

# Question Bank

---

## Problem 1:

Evaluate the integrals

$$\int \sqrt{\frac{1}{8x - x^2}} dx$$

---

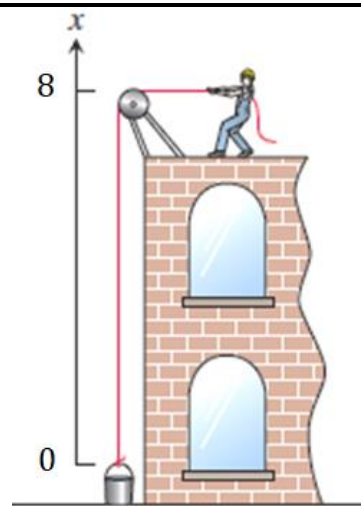
## Problem 2:

A vertical right-circular cylindrical tank measures 3 *m* high and 1 *m* in diameter. It is full of kerosene weighing 8200 *kg/m*<sup>3</sup>. How much work does it take to pump the kerosene to the level of the top of the tank?

---

## Problem 3:

A 100-N bucket is lifted from the ground into the air by pulling in 8 *meter* of rope at a constant speed. The rope weighs 0.4 *N/m*. How much work was spent lifting the bucket and the rope?



## Problem 4:

Find the area, on the right by  $x + y = 2$  on the left by  $y = x^2$  and below by  $x - axis$ .

---

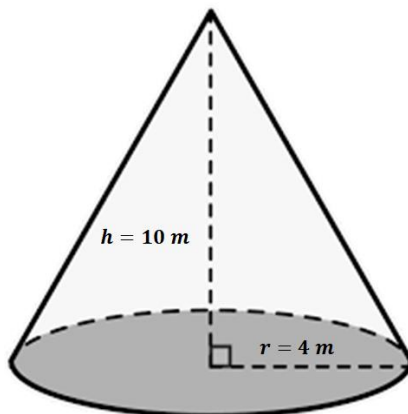
## Problem 5:

Find the volume of the solid generated by revolving the region bounded on the right by  $f(x) = x^4 - 2x^2$ , on the left by  $g(x) = 2x^2$ , and below by  $x - axis$  about  $x = 3$ .

---

## Problem 6: (30 Marks)

Use cross-section method to find the volume of the cone (see accompanying figure).



## Problem 7:

Find the area bounded on the top by  $g(y) = \cos y$ , below by  $f(y) = \sin y$ , and on the left by  $y - axis$  in the first quadrant.

---

# Question Bank

---

## **Problem 8:**

Evaluate the integral

$$\int \cot^4 x \, dx$$

---

## **Problem 9:**

Evaluate the integral

$$\int \cot x \sqrt{\frac{\sin x - 1}{\sin^5 x}} \, dx$$

---

## **Problem 10:**

Find the area bounded on the right by  $y = \frac{3}{2}x - 3$ , on top by  $x$  - *axis*, on the left by  $y = 2x - x^2$ , and below by  $y = -3$ .

---

## **Problem 11:**

Evaluate the integral

$$\int \frac{\cot y}{(\log_2 \sin y)^2} \, dy$$

---

## **Problem 12:**

Find  $dy/dx$  if

$$y = (\csc x)^{\ln(\cos^{-1} x^2)}$$

---

## **Problem 13:**

Evaluate the integral

$$\int \tan x \, dx$$

---

## **Problem 14:**

Evaluate the integrals

$$\int \sqrt{\frac{x-3}{x-2}} \, dx$$

---

## **Problem 15:**

Find  $dy/dt$  at any value of  $x$ , if

$$y = t \log_3(e^{(\sin t)(\ln 3)})$$

---

## **Problem 16:**

Simplify

$$\frac{x^4 + 81}{x(x^2 + 9)^2}$$

---

# Question Bank

---

## **Problem 17:**

Evaluate the integral

$$\int_{-\pi/4}^{\pi/4} \cosh(\tan \theta) \sec^2 \theta \, d\theta$$

---

## **Problem 18:**

Estimate the minimum number of subintervals needed to approximate  $\int_0^\pi \sin x \, dx$  in using **Simpson's Rule** with an error of magnitude less than  $10^{-4}$ .

---

## **Problem 19:**

Evaluate the integral

$$\int_0^3 x^3 \sqrt{x^2 + 1} \, dx$$

---

## **Problem 20:**

Evaluate the integral

$$\int_{-\infty}^0 \frac{1}{1+x^2} \, dx$$

---

## **Problem 21:**

Find the slope at any value of  $x$ , if

$$y = \ln \sqrt{\frac{(x+1)^5}{(x+2)^{20}}}$$

---

## **Problem 22:**

Find  $dy/dx$  at any value of  $x$ , if

$$y = \log_7 \left( \frac{\sin x \cos x}{e^x 2^x} \right)$$

---

## **Problem 23:**

Evaluate the integral

$$\int \frac{\operatorname{sech} \sqrt{t} \tanh \sqrt{t}}{\sqrt{t}} \, dt$$

---

# Question Bank

---

## **Problem 24:**

Evaluate the integral

$$\int_2^4 (1 + \ln y)y \ln y \, dy$$

---

## **Problem 25:**

Evaluate the integral

$$\int \frac{1}{(3r + 1)\sqrt{9r^2 + 6r}} \, dr$$

---

## **Problem 26:**

Evaluate the integral

$$\int \ln(x + x^2) \, dx$$

---

## **Problem 27:**

If  $f(x) = x^4 - 2x^2$  and  $g(y) = 2x^2$ , find the following:

1. The area bounded on the right by  $f(y)$ , below by  $x - axis$ , and on the left by  $g(x)$ .
  2. The volume of the solid generated by revolving the region enclosed between  $f(x)$  and  $x - axis$  about  $x = 3$ .
- 

## **Problem 28:**

Find the area of the surface generated by revolving a semi-circle of radius 1 centered at origin,  $-0.9 \leq x \leq 0.9$  about  $x - axis$ .

---

## **Problem 29:**

Find the slope at any value of  $x$ , if  $x^2(x - y)^2 = \tan(x^2 - y^2)$

---

## **Problem 30:**

Find the area bounded on the right by  $f(x) = -x^2 + 3x$ , on the left by  $g(x) = 2x^3 - x^2 - 5x$ , and below by  $y = -4$ .

---

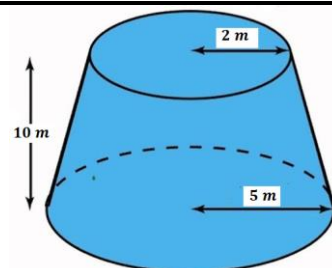
## **Problem 31:**

An open cylinder is to be made by rolling a rectangular golden sheet of perimeter 6 cm. what dimensions will make the cylinder to give the largest volume.

---

## **Problem 32:**

Use cross-section method to find the volume of the conical frustum (see accompanying figure).



# Question Bank

---

## **Problem 33:**

Evaluate the integral

$$\int \frac{\sin^3 x \sqrt{\sin^2 x - 1}}{\cos x} dx$$

---

## **Problem 34:**

Use the following information to find the curve  $y = f(x)$ , if:

- $\frac{d^2y}{dx^2} = \frac{1}{2} \sin\left(1 + \frac{x}{6}\right)$
  - The point  $(-6, -18)$  satisfy it and has a horizontal tangent at  $x = -6$ .
- 

## **Problem 35**

Evaluate the integral

$$\int \cot x \sqrt{\frac{\sin x - 1}{\sin^5 x}} dx$$

---

## **Problem 36:**

Evaluate the integral

$$\int \frac{(2r - 1) \cos \sqrt{3(2r - 1)^2 + 6}}{\sqrt{3(2r - 1)^2 + 6}} dr$$

---

## **Problem 37:**

Find the slope at  $x = 1$ , if  $\sqrt{xy} = 1$

---

## **Problem 38:**

Find the area bounded on the right by  $x = \sin y$ , on the left by  $y$  - axis, and on the top by  $y = \pi/2$ .

---

## **Problem 39:**

Find the area of the surface generated by revolving the curve  $y = 2\sqrt{x}$ ,  $-1 \leq x \leq 1$  about the  $x$  - axis.

---

## **Problem 40:**

Evaluate the integral

$$\int \sin^2 x \cos^4 x dx$$

---

## **Problem 41:**

Evaluate the integral

$$\int \frac{1}{\cos^2 y \sqrt{1 - \tan^2 y}} dy$$

---

# Question Bank

---

**Problem 42:**

Find  $dy/dx$  at  $x = 1$  and  $y = 1$  if

$$\sin^{-1}(\cos \sqrt{y}) \sqrt{y} = \cos^{-1}(\sin \sqrt{x})$$

---

**Problem 43:**

Evaluate the integral

$$\int \cos^{-1} x \, dx$$

---

**Problem 44:**

Evaluate the integral

$$\int \frac{\cosh(\ln t)}{t^2} \, dt$$

---

**Problem 45:**

Evaluate the integral

$$\int \frac{\ln(r) \sin(\sin^{-1} r)}{\sqrt{1-r^2}} \, dr$$

---

**Problem 46:**

Find the slope at  $x = 2$  and  $y = 1$ , if  ${}^x\sqrt{y} = {}^y\sqrt{x}$

---

**Problem 47:**

Evaluate the integral

$$\int \frac{\sin^3 x \sqrt{\sin^2 x - 1}}{\cos x} \, dx$$

---

**Problem 48:**

Use the integral to find the area bounded at top by a circle of radius 2 centered at (2,2), on the left by  $y - axis$ , and below by  $y - axis$ .

---

**Problem 49:**

Use Simpson's Rule with  $n = 4$  to approximate to three decimal places

$$\int_{\pi/6}^{3\pi/2} \sin \sqrt{x} \, dx$$

---

**Problem 50:**

Evaluate the integral

$$\int_0^{\infty} \frac{16 \tan^{-1} y}{1+y^2} \, dy$$

---

# Question Bank

---

**Problem 51:**

Evaluate the integral

$$\int \sqrt{\frac{x-2}{x-1}} dx$$

---

**Problem 52:**

Use Simpson's Rule with  $n = 4$  to approximate to three decimal places

$$\int_{\pi/6}^{3\pi/2} \frac{\sin x}{x} dx$$

---

**Problem 53:**

Evaluate the integral

$$\int \sec^{-1} y dy$$

---

**Problem 54:**

Evaluate the integral

$$\int \frac{\ln(x) \sec(\sec^{-1} x)}{\sqrt{x^2 - 1}} dx$$

---

**Problem 55:**

Evaluate the integrals

$$\int \sqrt{\frac{x-2}{x-1}} dx$$

---

**Problem 56:**

Find the slope at  $x = 1$ , if

$$y = x^x$$

---

**Problem 57:**

Evaluate the integral

$$\int \tan x dx$$

---

**Problem 58:**

Evaluate the integral

$$\int \tanh y \coth y dy$$

---

# Question Bank

---

## **Problem 59:**

Estimate the minimum number of subintervals needed to approximate  $\int_0^{\pi} \sin x \, dx$  in using **Simpson's Rule** with an error of magnitude less than  $10^{-4}$ .

---

## **Problem 60:**

Evaluate the integral

$$\int_0^3 \frac{dx}{(x-1)^{2/3}}$$

---

## **Problem 61:**

Evaluate the integral

$$\int \csc^3 x \, dx$$

---

## **Problem 62:**

Find the slope at any value of  $x$ , if  $\log_3 3^y = \ln e^{(1/x)^{\frac{\cosh x}{\sinh x}}}$

---

## **Problem 63:**

Find the area bounded on the left by  $y$  - **axis**, on the top by  $y = 3 - x$ , and below by  $x = 3y - y^2$ .

---

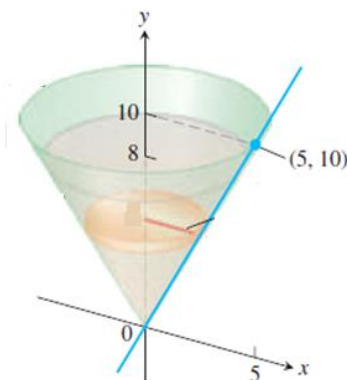
## **Problem 64:**

The region bounded by the curve  $y = x^2 + 1$  and the line  $y = 3 - x$  is revolved about the  $x = -3$  to generate a solid. Use **Shell Method** to find the volume of the solid.

---

## **Problem 65:**

The conical tank in Figure shown is filled to within  $2 \, m$  of the top with olive oil weighing  $914 \, kg/m^3$ . How much work does it take to pump the oil to the rim of the tank?



## **Problem 66:**

Evaluate the integral

$$\int \frac{(\tan^{-1} x)^2}{\sqrt{\tan^{-1} x - 1}} \frac{1}{(1+x^2)} dx$$

---



# Question Bank

---

**Problem 67:**

Find  $dy/dx$  at  $x = 1$  and  $y = 1$  if  $\sin^{-1}(\cos \sqrt{y}) \sqrt{y} = \cos^{-1}(\sin \sqrt{x})$

---

**Problem 68:**

Find the area bounded on the left by  $x = 3y - y^2$ , on the right by  $y = 3 - x$ , and below by  $x$  - *axis*.

---

**Problem 69:**

The region bounded by the curve  $y = x^2 + 1$  and the line  $y = 3 - x$  is revolved about the  $x = 1$  to generate a solid. Use **Shell Method** to find the volume of the solid.

---

**Problem 70:**

Evaluate the integral

$$\int (e^{2x} \sinh 2x + \cos x) dx$$

---

**Problem 71:**

A **5-N** bucket is lifted from the ground into the air by pulling in **6 m** of rope at a constant speed. The rope weighs **0.2 N/m**. How much work was spent lifting the bucket and the rope?

---

**Problem 72:**

Evaluate the integrals

$$\int \sqrt{\frac{1}{4 - (\sin^{-1} \sqrt{x})^2} \cdot \frac{1}{x(1-x)}} dx$$

---

**Problem 73:**

Evaluate the integrals

$$\int \cos^3 x dx$$

---

**Problem 74:**

Evaluate the integrals

$$\int \cot x \sqrt{\frac{\sin x - 1}{\sin^5 x}} dx$$

---

**Problem 75:**

Evaluate the integrals

$$\int \frac{1}{\cos^{-1} x} \sqrt{\frac{1}{(\cos^{-1} x)^2 - 4} \cdot \frac{1}{(1-x^2)}} dx$$

---

# Question Bank

---

**Problem 76:**

Evaluate the integrals

$$\int \frac{1}{\csc^2 y \sqrt{1 - \tan^2 y}} dy$$

---

**Problem 77:**

Evaluate the integrals

$$\int \cos^{-1} x dx$$

---

**Problem 78:**

Evaluate the integrals

$$\int \frac{\cosh(\ln t)}{t^2} dt$$

---

**Problem 79:**

Evaluate the integrals

$$\int \frac{\ln(r) \sin(\sin^{-1} r)}{\sqrt{1 - r^2}} dr$$

---

**Problem 80:**

Find  $dy/dx$  at  $x = 1$  and  $y = 1$  if  $\sin^{-1}(\cos \sqrt{y}) \sqrt{y} = \cos^{-1}(\sin \sqrt{x})$

---