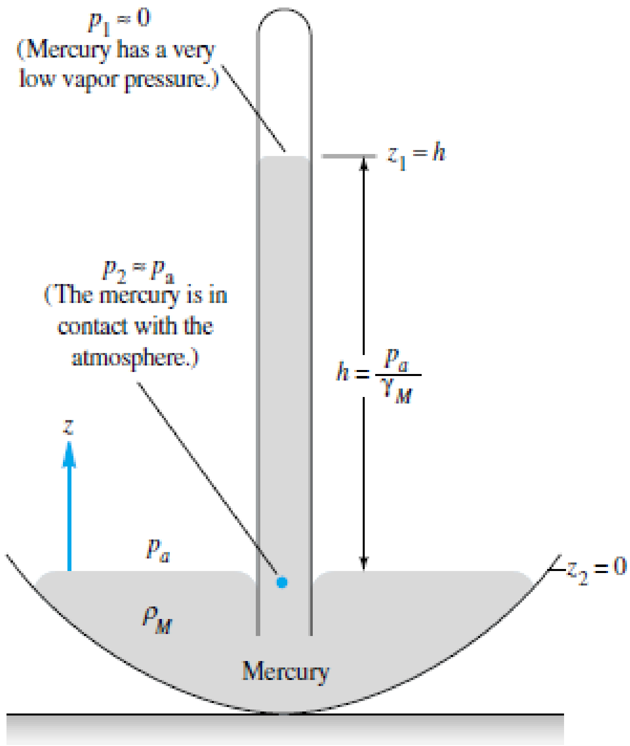
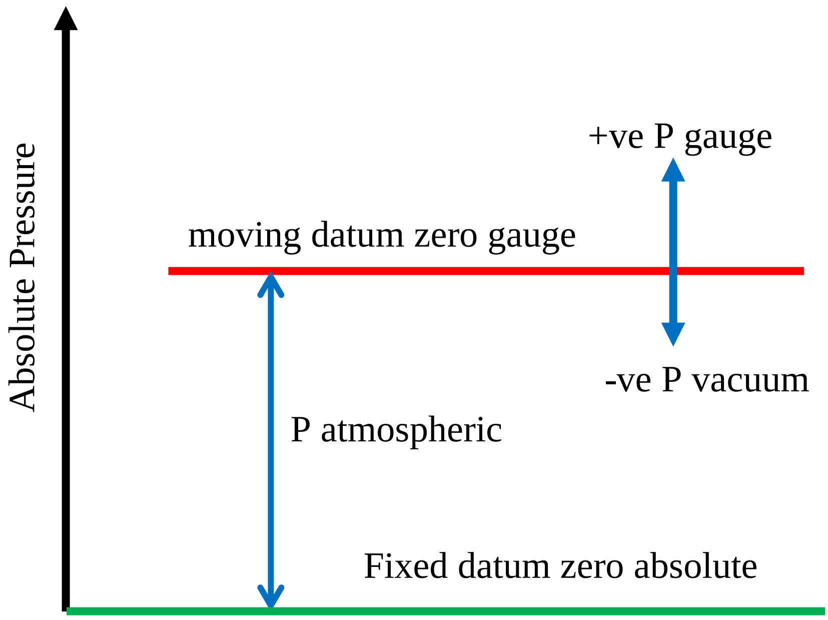
The mercury barometer

It is used to measure the atmospheric pressure, relative to absolute zero.



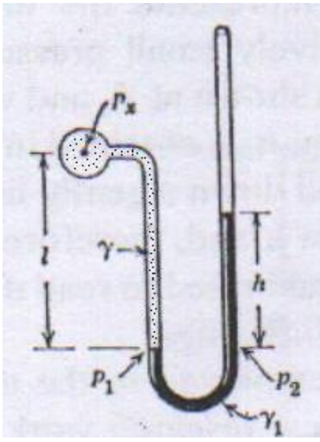
$$Absolute\ Pressure\ =\ P_{atm}\begin{cases}+\ p_{gauge}\\ -\ p_{vacuum}\end{cases}$$



Manometers:

A manometer is a device used to measure gauge pressure it consists of a tube containing a liquid.

The U - tube manometer:



$$P_1 = P_2 \dots\dots\dots\dots\dots\dots [1]$$

but

$$P_1 = P_x + \rho_f g l \dots\dots\dots\dots [2]$$

and

$$P_2 = P_{atm} + \rho_m g h \dots\dots\dots\dots [3]$$

Substitute 2 and 3 in 1

$$P_x + \rho_f g l = P_{atm} + \rho_m g h$$

$$P_x = \rho_m g h - \rho_f g l$$

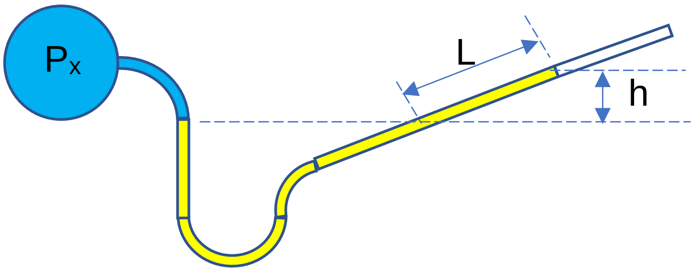
Since

$$P_{atm} = 0 \qquad [by\ gauge\ pressure]$$

The inclined manometer:

$$P_x = \rho g h$$

$$P_x = \rho g L \sin\theta$$

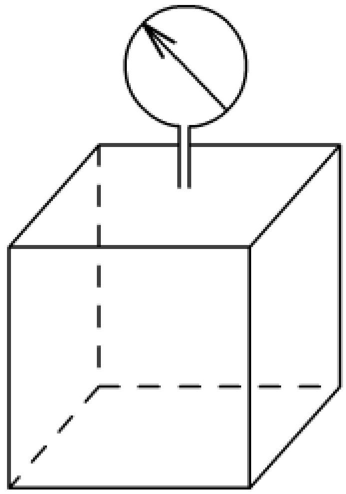


Example

A Bourdon gauge attached to a closed tank of air reads 141.1 kPa when the barometer was indicating 775 mm of mercury. If the barometric pressure drops to 741.2 mm Hg. What will the gauge read?

Solution

$$P_{atm1} = \rho_{Hg} g h_{Hg}$$



$$P_{atm1} = 1000 * 13.6 * 9.81 * 0.775$$

$$P_{atm1} = 103.397\ kPa$$

$$P_{abs} = P_g + P_{atm1}$$

$$= 141.1 + 103.397$$

$$= 244.5\ kPa.$$

$$P_{atm2} = \rho_{Hg} g h_{Hg}$$

$$= 1000 * 13.6 * 9.81 * 0.7412$$

$$= 98.88\ kPa.$$

$$P_{g_2} = P_{abs} - P_{atm2}$$

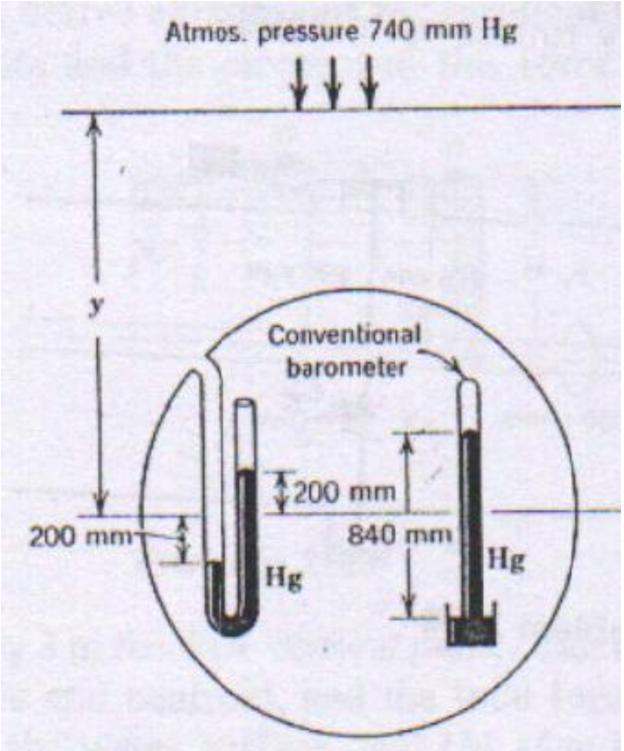
$$= 244.5 - 98.888$$

$$= 145.612\ kPa.$$

Example

The barometer reads 840 mmHg inside a submarine. Determine submarine's depth (y) when atmospheric pressure is 740 mmHg. Assume the weight density of sea water is 10 kN/m^3.

Solution



$$P_1 = \rho_{Hg} g(0.84)$$

$$P_2 = P_1 + \rho_{Hg} g(0.4)$$

$$P_2 = \rho_{Hg} g(0.84 + 0.4)$$

$$P_3 = \rho_w g(y + 0.2) + \rho_{Hg} g(0.74)$$

$$P_2 = P_3$$

$$\rho_{Hg} g(0.84 + 0.4) = \rho_w g(y + 0.2) + \rho_{Hg} g(0.74)$$

$$\rho_w g(y + 0.2) = \rho_{Hg} g(0.84 + 0.4 - 0.74)$$

$$y = \frac{\rho_{Hg}}{\rho_w}(0.84 + 0.4 - 0.74) - 0.2$$

$$y = 13.6(0.5) - 0.2$$

$$y = 6.6\, m$$

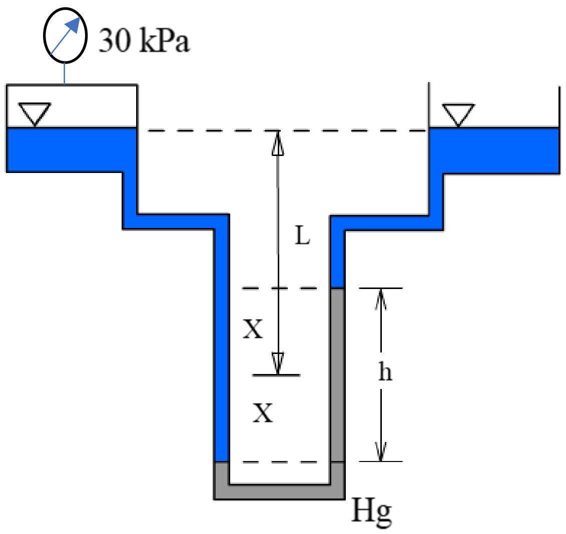
Example

Calculate the manometer reading if the pressure in the closed tank is raised to 30 kPa.

Solution:

$$P_g + \rho_w g(L + x) = \rho_w g(L - x) + \rho_{Hg} gh$$

$$P_g = \rho_{Hg} gh - \rho_w g(2x)$$



But

$$2x = h$$

$$h = \frac{P_g}{g(\rho_{Hg} - \rho_w)}$$

$$h = \frac{30\ 000}{9.81(13\ 600 - 1\ 000)}$$

$$h = 0.2427\ m$$

A second solution

$$P_g + \rho_w gh = \rho_{Hg} gh$$

$$P_g = gh(\rho_{Hg} - \rho_w)$$

$$h = \frac{P_g}{g(\rho_{Hg} - \rho_w)}$$

$$h = \frac{30\ 000}{9.81(13\ 600 - 1\ 000)}$$

$$h = 0.2427\ m$$