Programme Evaluation and Review Technique (PERT)

Introduction:

Program evaluation and review technique (PERT) is a project management tool Used to schedule. Organize. And coordinate tasks within a project. it is basically a method to analyze the tasks involved in completing a given project Especially the time needed to complete each task. And to identify the minimum needed to complete the total project. PERT planning involves the following steps

1-Idewtify the specific activities and milestones

2-Determine the proper sequence of the activities

3-Construct a network diagram

4-Estimate the time required for each activity

5-Determine the critical path

6-Update the PERT chart as the project progresses

Assumptions for PERT:

Note that in CPM, the assumption is that precise time estimate is available for each activity in a project. However, one finds most of the times that this is not practically possible.

In PERT, we assume that it is not possible to have precise time estimate for each activity and instead, probabilistic estimates of time alone are possible. A multiple time estimate approach is followed here. In probabilistic time estimate, the following 3 types of estimate are possible:

1. Pessimistic time estimate (t_p)
2. Optimistic time estimate (t_o)
3. Most likely time estimate (t_m)

The optimistic estimate of time is based on the assumption that an activity will not involve any difficulty during execution and it can be completed within a short period. On the other hand, a pessimistic estimate is made on the assumption that there would be unexpected problems during the execution of an activity and hence it would consume more time. The most likely time estimate is made in between the optimistic and the pessimistic estimates of time. Thus the three estimates of time have the relationship

$$t_o \le t_m \le t_p .$$

Practically speaking, neither the pessimistic nor the optimistic estimate may hold in reality and it is the most likely time estimate that is expected to prevail in almost all cases. Therefore, it is preferable to give more weight to the most likely time estimate.

We give a weight of 4 to most likely time estimate and a weight of 1 each to the pessimistic and optimistic time estimates. We arrive at a time estimate (t_e) as the weighted average of these estimates as follows:

$$t_e = \frac{t_o + 4\,t_m + t_p}{6}$$

Since we have taken 6 units (1 for t_p, 4 for t_m and 1 for t_o), we divide the sum by 6. With this time estimate, we can determine the project completion time as applicable for CPM.

Since PERT involves the average of three estimates of time for each activity, this method is very practical and the results from PERT will be have a reasonable amount of reliability.

Measure of Certainty

The 3 estimates of time are such that

$$t_o \leq t_m \leq t_p .$$

Therefore the range for the time estimate is $t_p - t_o$.

The time taken by an activity in a project network follows a distribution with a standard deviation of one sixth of the range, approximately.

i.e., The standard deviation = $\sigma = \dfrac{t_p - t_o}{6}$

and the variance = $\sigma^2 = \left(\dfrac{t_p - t_o}{6}\right)^2$

The certainty of the time estimate of an activity can be analysed with the help of the variance. The greater the variance, the more uncertainty in the time estimate of an activity.

Problem:

Two experts A and B examined an activity and arrived at the following time estimates.

Expert	Time Estimate

	t_o	t_m	t_p
A	4	6	8
B	4	7	10

Determine which expert is more certain about his estimates of time:

Solution:

$$\text{Variance } (\sigma^2) \text{ in time estimates} = \left(\frac{t_p - t_o}{6}\right)^2$$

In the case of expert A, the variance = $\left(\frac{8-4}{6}\right)^2 = \frac{4}{9}$

As regards expert B, the variance = $\left(\frac{10-4}{6}\right)^2 = 1$

So, the variance is less in the case of A. Hence, it is concluded that the expert A is more certain about his estimates of time.

Problem 2:

Find out the time required to complete the following project and the critical activities:

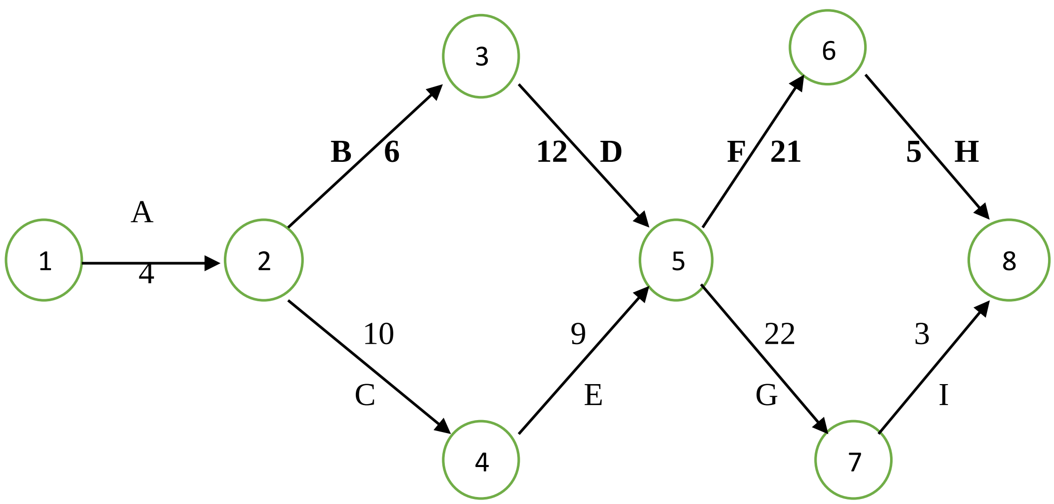
Activity	Predecessor Activity	Optimistic time estimate (t_o days)	Most likely time estimate (t_m days)	Pessimistic time estimate (t_p days)
A	-	2	4	6
B	A	3	6	9
C	A	8	10	12
D	B	9	12	15
E	C	8	9	10
F	D, E	16	21	26
G	D, E	19	22	25
H	F	2	5	8
I	G	1	3	5

Solution:

From the three time estimates t_p, t_m and t_o, calculate t_e for each activity. We obtain the following table:

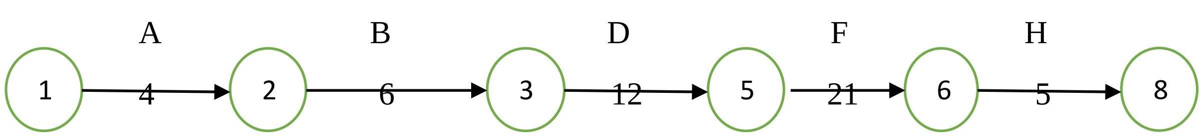
Activity	Optimistic time estimate (t_o)	4 x Most likely time estimate	Pessimistic time estimate (t_p)	t_o+ 4t_m+ t_p	Time estimate $t_e = \frac{t_o + 4\,t_m + t_p}{6}$
A	2	16	6	24	4
B	3	24	9	36	6
C	8	40	12	60	10
D	9	48	15	72	12
E	8	36	10	54	9
F	16	84	26	126	21
G	19	88	25	132	22
H	2	20	8	30	5
I	1	12	5	18	3

Using the single time estimates of the activities, we get the following network diagram for the project.



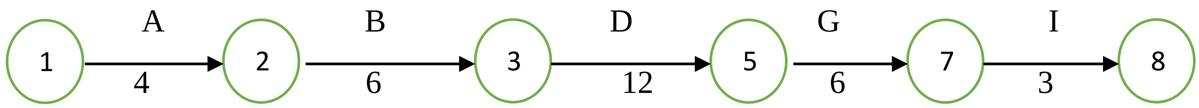
Consider the paths, beginning in the start node and stopping in the end node. There are four such paths for the given project. They are as follows:

Path I



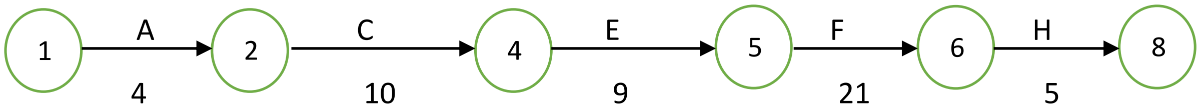
Time for the path: 4+6+12+21+5 = 48 days.

Path II



Time for the path: 4+6+12+ 6+3 = 31 days.

Path III



Time for the path: 4+10+9+ 21+5 = 49 days.

Path IV

Time for the path: 4+10+9+ 6+3 = 32 days.
Compare the times for the four paths.
Maximum of {48, 31, 49, 32} = 49.
We see that Path III has the maximum time.
Therefore, the critical path is Path III. i.e., 1 $\rightarrow$ 2 $\rightarrow$ 4 $\rightarrow$ 5 $\rightarrow$ 6 $\rightarrow$ 8.
The critical activities are A, C, E, F and H.
The non-critical activities are B, D, G and I.
Project time (Also called project length) = 49 days.

Problem 3:

Find out the time, variance and standard deviation of the project with the following time estimates in weeks:

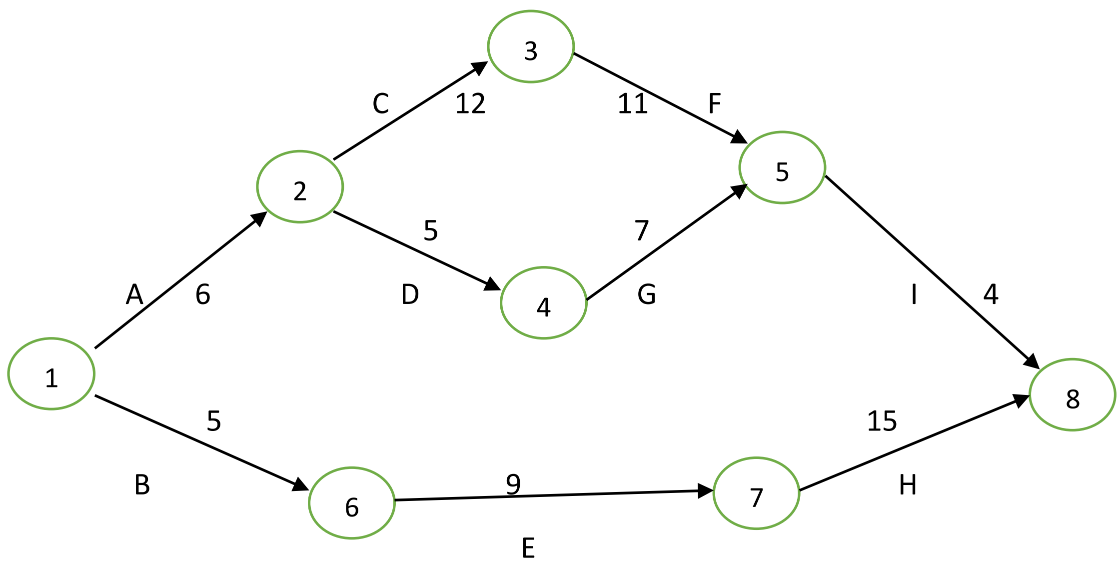
Activity	Optimistic time estimate (t_o)	Most likely time estimate (t_m)	Pessimistic time estimate (t_p)
1-2	3	6	9
1-6	2	5	8
2-3	6	12	18
2-4	4	5	6
3-5	8	11	14
4-5	3	7	11
6-7	3	9	15
5-8	2	4	6
7-8	8	16	18

Solution:

From the three time estimates t_p, t_m and t_o, calculate t_e for each activity. We obtain the following table:

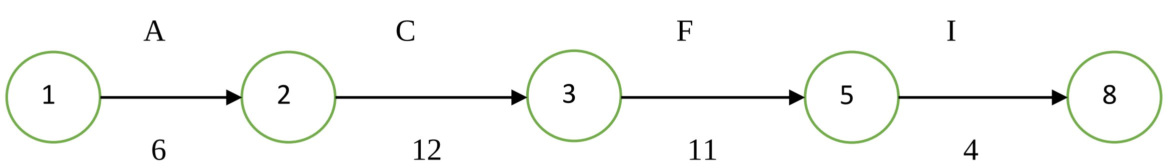
Activity	Optimistic time estimate (t_o)	4 x Most likely time estimate	Pessimistic time estimate (t_p)	t_o+ $4t_m$+ t_p	Time estimate $t_e = \frac{t_o + 4\,t_m + t_p}{6}$
1-2	3	24	9	36	6
1-6	2	20	8	30	5
2-3	6	48	18	72	12
2-4	4	20	6	30	5
3-5	8	44	14	66	11
4-5	3	28	11	42	7
6-7	3	36	15	54	9
5-8	2	16	6	24	4
7-8	8	64	18	90	15

With the single time estimates of the activities, we get the following network diagram for the project.



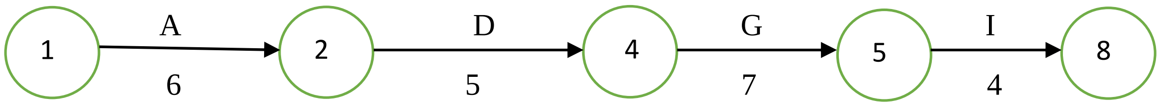
Consider the paths, begging the start node stopping with the end node. There are three such paths for the given project. They are as follows:

Path I



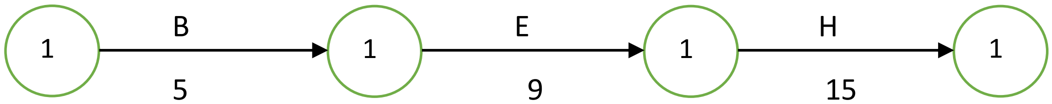
Time for the path: 6+12+11+4 = 33 weeks.

Path II



Time for the path: 6+5+7+ 4= 22 weeks.

Path III



Time for the path: 5+9+15 = 29 weeks.

Compare the times for the three paths.
Maximum of $\{33, 22, 29\} = 33$.
It is noticed that Path I has the maximum time.
Therefore the critical path is Path I. i.e., $1 \to 2 \to 3 \to 5 \to 8$
The critical activities are A, C, F and I.
The non-critical activities are B, D, G and H.
Project time $= 33$ weeks.

Calculation of Standard Deviation and Variance for the Critical Activities:

Critical Activity	Optimistic time estimate (t_o)	Most likely time estimate (t_m)	Pessimistic time estimate (t_p)	Range ($t_p - t_o$)	Standard deviation = $\sigma = \frac{t_p - t_o}{6}$	Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
A: 1→2	3	6	9	6	1	1
C: 2→3	6	12	18	12	2	4
F: 3→5	8	11	14	6	1	1
I: 5→8	2	4	6	4	2/3	4/9

Variance of project time (Also called Variance of project length) =
Sum of the variances for the critical activities = 1+4+1+ 4/9 = 58/9 Weeks.
Standard deviation of project time = $\sqrt{\text{Variance}} = \sqrt{58/9} = 2.54$ weeks.

Problem 4
A project consists of seven activities with the following time estimates. Find the probability that the project will be completed in 30 weeks or less.

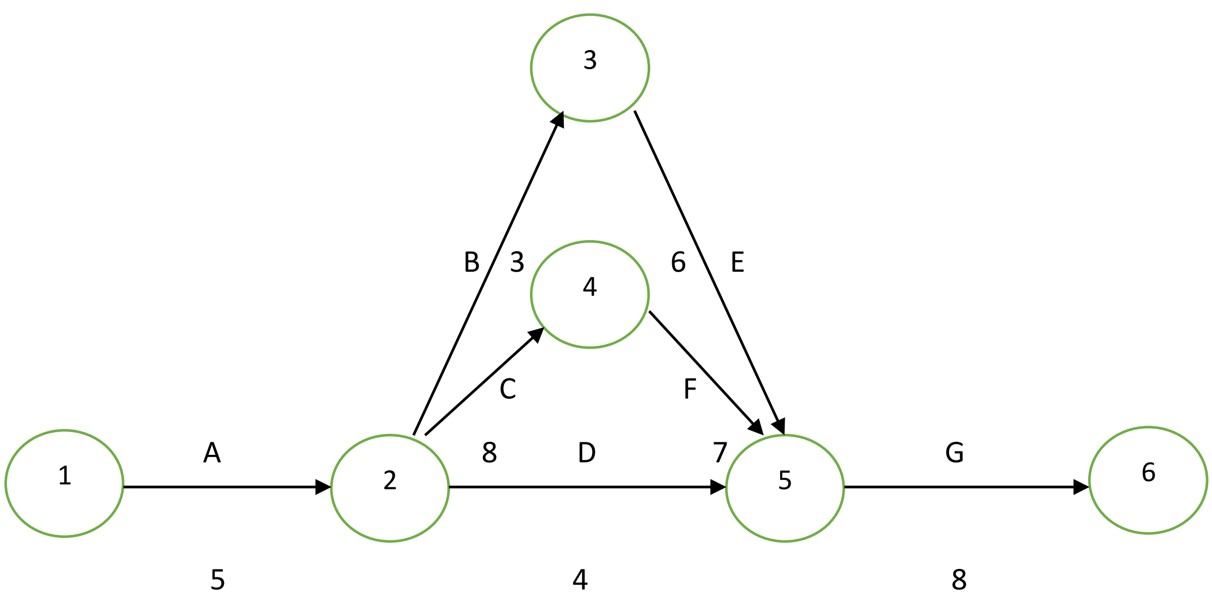
Activity	Predecessor Activity	Optimistic time estimate (t_o days)	Most likely time estimate (t_m days)	Pessimistic time estimate (t_p days)
A	-	2	5	8
B	A	2	3	4
C	A	6	8	10
D	A	2	4	6
E	B	2	6	10
F	C	6	7	8
G	D, E, F	6	8	10

Solution:

From the three time estimates t_p, t_m and t_o, calculate t_e for each activity. The results are furnished in the following table:

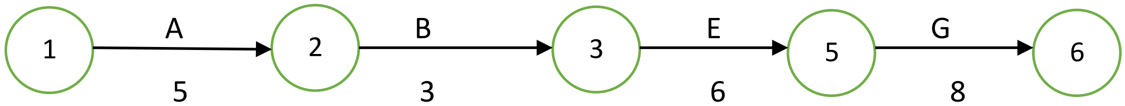
Activity	Optimistic time estimate (t_o)	4 x Most likely time estimate	Pessimistic time estimate (t_p)	$t_o + 4t_m + t_p$	Time estimate $t_e = \frac{t_o + 4\,t_m + t_p}{6}$
A	2	20	8	30	5
B	2	12	4	18	3
C	6	32	10	48	8
D	2	16	6	24	4
E	2	24	10	36	6
F	6	28	8	42	7
G	6	32	10	48	8

With the single time estimates of the activities, the following network diagram is constructed for the project.



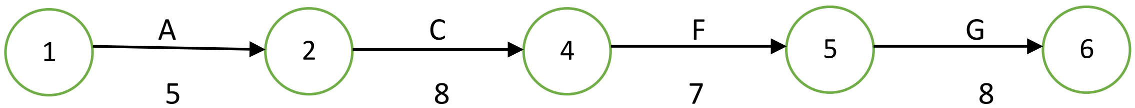
Consider the paths, beginning with the start node and stopping with the end node. There are three such paths for the given project. They are as follows:

Path I



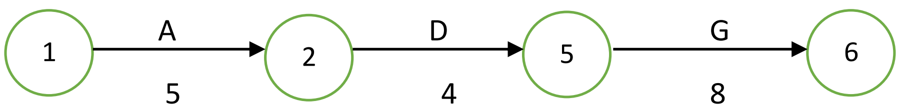
Time for the path: 5+3+6+8 = 22 weeks.

Path II



Time for the path: 5+8+7+ 8 = 28 weeks.

Path III



Time for the path: 5+4+8 = 17 weeks.

Compare the times for the three paths.
Maximum of {22, 28, 17} = 28.
It is noticed that Path II has the maximum time.

Therefore the critical path is Path II. i.e., $1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6$.
The critical activities are A, C, F and G.
The non-critical activities are B, D and E.
Project time = 28 weeks.

Calculation of Standard Deviation and Variance for the Critical Activities:

Critical Activity	Optimistic time estimate (t_o)	Most likely time estimate (t_m)	Pessimistic time estimate (t_p)	Range (t_p -t_o)	Standard deviation = $\sigma = \frac{t_p - t_o}{6}$	Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
A: $1 \rightarrow 2$	2	5	8	6	1	1
C: $2 \rightarrow 4$	6	8	10	4	$\frac{2}{3}$	$\frac{4}{9}$
F: $4 \rightarrow 5$	6	7	8	2	$\frac{1}{3}$	$\frac{1}{9}$
G: $5 \rightarrow 6$	6	8	10	4	$\frac{2}{3}$	$\frac{4}{9}$

Standard deviation of the critical path = $\sqrt{2} = 1.414$

The standard normal variate is given by the formula

$$Z = \frac{\text{Given value of } t - \text{Expected value of } t \text{ in the critical path}}{\text{SD for the critical path}}$$

So we get $Z = \frac{30 - 28}{1.414} = 1.414$

We Know P (Z < Z Network) = 0.5 + Z (1.414) (from normal table Z (1.414) =0.4207)

=0.5 + 0.4207 = 0.9207

Therefore, the required probability is 0.92 i.e., There is 92% chance that the project will be completed before 30 weeks. In other words, the chance that it will be delayed beyond 30 weeks is 8%

Problem 5

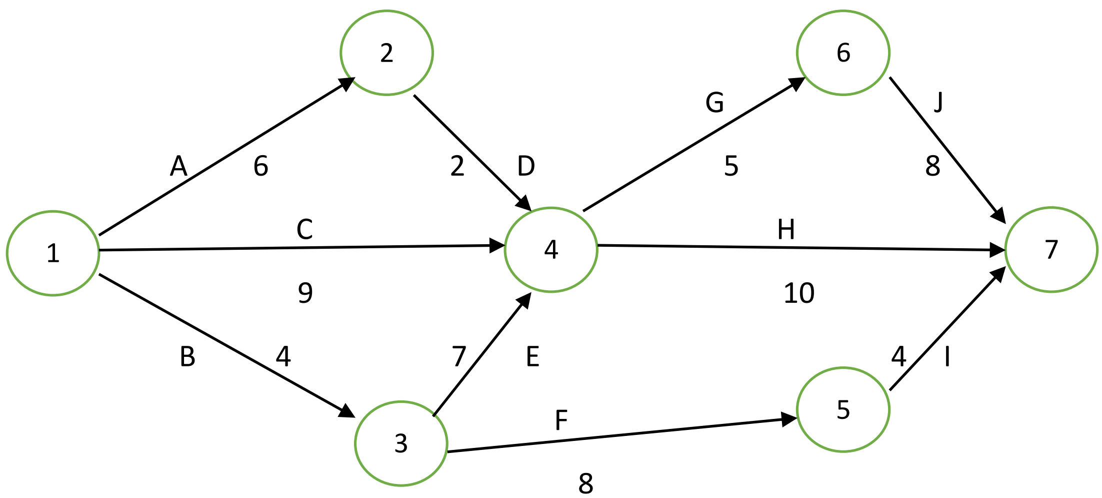
Find the probability that the project is completed in 19 days. If the probability is less than 20%, find the probability of completing it in 24 days

Activity	Activity Name	Optimistic time estimate (t_o)	Most likely time estimate (t_m)	Pessimistic time estimate (t_p)	T_e	Variance
1-2	**A**	**4**	**6**	**8**	**6**	**0.444**
1-3	**B**	**2**	**3**	**10**	**4**	**1.777**
1-4	**C**	**6**	**8**	**16**	**9**	**2.777**
2-4	**D**	**1**	**2**	**3**	**2**	**0.111**
3-4	**E**	**6**	**7**	**8**	**7**	**0.111**
3-5	**F**	**6**	**7**	**14**	**8**	**1.777**
4-6	**G**	**3**	**5**	**7**	**5**	**0.444**
4-7	**H**	**4**	**11**	**12**	**10**	**1.777**
5-7	**I**	**2**	**4**	**6**	**4**	**5.444**
6-7	**J**	**2**	**9**	**10**	**8**	**1.777**

$$T_e = \frac{to + 4\,tm + te}{6}$$

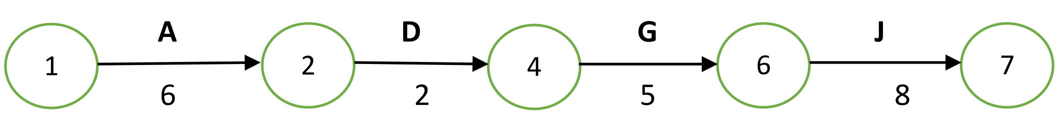
$$\sigma^2 = \left(\frac{te - to}{6}\right)^2$$

With the single time estimates of the activities, we get the following network diagram for the project.



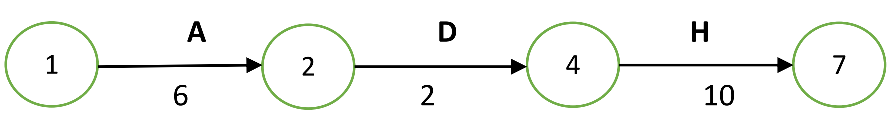
Consider the paths, beginning with the start node and stopping with the end node. There are three such paths for the given project. They are as follows:

Path I



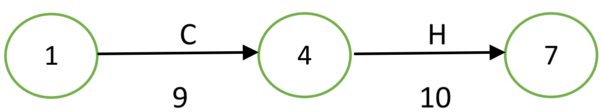
Time for the path: 6+2+5+8=21 days

Path 2



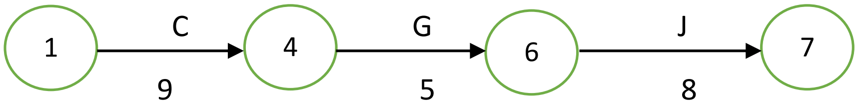
Time for the path: 6+2+10=18 days

Path 3



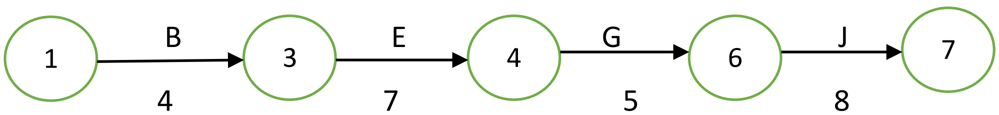
Time for the path: 9+10=19 days

Path 4



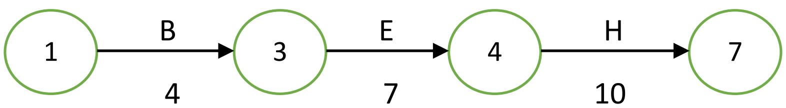
Time for the path: 9+5+8=22 days

Path 5



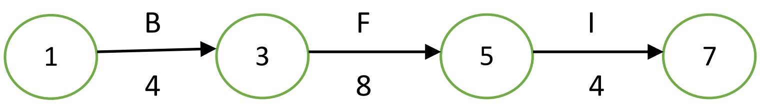
Time for the path: 4+7+5+8=24 days

Path 6



Time for the path: 4+7+10=21 days

Path 7



Time for the path: 4+8+4=16 days

Compare the times for the seven paths.
Maximum of {21,18,19,22,24,21,16} = 24.
It is noticed that Path 5 has the maximum time.

Therefore the critical path is Path 5. i.e., 1 → 3 → 4 → 6 → 7.
The critical activities are B, E, G and J.
Project time = 24 days.

Calculation of Standard Deviation and Variance for the Critical Activities:

Activity	Activity name	T_o	T_m	T_p	T_e	σ^2
1-3	**B**	**2**	**3**	**10**	**4**	**1.777**
3-4	**E**	**6**	**7**	**8**	**7**	**0.111**
4-6	**G**	**3**	**5**	**7**	**5**	**0.444**
6-7	**J**	**2**	**9**	**10**	**8**	**1.777**

Standard deviation of the critical path is = $\sqrt{4.109}$ = 2. 027

$$Z = \frac{Given\ value\ of\ t\ -\ Expected\ value\ of\ t\ \text{in}\ the\ critical\ path}{SD\ for\ the\ critical\ path}$$

$Z = \frac{19-24}{2.027} =$ -2.370

We know ، P (Z < Z network)
= 0.5 + Z (2.370) (From normal table, Z (2.370) =0. 4911)

=0.5 - 0.4911 = 0.0089
Therefore, the required probability is 0.0089, It means that 0.89%of the project completed in 19 days.
Since the probability of completing the project in 19 days is less than 20% as in question, we find the probability of completing it in 24 days

$Z = \frac{26-24}{2.027} = 0$

We know ، P (Z < Z network)
=0.5 +Z (0) (from normal table, Z (0) = 0.0000)
= 0.5 +0.0000= 0.5 Therefore, the required probability is 0.5, It means that 50% of the project completed in 24 days.

NORMAL DISTRIBUTION TABLE

Area Under Standard Normal Distribution

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990