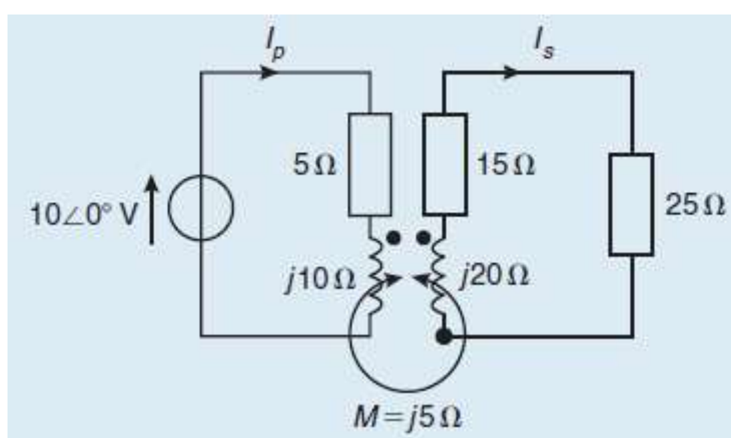
A) For the magnetically coupled circuit shown in figure below, determine (a) the self impedance of the primary circuit, (b) the self impedance of the secondary circuit, (c) the primary current and (d) the secondary current.



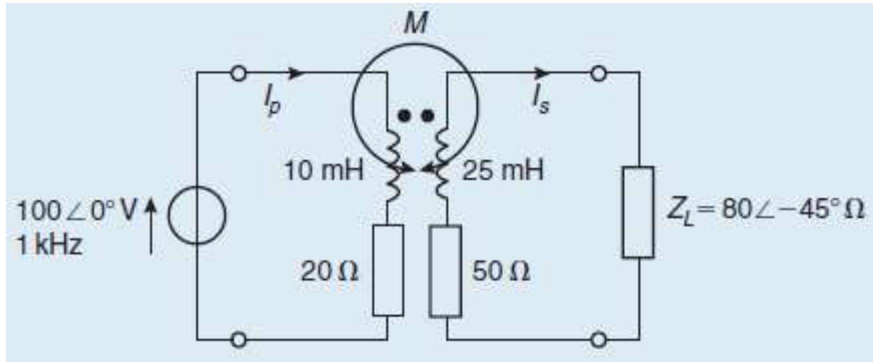
B) In the coupled circuit shown in figure below, each winding is tuned to resonance at the same frequency. Calculate (a) the resonant frequency, (b) the value of *CS*, (c) the primary current, (d) the secondary current, (e) the voltage across capacitor *CS* and (f) the coefficient of coupling.



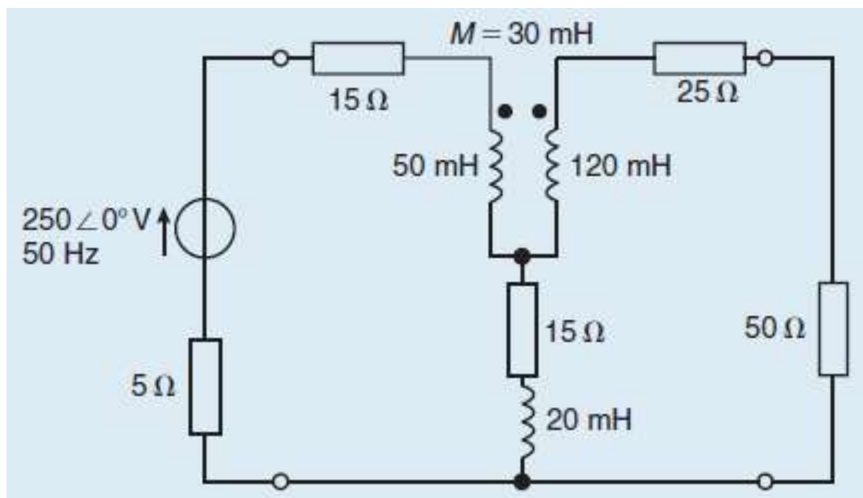
C) Determine the values of currents I_p and I_s in the coupled circuit shown in figure below.



D) The **coefficient of coupling** between the primary and secondary windings for the air-cored transformer shown in figure below is 0.84. Calculate for the circuit (a) the mutual inductance $\boldsymbol{M}$, (b) the primary current $\boldsymbol{I_p}$ and (c) the secondary current $\boldsymbol{I_s}$.



E) For the magnetically coupled circuit shown in figure below. Determine (a) the source current and (b) the load current. (c) If one of the windings is reversed, determine the new value of source and load currents.

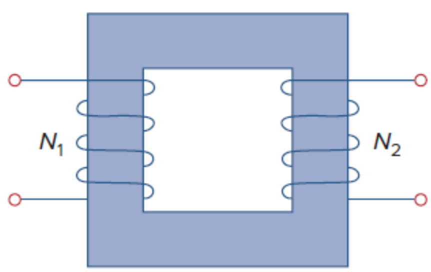


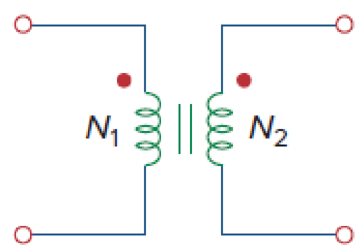
1.6 Applications

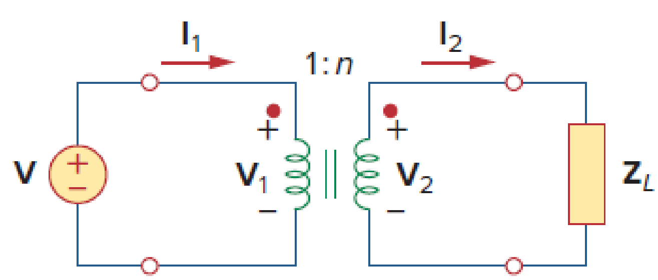
Transformer is an electrical device designed on the basis of the concept of magnetic coupling. They have numerous applications. For example:

* To step up or step down voltage and current, making them useful for power transmission and distribution.
* To isolate one portion of a circuit from another (transfer power without any electrical connection).
* As an impedance-matching device for maximum power transfer.
* In frequency-selective circuits whose operation depends on the response of inductances.

An ideal transformer is one with perfect coupling (***k* =1**). It consists of two (or more) coils with a large number of turns wound on a common core of high permeability. Because of this high permeability of the core, the flux links all the turns of both coils, thereby resulting in a perfect coupling.







A transformer is said to be ideal if it has the following properties:
1. Coils have very large reactances **(L_1, L_2, $M \rightarrow \infty$)**.
2. Coupling coefficient is equal to unity **(k = 1)**.
3. Primary and secondary coils are lossless **($R_1 = 0 = R_2$)**.

$$v_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n$$

For the reason of power conservation, the energy supplied to the primary must equal the energy absorbed by the secondary, since there are no losses in an ideal transformer. This implies that

$$v_1 i_1 = v_2 i_2$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = n$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \frac{\mathbf{V}_2}{n}(n\mathbf{I}_2)^* = \mathbf{V}_2\mathbf{I}_2^* = \mathbf{S}_2$$

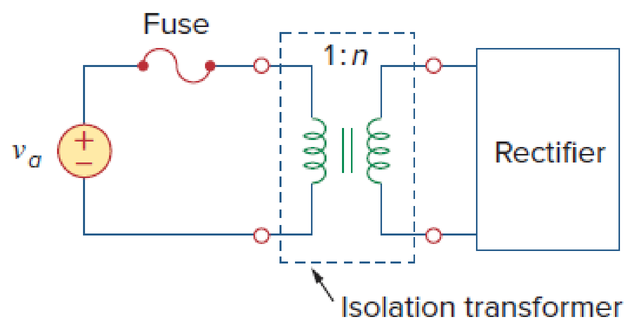
Example 1.11: An ideal transformer is rated at **2400/120 V, 9.6 kVA**, and has **50 turns** on the secondary side. Calculate: (a) the turns ratio, (b) the number of turns on the primary side, and (c) the current ratings for the primary and secondary windings.

Solution:

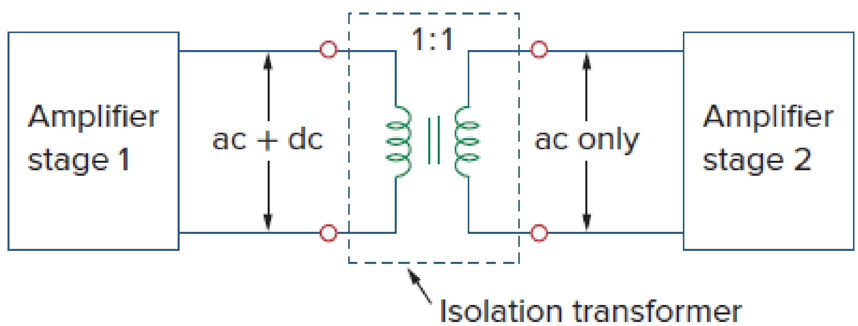
Three important applications are described in the following: transformer as an isolation device, transformer as a matching device, and power distribution system.

A) Transformer as an Isolation Device

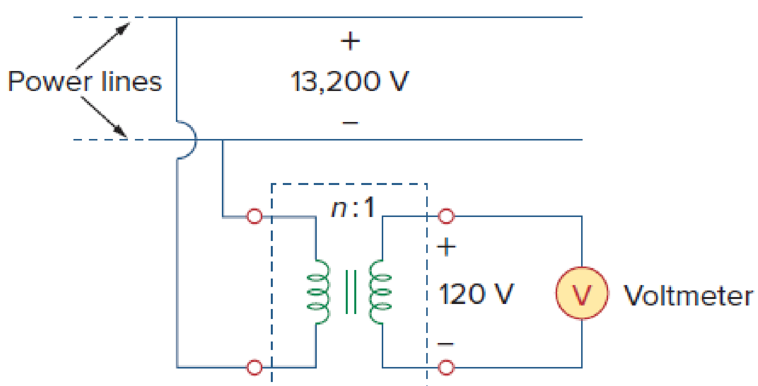
Electrical isolation is said to exist between two devices when there is no physical connection between them. In a transformer, energy is transferred by magnetic coupling, without electrical connection between the primary circuit and secondary circuit.



A transformer used to isolate an ac supply from a rectifier



A transformer providing dc isolation between two amplifier stages

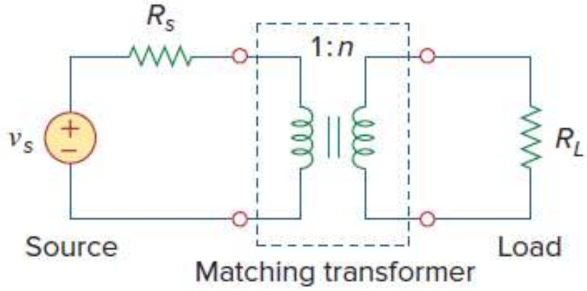


A transformer providing isolation between the power lines and the voltmeter

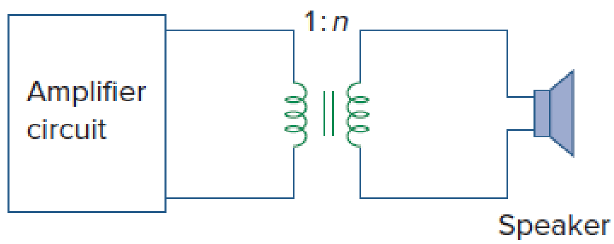
B) Transformer as a Matching Device

For maximum power transfer, the load resistance R_L must be matched with the source resistance R_s.

$$R_s = \frac{R_L}{n^2}$$



For example, to connect a loudspeaker to an audio power amplifier requires a transformer, because the speaker's resistance is only a few ohms while the internal resistance of the amplifier is several thousand ohms.



C) Power Distribution

A power system basically consists of three components: generation, transmission, and distribution.