Transportation Problem:

The transportation problem is a special case of the linear programming problem in transportation problem we have a set of sources and a set of destination, suppose that we have M sources and N destination, let a_i (i =1,2,…,m) be the number of supply units at source i, and b_j (j =1,2,…,n) be the number of demand units required at the destination j, c_{ij} be the per unit transportation cost for transporting one unit from source i to destination j.

The objective is to find the number (x_{ij}) of unit to be shipped from source to destination, such as the total transportation cost (or distance or time) is minimum.

And $a_1 + a_2 + ... + a_m = b_1 + b_2 + ... + b_n$, then the problem is called balanced, otherwise is called unbalanced

	Destination $d_1 \quad d_2 \quad . \quad . \quad . \quad d_3$	Supply units
Sources s_1	$c_{11} + c_{12} + ... + c_{1n}$	a_1
s_2	$c_{21} + c_{22} + ... + c_{2n}$	a_2
.	.	.
.	.	.
.	.	.
s_m	$c_{m1} + c_{m2} + ... + c_{mn}$	a_m
Demand units	$b_1 \quad b_2 \quad . \quad . \quad . \quad b_n$	

Table of costs

	Destination $d_1 \quad d_2 \quad . \quad . \quad . \quad d_3$	Supply units
Sources s_1	$x_{11} + x_{12} + ... + x_{1n}$	a_1
s_2	$x_{21} + x_{22} + ... + x_{2n}$	a_2
.	.	.
.	.	.
.	.	.
s_m	$x_{m1} + x_{m2} + ... + x_{mn}$	a_m
Demand units	$b_1 \quad b_2 \quad . \quad . \quad . \quad b_n$	

Table of transporting units from sources I to destination j

Minimize $Z=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad (i = 1,2,\ldots,n),$$

$$\sum_{i=1}^{n} x_{ij} = b_j \qquad (j = 1,2,\ldots,n),$$

$$x_{ij} \ge 0$$

Transportation Algorithm

Step 1: Determine a starting basic feasible solution, using any one of the following three methods

1. North West Corner Method
2. Least Cost Method
3. Vogel Approximation Method

Step 2: Determine the optimal solution using the following method
MODI (Modified Distribution Method) or UV Method.

Methods for finding Basic Feasible Solution of a Transportation Problem:

1)North West Corner Method:

The method starts at the North West (upper left) corner cell of the tableau (variable x_{11}).

Step 1: the first assignment is made in the cell occupying the upper left-hand (north-west) corner of the transportation table.
$x_{11} = \min(a_1, b_1)$, this value of x_{11} is then entered in the cell (1,1) of the transportation table.

Step 2: i) if $b_1 > a_1$, move vertically down wards to the second row and make the second allocation of amount $x_{21} = \min(a_2, b_1 - x_{11})$ in the cell (2,1).
ii) if $b_1 < a_1$, move horizontally right-side to the second column and make the second allocation of amount $x_{12} = \min(a_1 - x_{11}, b_2)$ in the cell (1,2).
iii) if $b_1 = a_1$, there is a tie for the second allocation one can make the second allocation of magnitude

$x_{12} = \min(a_1 - a_1, b_2) = 0$ in the cell (1,2)
Or
$x_{21} = \min(a_2, b_1 - b_1) = 0$ in the cell (2,1)
Step 3: repeat steps 1 and 2.

Example: determining basic feasible solution by using North West Corner Method

	Retail Agency					
Factories	1	2	3	4	5	Capacity
1	1	9	13	36	51	50
2	24	12	16	20	1	100
3	14	33	1	23	26	150
Requirement	100	60	50	50	40	300

Solution:

2) Least Cost Method (Minimum Matrix Method):

Step 1: Determine the smallest cost in the cost matrix of the transportation table.

Step 2: i) If $X_{ij} = a_i$, gross-out the ith row of the transportation table and decrease b_j by a_i.

ii) if $X_{ij} = b_j$, gross-out the jth column of the transportation table and decrease a_i by b_j.

iii) if $X_{ij} = a_i = b_j$, gross-out either ith row or jth column but not both

Step 3: repeat steps 1 and 2.

Example: determining basic feasible solution by using Least Cost Method

	Retail Agency					
Factories	1	2	3	4	5	Capacity
1	1	9	13	36	51	50
2	24	12	16	20	1	100
3	14	33	1	23	26	150
Requirement	100	60	50	50	40	300

Solution:

3) Vogel Approximation Method (VAM):

Step 1: for each row and column of the transportation table identify the two smallest costs and determine the difference (penalty) between them.

Step 2: determine the largest difference of the penalties computed in step 1, and select the minimum cost of that row or column.

If the penalties corresponding to two or more rows or columns are equal, select the top most row and the extreme left column.

Step 3: reduce the largest difference row or column from the transportation table and recomputed the procedure above until all the requirements are satisfied.

Example:

Origin	Destination				a_i
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
b_j	60	40	30	110	240

Note: a_i=capacity (supply)
b_j=requirement (demand)

Solution:

MODI (Modified Distribution Method) or UV Method:

Step 1: we find initial solution by methods (1,2,3), u_i to the ith source (i =1,2,…,m) and v_j to the jth destination (j=1,2,…,n).

Step 2: the number of allocations in initial solution should be m+n-1, if not the solution will be degenerate problem.

Step 3: for the basic variables compute u_i and v_j such that $c_{ij} = u_i + v_j$, by setting u_i or v_j equal to zero.

Step 4: for the non-basic variable compute the values $d_{ij} = c_{ij} - u_i - v_j$, if all the $d_{ij} \geq 0$, then solution will be optimal, otherwise not optimal.

Step 5: add a variable θ to one of the negative cells such that form a closed loop, then compute θ by the following relation

$$\theta = \min\{x_{ij} : which contains -\theta\}$$

Example: Find the optimum Solution of the following Problem using MODI method.

						Supply
	1	9	13	36	51	50
	24	12	16	20	1	100
	14	33	1	23	26	150
Demand	**100**	**70**	**50**	**40**	**40**	**300**

Solution:

Example: Find the optimum Solution of the following Problem using MODI method.

	1	2	3	4	Supply units
1	17	28	48	8	7
2	68	28	38	58	9
3	38	6	68	18	17
Demand units	5	8	7	13	33=33

Solution: in this example we suppose that we start with the initial solution obtained by minimum matrix method

			7(8)	7
2(68)		7(38)		9
3(38)	8(6)		6(18)	17
5	8	7	13	

Degeneracy Transportation Problem:

If the number of all allocations in initial solution are less than m+n-1, then this problem is said to be degenerate transportation problem, then we put a very small quantity Δ in least cost independent cell.

Example: Find the optimum Solution of the following Problem using MODI method.

10	**9**	**5**	**50**
5	**10**	**11**	**60**
13	**5**	**7**	**70**
40	**70**	**70**	

Solution:

Unbalanced Transportation Problem:

In same cases of transportation problem we may have unbalanced problem that is the number of supply units is not equal to the number of demand units ($\sum a_i = \sum b_j$), then we can find basic feasible solution by adding dummy lines or columns that is by adding dummy sources or dummy destinations with zero costs.

Example: Find the optimum Solution of the following Problem using MODI method.

	Destination 1	2	3	4	5	Supply
Sources 1	4	2	3	2	6	8
2	5	4	5	2	1	12
3	6	5	4	7	3	14
Demand	4	4	6	8	8	

Solution:

To find the initial basic solution by using (Vogel Approximation Method (VAM))

	4(2)		4(2)			8
			4(2)	8(1)		12
4(6)		6(4)			4(0)	14
4	4	6	8	8	4	

Example: Find the optimum Solution of the following Problem using MODI method.

	Destination					Supply
	1	2	3	4	5	
Sources 1	5	8	6	6	3	800
2	4	7	7	6	5	500
3	8	4	6	6	4	900
Demand	400	400	500	400	800	

Solution:

To find the initial basic solution by using (Vogel Approximation Method (VAM))

		500(6)		800(3)	800
400(4)			100(6)		500
	400(4)			500(4)	900
			300(0)		300
400	400	500	400	800	