Chapter (3)

Fluid Dynamic
Part (1)

3.1 ***Fluid Kinematics:*** Steady and unsteady flow, laminar and turbulent flow, uniform and non-uniform flow. Path-line, streamlines and stream tubes. Velocity and discharge. Control volume, Equation of continuity for compressible and incompressible fluids.

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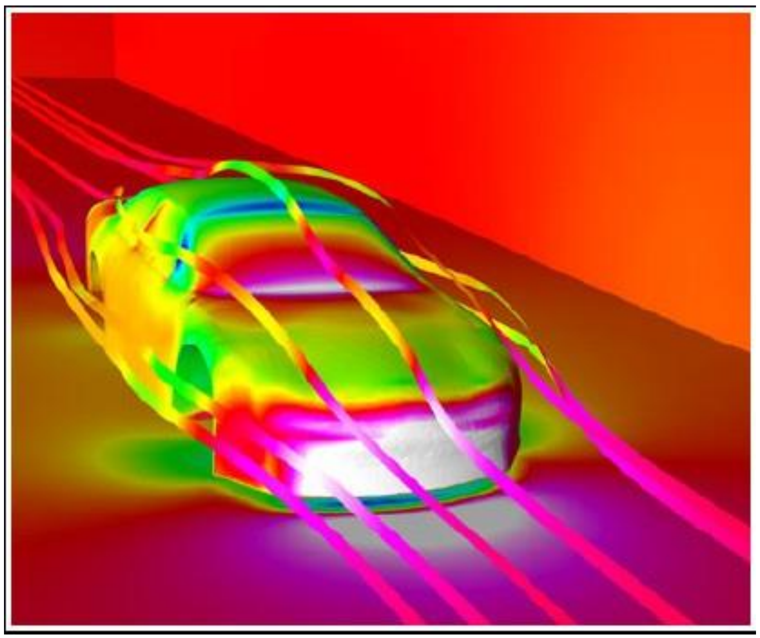
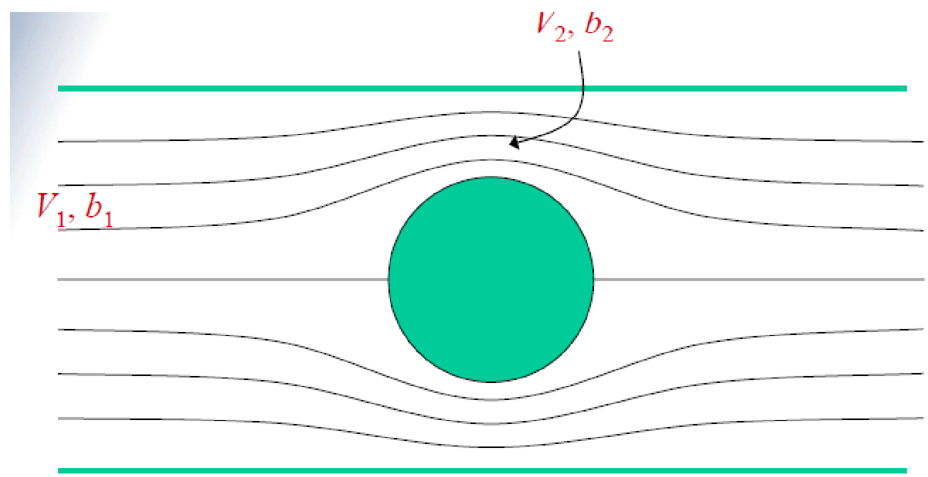
Fluid Kinematics

- Branch of fluid mechanics which deals with response of fluids in motion without considering forces and energies in them.
- The study of *kinematics* is often referred to as the *geometry of motion.*



CAR surface pressure contours and streamlines



Flow around cylindrical object

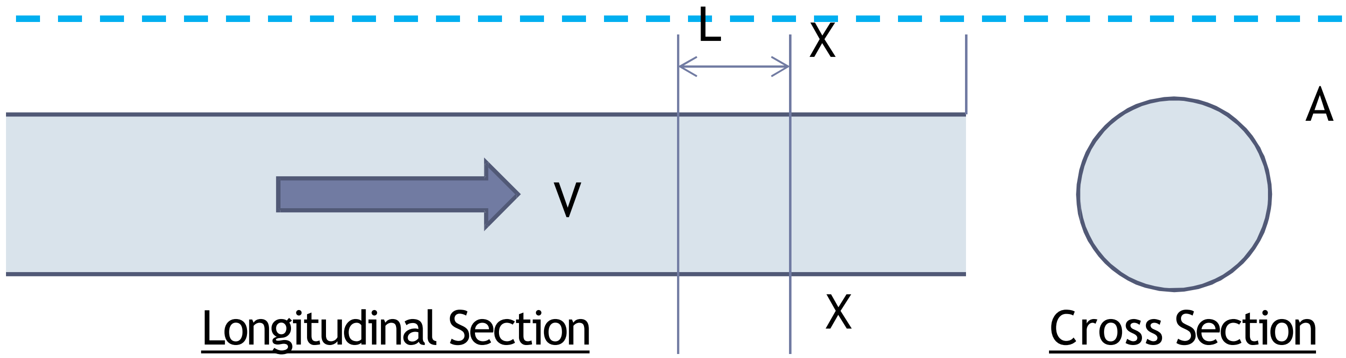
Fluid Flow

- **Rate of flow:** Quantity of fluid passing through any section in a unit time.

$$\text{Rate of flow} = \frac{\text{Quantity of fluid}}{\text{time}}$$

- **Type:**
  - 1. Volume flow rate: $= \frac{\text{volume of fluid}}{\text{time}}$
  - 2. Mass flow rate $= \frac{\text{mass of fluid}}{\text{time}}$
  - 3. Weigh flow rate $= \frac{\text{weight of fluid}}{\text{time}}$

Fluid Flow



- Let's consider a pipe in which a fluid is flowing with mean velocity, V.
- Let, in unit time, t, volume of fluid (AL) passes through section X-X,

  - 1. Volume flow rate:

$$Q = \frac{\text{volume of fluid}}{\text{time}} = \frac{AL}{t}$$

  - 2. Mass flow rate

$$M = \frac{\text{mass of fluid}}{\text{time}} = \frac{\rho(AL)}{t}$$

  - 3. Weigh flow rate

$$G = \frac{\text{weight of fluid}}{\text{time}} = \frac{\rho g(AL)}{t} = \frac{\gamma(AL)}{t}$$

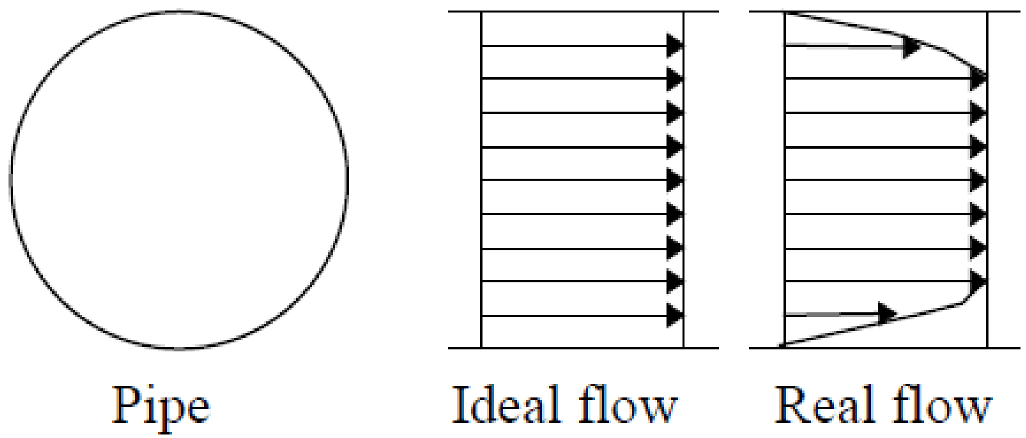
Types of Flow

- **Depending upon fluid properties**
- Ideal and Real flow
- Incompressible and compressible

- **Depending upon properties of flow**
- Laminar and turbulent flows
- Steady and unsteady flow
- Uniform and Non-uniform flow

Ideal and Real flow

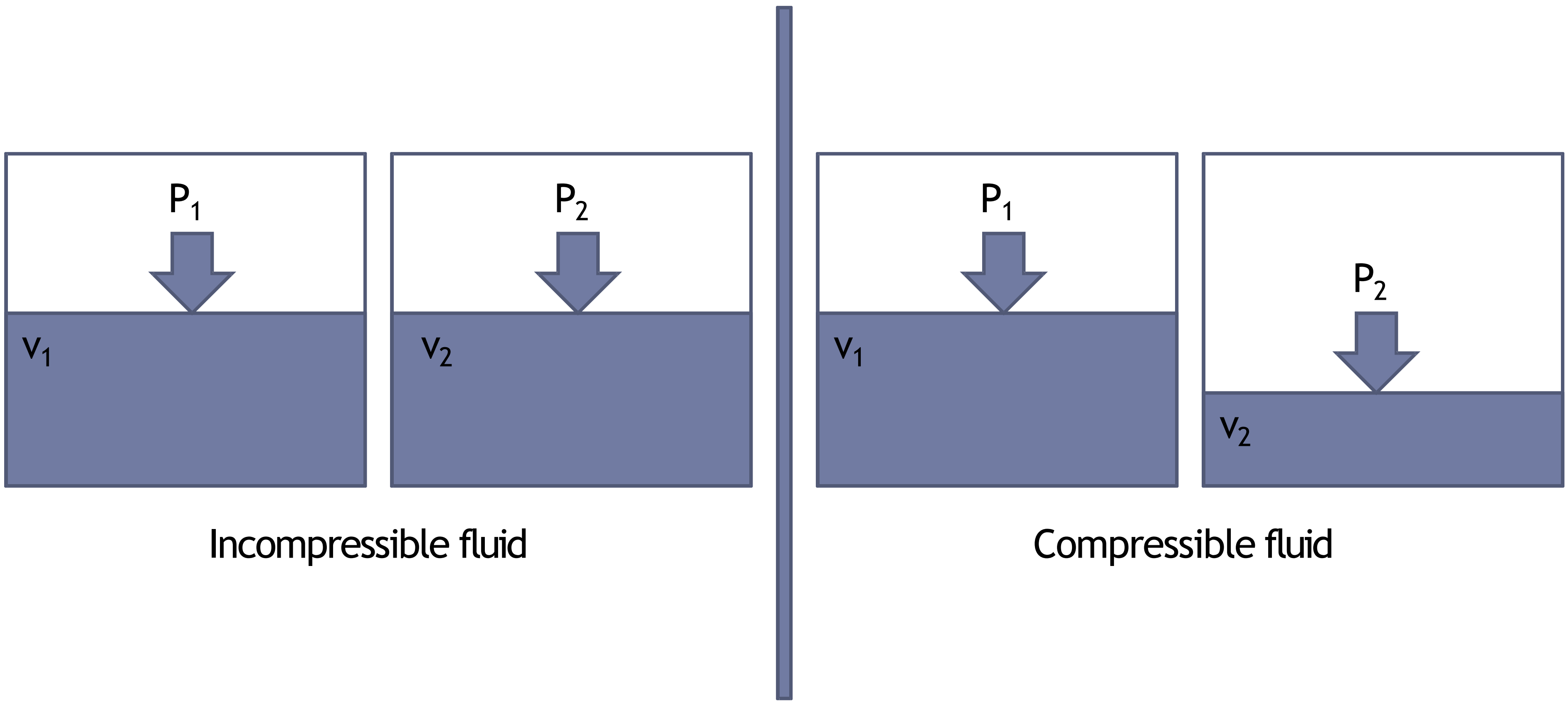
- Real fluid flows implies friction effects. Ideal fluid flow is hypothetical; it assumes no friction.



Velocity distribution of pipe flow

Compressible and incompressible flows

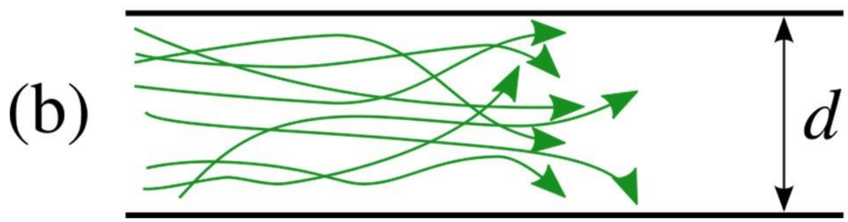
- Incompressible fluid flows assumes the fluid have constant density while in compressible fluid flows density is variable and becomes function of temperature and pressure.



Laminar and turbulent flow

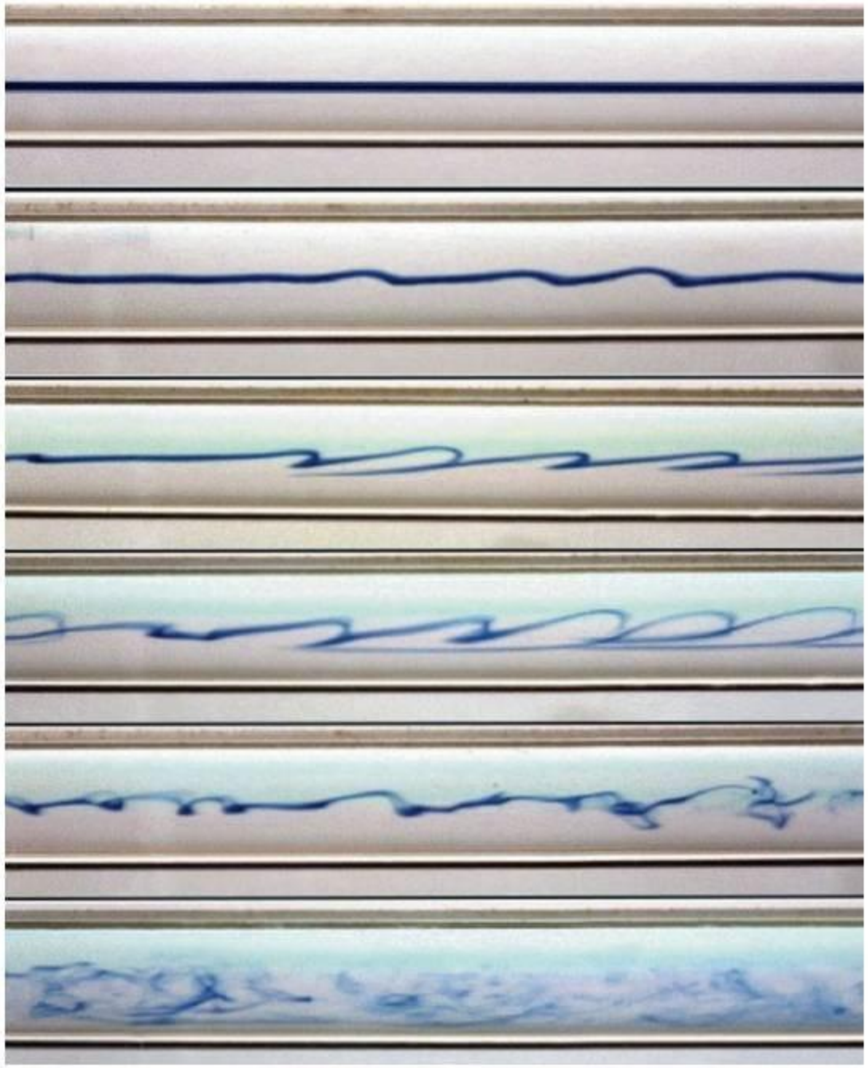
- The flow in laminations (layers) is termed as laminar flow while the case when fluid flow layers intermix with each other is termed as turbulent flow.

Laminar flow



Turbulent flow

- Reynold's number is used to differentiate between laminar and turbulent flows.



Transition of flow from Laminar to turbulent

Osborne Reynolds concept (1842-1912)

- **Reynold's Number(*Re*):** It is ratio of inertial forces (Fi) to viscous forces (Fv) of flowing fluid

$$\mathrm{Re} = \frac{Fi}{Fv} = \frac{Mass.\dfrac{Velocity}{Time}}{Shear\ Stress.\ Area} = \frac{\rho\dfrac{Volume}{Time}.\ Velocity}{Shear\ Stress.\ Area}$$

$$= \frac{\rho Q.V}{\tau.A} = \frac{\rho AV.V}{\mu\dfrac{du}{dy}.A} = \frac{\rho AV.V}{\mu\dfrac{V}{L}.A} = \frac{\rho VL}{\mu} = \frac{VL}{\upsilon}$$

$$R_e = \frac{\rho VD}{\mu} = \frac{VD}{\nu}$$

Where ;
V is avg. velocity of flow in pipe
ν is kinematic viscosity
L is characteristic/representative linear dimension of pipe. It is diameter of pipe (circular conduits) or hydraulic radius (non-circular conduits).

- For laminar flow: Re<=2000
- For transitional flow: 2000<Re<4000
- For Turbulent flow: Re>= 4000

Values of critical Reynolds no.

Note: For non-circular section, we need to use hydraulic radius (R_h) instead of diameter (D) for the linear dimension (L).

Critical Reynolds number

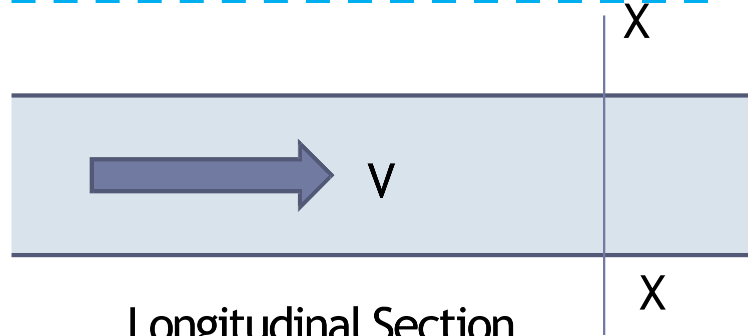
The advantage of using a critical Reynolds number, instead of critical velocity, is that the results of the experiments are applicable to all Newtonian fluid flows in pipes with a circular cross-section.

Table (1) shows the kinematic viscosity of water at different temperature states.

Temp. C°	ν cm^2/sec	Temp. C°	ν cm^2/sec	Temp. C°	ν cm^2/sec	Temp. C°	ν cm^2/sec
10	0.01308	14	0.01172	18	0.01057	22	0.00960
11	0.01271	15	0.01141	19	0.01032	23	0.00936
12	0.01237	16	0.01112	20	0.01007	24	0.00917
13	0.01204	17	0.01048	21	0.00983	25	0.00899

Steady and Unsteady flows

- **Steady flow:** It is the flow in which conditions of flow remains constant with respect to time at a particular section but the condition may be different at different sections.
- Flow conditions: velocity, pressure, density or cross-sectional area etc.
- e.g.,A constant discharge through a pipe.



Longitudinal Section

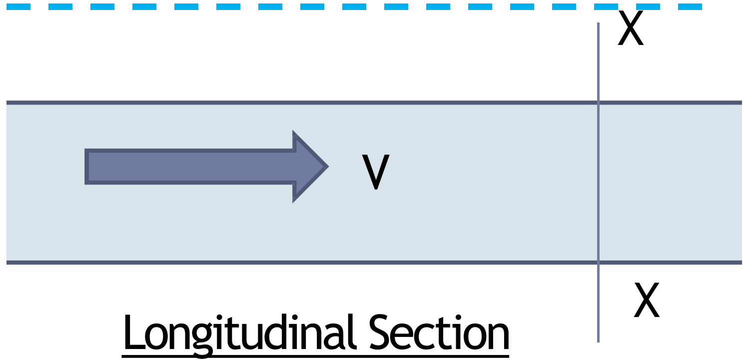
$$\frac{\partial V}{\partial t} = 0; \Rightarrow V = contt$$

- **Unsteady flow:** It is the flow in which conditions of flow changes With respect to time at a particular section.
- e.g.,A variable discharge through a pipe

$$\frac{\partial V}{\partial t} \neq 0; \Rightarrow V = \text{variable}$$

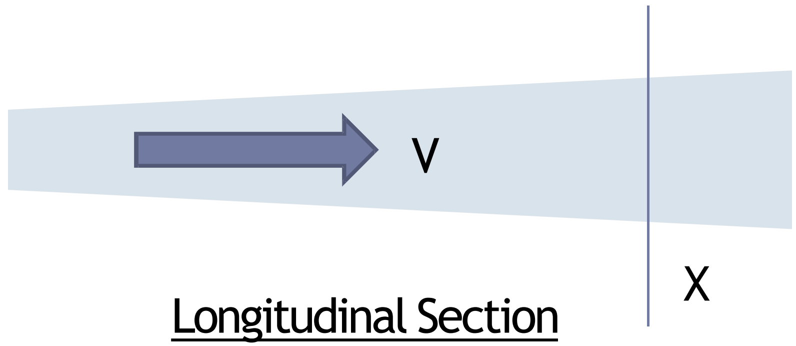
Uniform and Non-uniform flow

- **Uniform flow:** It is the flow in which conditions of flow remains constant from section to section.
- e.g., Constant discharge though a constant diameter pipe



$$\frac{\partial V}{\partial x} = 0; \Rightarrow V = contt$$

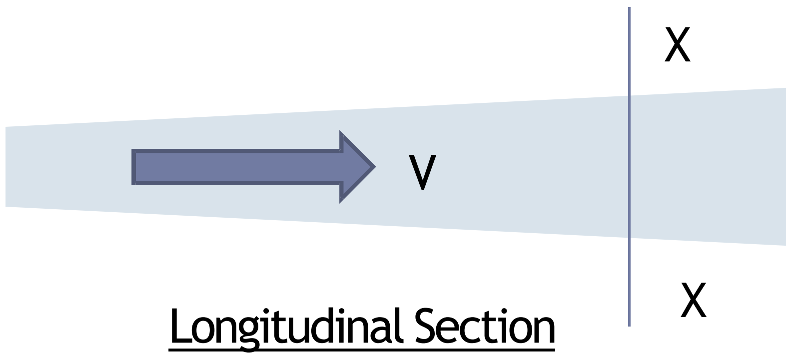
- **Non-uniform flow:** It is the flow in which conditions of flow does not remain constant from section to section.
- e.g., Constant discharge through variable diameter pipe



$$\frac{\partial V}{\partial x} \neq 0; \Rightarrow V = \text{variable}$$

Describe flow condition

- Constant discharge though non variable diameter pipe

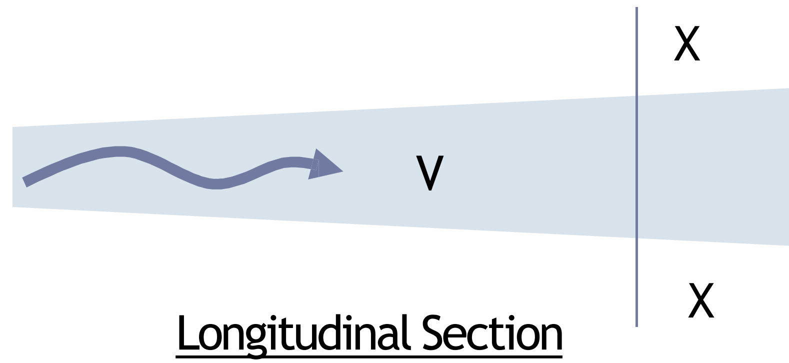


Steady flow !!

Non-uniform flow !!

Steady-non-uniform flow

- Variable discharge though non variable diameter pipe



Unsteady flow !!

Non-uniform flow !!

unsteady-non-uniform flow

Flow Combinations

Type	Example
1. Steady Uniform flow	Flow at constant rate through a duct of uniform cross-section
2. Steady non-uniform flow	Flow at constant rate through a duct of non-uniform cross-section (tapering pipe)
3. Unsteady Uniform flow	Flow at varying rates through a long straight pipe of uniform cross-section.
4. Unsteady non-uniform flow	Flow at varying rates through a duct of non-uniform cross-section.

Critical, sub-critical and super-critical flow

The criterion used in this classification is what is known by Froude number, Fr, which is the measure of the relative effects of inertia forces to gravity force:

- $F_r = 1$: **Critical Flow**
- $F_r < 1$: **Subcritical Flow** – slow flowing water
- $F_r > 1$: **Supercritical Flow** – fast flowing water

$$Fr = \frac{V}{\sqrt{gD}}$$

V = average channel velocity
g = gravity acceleralation
D = hydraulics water depth

William's Froude number (Fr)

Named after William Froude is a dimensionless number defined as the ratio of characteristic velocity to the gravity wave velocity. It is used to determine the resistance of an body which is submerged partially moving along with water.

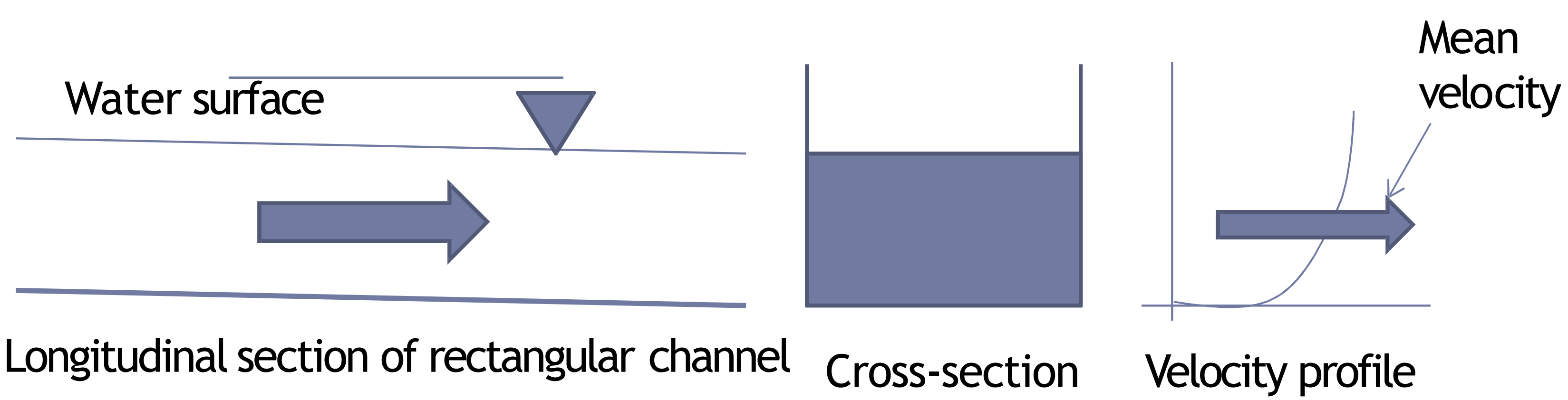
$$Fr = \sqrt{\frac{Fi}{Fg}} = \sqrt{\frac{Mass.\frac{Velocity}{Time}}{Mass.Gavitational\ Acceleraion}} = \sqrt{\frac{\rho\frac{Volume}{Time}.Velocity}{Mass.\ Gavitational\ Acceleraion}}$$

$$= \sqrt{\frac{\rho Q.V}{\rho Volume.g}} = \sqrt{\frac{\rho AV.V}{\rho AL.g}} = \sqrt{\frac{V^2}{gL}} = \frac{V}{\sqrt{gL}}$$

Where,
Fr is Froude number, v is velocity,
g is gravity,
L is characteristic length.

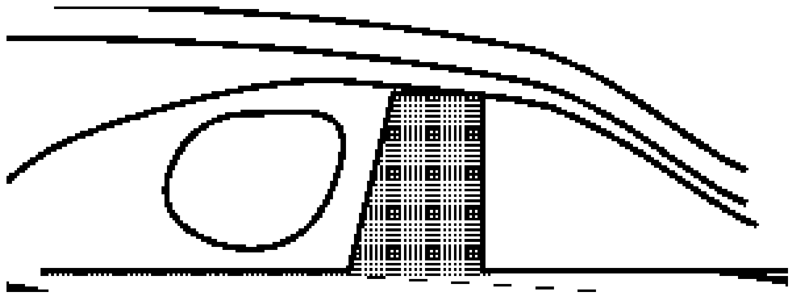
One, Two and Three-Dimensional Flows

- Although in general all fluids flow three-dimensionally, with pressures and velocities and other flow properties varying in all directions, in many cases the greatest changes only occur in two directions or even only in one. In these cases changes in the other direction can be effectively ignored making analysis much more simple.
- Flow is *one dimensional if the flow parameters (such as velocity, pressure, depth etc.) at a given instant in* time only vary in the direction of flow and not across the cross-section



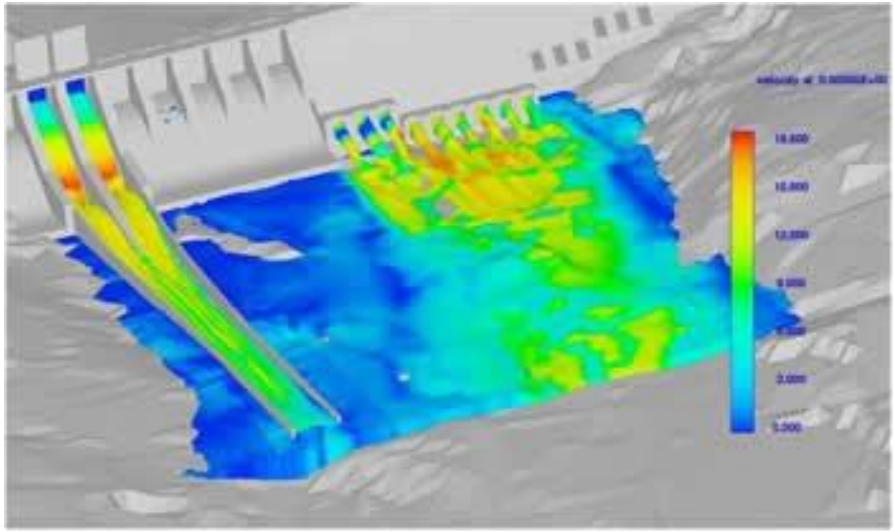
One, Two and Three Dimensional Flows

- Flow is *two-dimensional if it can be assumed that the flow parameters vary in the direction of flow and in* one direction at right angles to this direction



Two-dimensional flow over a weir

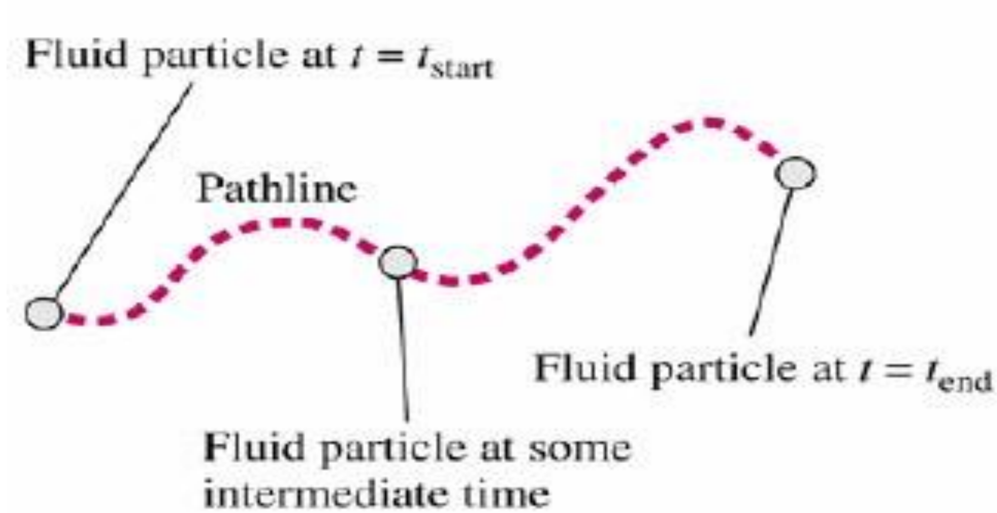
- Flow is *three-dimensional if the flow parameters vary in all three directions of flow*



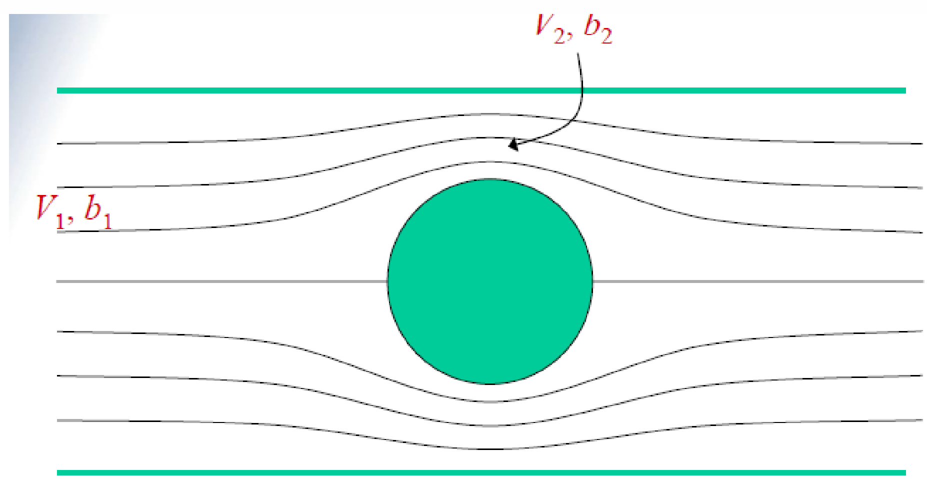
Three-dimensional flow in stilling basin

Path line and stream line

- **Pathline:** It is trace made by single particle over a period of time.
- **Streamline** show the mean direction of a number of particles at the same instance of time.



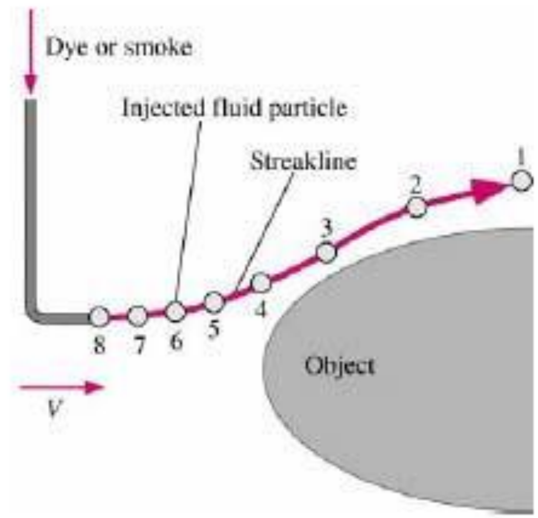
- **Character of Streamline**
- 1. Streamlines can not cross each other. (otherwise, the cross point will have two tangential lines.)
- 2. Streamline can't be a folding line, but a smooth curve.
- 3. Streamline cluster density reflects the magnitude of velocity. (Dense streamlines mean large velocity; while sparse streamlines mean small velocity.)



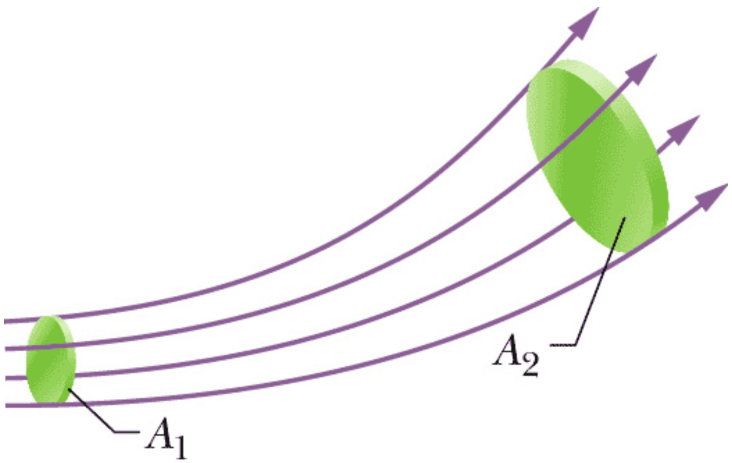
Flow around cylindrical object

Streakline and streamtubes

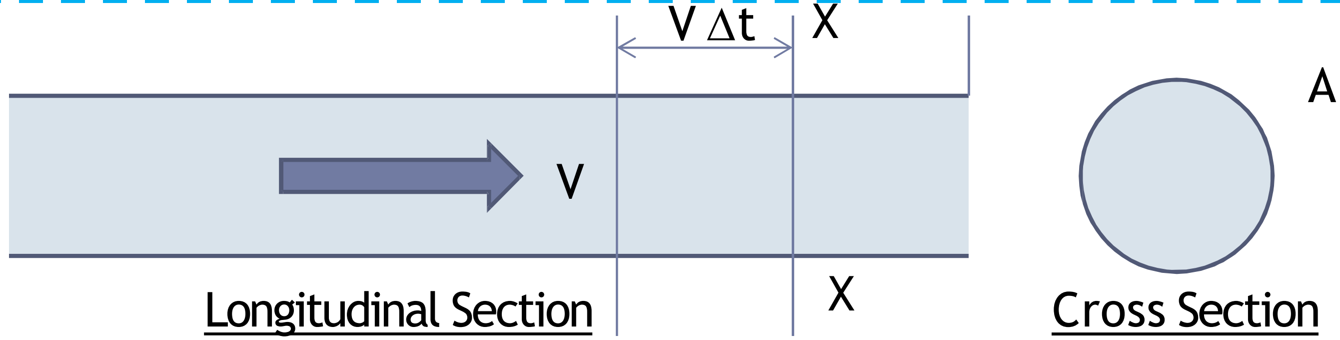
- A **Streakline** is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.
- It is an instantaneous picture of the position of all particles in flow that have passed through a given point.



- **Streamtube** is an imaginary tube whose boundary consists of streamlines.
- The volume flow rate must be the same for all cross sections of the stream tube.



Mean Velocity and Discharge



- Let's consider a fluid flowing with mean velocity,V, in a pipe of uniform cross-section.Thus volume of fluid that passes through section X X in unit time ,Δt, becomes;

$$\text{Volume of fluid} = VA * (\Delta t)$$

- Volume flow rate: $Q = \dfrac{\text{volume of fluid}}{\text{time}} = \dfrac{(\Delta t)\,VA}{\Delta t}$

$$Q = AV$$

Similarly

$$M = \rho AV$$

$$G = \gamma AV$$

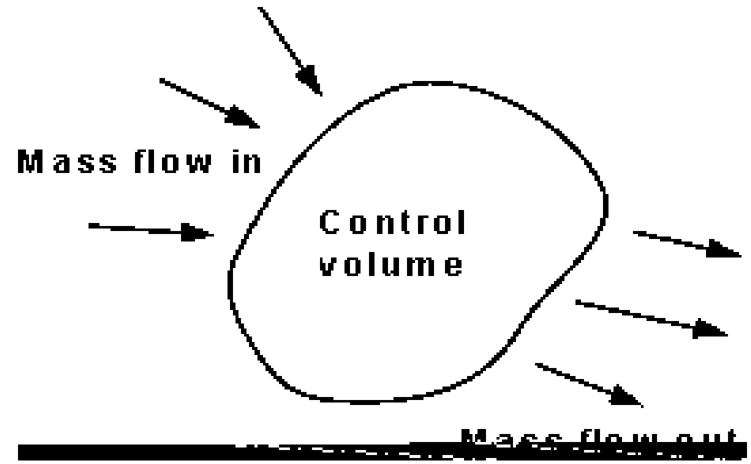
Fluid System and Control Volume

- **Fluid system** refers to a specific mass of fluid within the boundaries defined by close surface. The shape of system and so the boundaries may change with time, as when fluid moves and deforms, so the system containing it also moves and deforms.

- **Control volume** refers to a fixed region in space, which does not move or change shape. It is region in which fluid flow into and out.

Continuity

- Matter cannot be created or destroyed - (it is simply changed in to a different form of matter).
- This principle is know as the *conservation of mass* and we use it in the analysis of flowing fluids.
- The principle is applied to fixed volumes, known as control volumes shown in figure:



An arbitrarily shaped control volume.

For any control volume the principle of conservation of mass says

Mass entering per unit time -Mass leaving per unit time

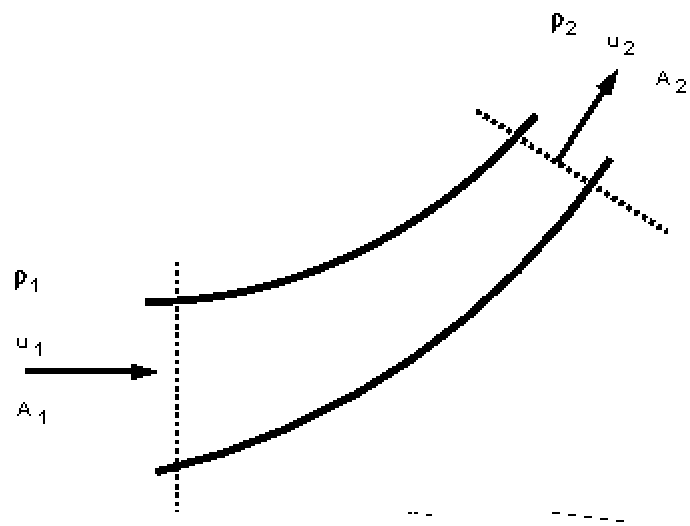
= Increase of mass in the control volume per unit time

Continuity Equation

- For steady flow there is no increase in the mass within the control volume, so

Mass entering per unit time = Mass leaving per unit time

- **Derivation:**
- Lets consider a stream tube.
- ρ_1, v_1 and A_1 are mass density, velocity and cross-sectional area at section 1. Similarly, ρ_2, v_2 and A_2 are mass density, velocity and cross-sectional area at section 2.
- According to mass conservation



A stream tube

$$M_1 - M_2 = \frac{d(M_{CV})}{dt}$$

$$\rho_1 A_1 V_1 - \rho_2 A_2 V_2 = \frac{d(M_{CV})}{dt}$$

$$M_1 = \rho_1 A_1 V_1$$

$$M_2 = \rho_2 A_2 V_2$$

Continuity Equation

- For steady flow condition $d\left(M_{CV}\right)/\, dt = 0$

$$\rho_1 A_1 V_1 - \rho_2 A_2 V_2 = 0 \Rightarrow \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$M = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

- Hence, for stead flow condition, mass flow rate at section 1= mass flow rate at section 2. i.e., mass flow rate is constant.
- Similarly $G = \rho_1 g A_1 V_1 = \rho_2 g A_2 V_2$

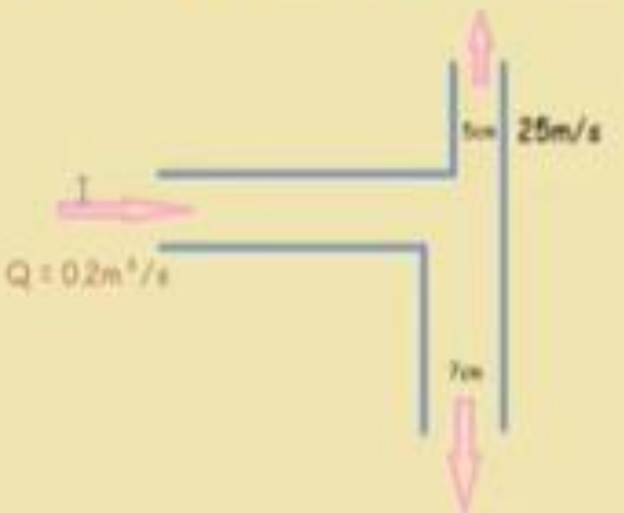
- Assuming incompressible fluid, $\rho_1 = \rho_2 = \rho$

$$A_1 V_1 = A_2 V_2 \Rightarrow Q_1 = Q_2 \Rightarrow Q_1 = Q_2 = Q_3 = Q_4$$

- Therefore, according to mass conservation for steady flow of incompressible fluids volume flow rate remains same from section to section.

Problem

0.2m³/s of water enters a pipe, the water then splits into a 5cm and 7cm pipe as shown below. If average velocity in the 5cm pipe is 25m/s what is the flow rate Q in the 7cm pipe?



$$Q_2 = V \cdot A$$

$$= \left(25\,\frac{m}{s}\right)\left(\frac{\pi r^2}{4}\right)$$

$$= (25)\left(\frac{\pi (0.05)^2}{4}\right)$$

$$= 0.0491\,\frac{m^3}{s}$$

$$Q_3 = Q_1 - Q_2$$

$$= 0.2 - 0.0491$$

$$= 0.151\,\frac{m^3}{s}$$