'Geometrical Geodesy'

Lecture-1

Geodesy and types of measuring distance

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Geodesy

Geodesy: is the science of measuring and monitoring the size and the shape of the earth, including its gravity field and determining the location of point on the earth surface.

- The surface of the earth is the shaped by the earth's gravity and most geodetic observations are referred to the earth's gravity field.
- To take measurement over a large distance like several kilometers in each direction, geodesy become more important and necessary, because over large distances curvature of the earth more and error resulted from that will increase significantly.

Why we study geodesy?

- For more accurate coordinate
- For more accurate distance.
- For more accurate angle.

Geodesy may be divided into the following areas:

1. Global geodesy.
2. National geodesic survey.
3. Plane surveying.

Global geodesy: Is responsible for the determination of the figure of the earth and of the external gravity field.

Geodetic surveying: Establishes the fundamentals for the determination of the surface and the gravity field of a country.

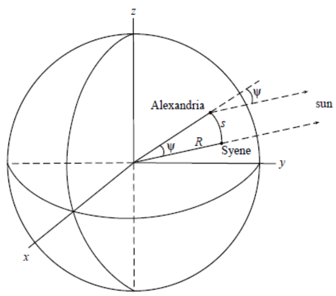
Plane surveying (topographic surveying): The details of the terrain are obtained the horizontal plane generally used as a reference surface.

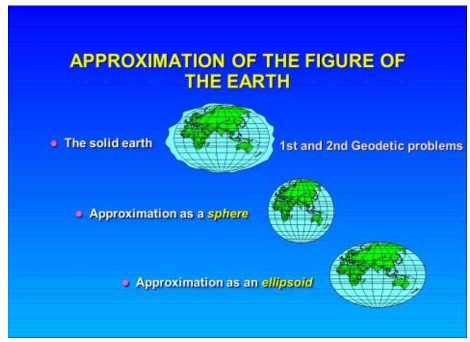
The geodesy can also be classified according to observation as follow:

1. Terrestrial geodesy.
2. Mathematical geodesy.
3. Physical geodesy.
4. Satellite geodesy.
5. Astronomical geodesy.

The figure of the earth:

The shape of the earth is not a sphere, it's a complicated irregular surface, there for its not possible to be represented mathematically, since you have to have a surface which could be described mathematically, and as near as possible to the actual surface of the earth's surface.

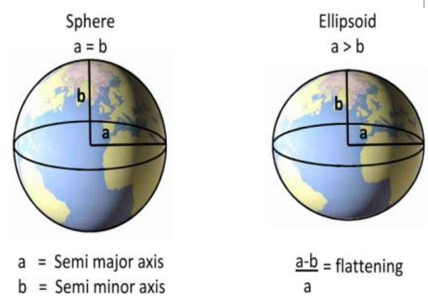




Reference surfaces

- **Solid earth**: approximately spherical, however irregular so, no simple mathematical model cannot be represented mathematically.
- **Ellipsoid**: it is one of the possible geometrical approximations (ellipsoidal geometry).
- **Geoid:** the equipotential surface of the earth's gravity field which best fits global mean sea level. Since the water is covering 70% of the earth's surface, it will be the best representation of the earth's surface.

Various arc measurements in the 17th and 18th centuries, as well as Newton's (and other's) arguments based on physical principles, gave convincing proof that the earth ellipsoidal in shape, flattened at the poles, with approximate rotational symmetry about the polar axis.



If the ellipse is rotated around minor axis it will form a surface called (**Ellipsoid**) which will be the nearest shape to the geoid.

Maximum difference between ellipsoid and geoid is ± **100 m**, and difference between semi-major (**a**) and semi-minor (**b**) of the ellipse is approximately **21 km**.

The geoid is not regular surface under the gravity forces its shape is changing from a place to another. Also it's not possible to be represented mathematically.

Geoid is used because it is a conventional reference surface of heights, but because it is irregular in shape, it cannot be used for determination of coordinates.

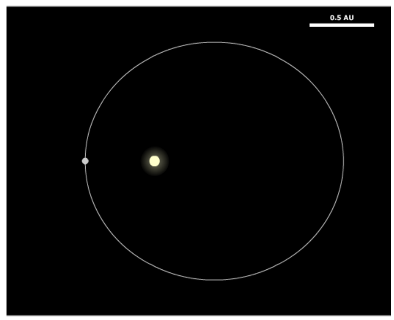
Scientists thought about a surface as close as possible to the geoid can be represented mathematically. **This was an ellipse.**

The earth's shape is closest to an ellipsoid, but sphere acceptable for many applications in case of a very high accuracy is not required.

Gravity: it is the centrifugal force caused by the earth's rotation, and because points on the equator are farthest from the center of the earth, and it is the direction perpendicular on the geoid, so we have less gravity. There are another gravity that was the direction perpendicular on the spheroid called (**normal gravity**).

SWF

9 kepler.swf



Equipotential surface: coincide with the direction of the force of gravity, that is, with the plumb line; an example on an equipotential surface is the surface liquid in equilibrium. The equipotential surface of the earth's gravity field coincides with the mean water level in the oceans.

- Equipotential surface are not parallel. And the plumb line, perpendicular to the equipotential surfaces is curved as shown in the figure.
- Equipotential Surface: is that imaginary surface on which at every point , the potential is same.
- The further the potential surface the lowest the gravity

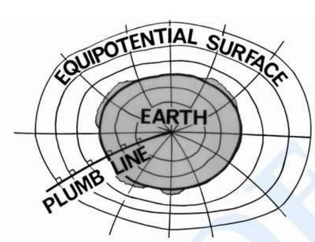
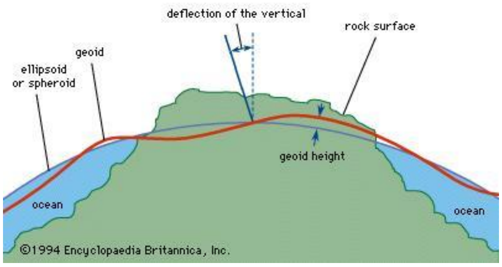
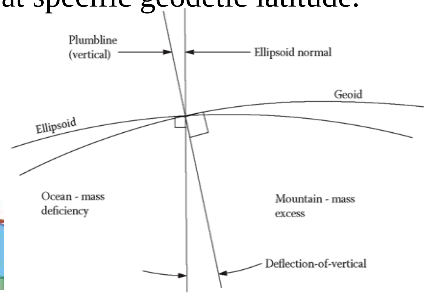


Fig. 1. Structure of Earth's gravity field.

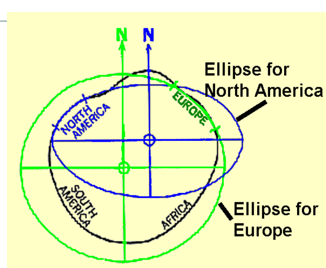
There are some intersection points between, spheroid and ellipsoid and the topography of the earth, these points are called Laplas points (standard points or reference points). In the laplas points N=V

Convergence angle (ɤ) (deflection from the vertical): the difference between grid north and the true north at specific geodetic latitude.





Every nation has its own the spheroid or ellipsoid was known (a) and (b), to make sure that this surface is best fit for their nation.



different ellipsoid for different country

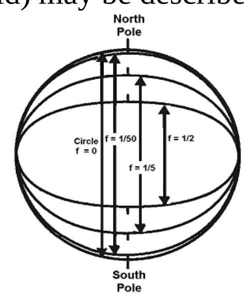
Why different models will be used in geodesy?

- Geoid has a connection with real life, but mathematically complicated.
- Ellipsoid good but math's still difficult for calculation.
- For simplicity a spherical system can be used.
  - Fits the shape of the geoid well (deviation small).
  - Simple to work with.
  - Can use spherical trigonometry in calculations.
  - Good basis for many surveying geodesy and navigation problems.

Earth's flattening

The ellipsoid provides relatively simple mathematical figure of the earth, it is defined by its major axis (a) and its minor axis (b), and these two different axes will make flattening.

Flattening: is squashing of the ellipsoidal poles downward towards its equator. And the reference surface (ellipsoid) may be described by its flattening (f).



Ellipses of Varing Flattening
Earth's Flattening Is About 1/300
Figure 4

$f = \frac{a-b}{a}$, f WGS84 = 1/298.275…

- If f = 0 it means that the ellipse becomes sphere a = b
- If f = 1 it means that b = 0 so, becomes flat surface
- So the value of flattening (f) is between zero and one

❑ This ellipsoid does not exactly represent the earth's shape. It is sufficient to measure coordinates (x, y) that surface to be represented on a map, but when you need to measure elevations the ellipsoid is not accurate surface would be used for elevation measurements. Therefore, it is necessary to have a surface that you can measure elevations accurately from this surface that is the geoid.

The Ellipse

Equation of ellipsoid

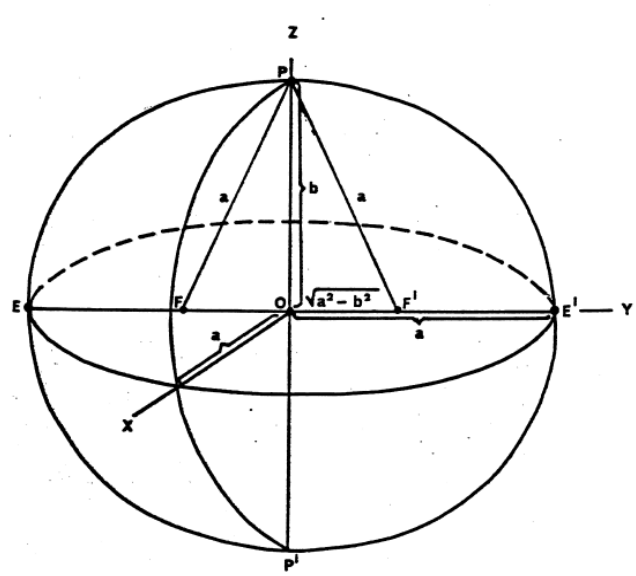
$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1 \text{ in case of 3D}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ in case of 2D}$$

The equation of any meridian curve is $\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1$

The surface of the ellipsoid of rotation is given by

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} = 1 \; in\ case\ of\ 3D$$



As shown in the figure, the points F and F' are the focus of the meridian ellipse through points P, E', P', and E. the focuses are equidistant from the geometric center (O) of the ellipse (this means that OF = OF') and the distances PF and PF' are equal to the semi-major axis (a).

$$PF = PF' = a \text{ and } PF + PF' = 2a$$

F1 and F2 are focuses of the ellipse so, for any P on the ellipse

$$F1P + F2P = 2a$$

This information is now used to help describe further properties of an ellipse.