Dual Problem and Dual Simplex Method

Original Problem: This is the original linear programming problem, also called as primal problem.
Dual Problem: A dual problem is a linear programming problem is another linear programming problem formulated from the parameters of the primal problem.

Dual Problem Formulation:
If the original problem is in the standard form, then the dual problem can be formulated using the following rules:

*The number of constraints in the original problem is equal to the number of dual variables.

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*The original problem profit coefficients appear on the right hand side of the dual problem constraints.

*If the original problem is a maximization problem, then the dual problem is a minimization problem.

*The original problem has less than or equal to (≤) type of constraints while the dual problem has greater than or equal to (≥) type constraints.

*The coefficients of the constraints of the original problem which appear from left to right are placed from top to bottom in the constraints of the dual problem and vice versa.

The original problem as the form:

Maximize

$$Z = C^t x$$

Subject to:

$$Ax \leq B$$

$$x \geq 0$$

We called the following problem duality problem

Minimize

$$Z = B^t y$$

Subject to:

$$A^t y \geq C$$

$$y \geq 0$$

Example: Three machine shops A, B, C produces three types of products X, Y, Z respectively. Each product involves operation of each of the machine shops. The time required for each operation on various products is given as follows:

Products	Machine Shops A	B	C	Profit per unit
X	10	7	2	\$12
Y	2	3	4	\$3
Z	1	2	1	\$1
Available Hours	100	77	80	

The available hours at the machine shops A, B, C are 100, 77, and 80 only. The profit per unit of products X, Y, and Z is \$12, \$3, and \$1 respectively.

Solution:

Example:

Maximize

$$22\,x_1 + 25\,x_2 + 19\,x_3$$

Subject to:

$$18\,x_1 + 26\,x_2 + 22\,x_3 \leq 350$$

$$14\,x_1 + 18\,x_2 + 20\,x_3 \geq 180$$

$$17\,x_1 + 19\,x_2 + 18\,x_3 = 205$$

$$x_1, x_2, x_3 \geq 0$$

Solution:

Dual Simplex Method:

*Rule of finding the vector (X_{Br}) to leave the basic
We always remove the vector Br for which r is obtained by

$$X_{Br} = \min(X_{Br}, X_{Br} < 0)$$

*Rule for finding the entering vector (X_K)
For predetermined value of r, we determine K by using

$$\frac{\Delta_K}{y_{rK}} = \underset{j}{Max}\ (\frac{\Delta_j}{y_{rj}}, y_{rj} < 0)$$

Then the vector S_K will enter the basic

Examples:

1) Minimize $Z=2x_1 + x_2$

Subject to:

$$3x_1 + x_2 \geq 3$$
$$4x_1 + 2x_2 \geq 6$$
$$x_1 + 2x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

2) Minimize

$$2x_1 + 2x_2 + 4x_3$$

Subject to:

$$2x_1 + 3x_2 + 5x_3 \geq 2$$
$$3x_1 + 1x_2 + 7x_3 \leq 3$$
$$1x_1 + 4x_2 + 6x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$