Chapter 4

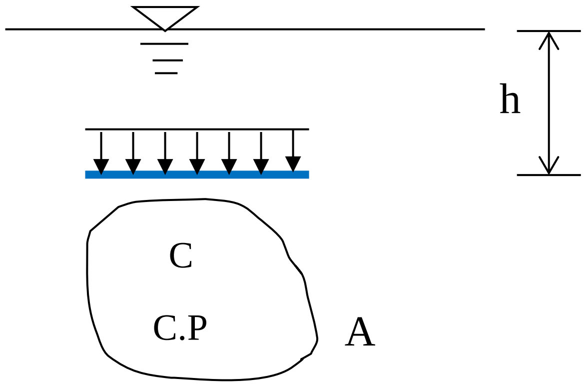
1. The hydrostatic force on a horizontal plane surface

The pressure on the horizontally immersed surface is uniformly distributed. It represents the weight of the liquid over the surface.

$$P = \rho.g.h$$

$$F = P.A = \rho gh.A$$

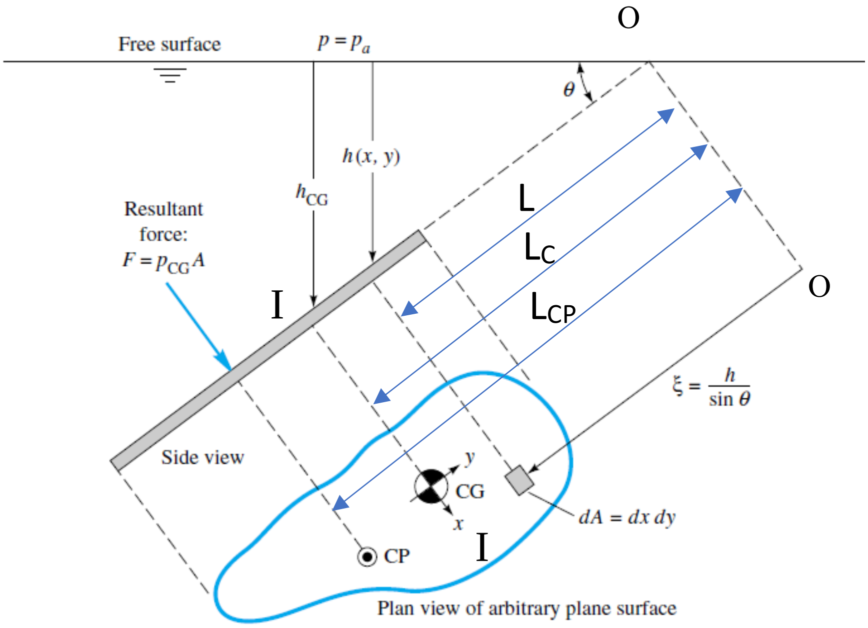
$$F = \rho gV$$



The resultant force F passes through a point on the plane surface defined as the center of pressure (CP), for a horizontal plane CP will coincide with the centroid (C) of the surface.

- Note that the force acting downwards on the upper face of the plane is equal to the force acting upward on the lower face.

2. The hydrostatic force on an Inclined Plane Surface:



Consider the force on a differential area (dA) as shown.

$$dF = \rho gh.dA$$

$$dF = \rho gL \sin\theta \,.dA$$

Integration over the area (A) of the surface gives:

$$F = \rho g.\sin\theta \int_A L\,dA$$

But the integral is the moment of the entire area (A) about the axis $O - O$ and it equals to:

$$\int_A LdA = L_c A$$

$$F = \rho g.\sin\theta \,.L_C A$$

$$F = \rho g.h_C A$$

But

$$\rho g h_c = P_c$$

Where

$$P_c \; is\ the\ pressure\ at\ the\ centroid\ of\ the\ surface.$$

The Center of pressure:

Consider the moment produced by the differential force dF about o-o.

$$dM = L.dF$$

$$dM = \rho g.L^2 \sin\theta \,.\, dA$$

$$M = \rho g.\sin\theta \,.\int_A L^2 dA$$

The quantity $\int_A L^2 dA$ is the second moment of area (A) about the axis o-o.

$$\int_A L^2 dA = I_{o-o}$$

$$M = \rho g.\sin\theta \,.\, I_{o-o}$$

But

$$M = F.L_p$$

$$M = \rho g \sin\theta \, L_c A.L_p$$

Thus

$$L_p = \frac{I_{o-o}}{L_c A}$$

Transferring the axis of rotation from (o-o) to the centroid of the plane surface using the parallel axis theorem,

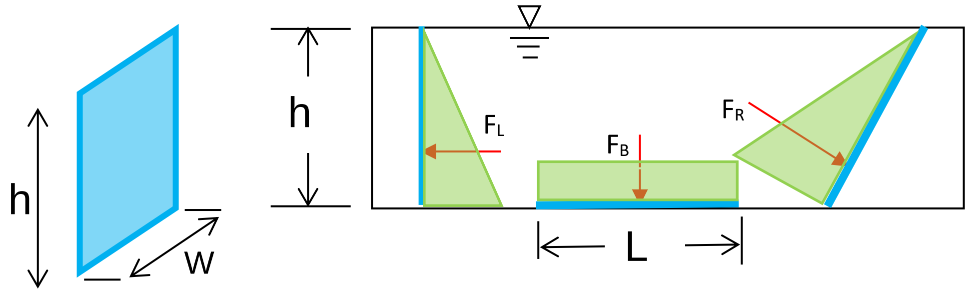
$$I_{o-o} = I_c + A.L_c^2$$

$$L_p = \frac{I_c}{L_c A} + L_c$$

or

$$L_p - L_C = \frac{I_c}{L_c A}$$

The Pressure Prism



- Force on the Left wall F_L

$$F_L = \rho g . h_c . A$$

$$F_L = \rho g \frac{h}{2} . hw$$

- Force on the Right wall F_R

$$F_R = \rho g . h_c . A$$

$$h_c = \frac{h}{2} \qquad , \qquad A = \frac{h}{\sin \theta} w$$

$$F_R = \rho g \frac{h}{2} * \frac{h}{\sin \theta} w$$

- Force on the Bottom F_B

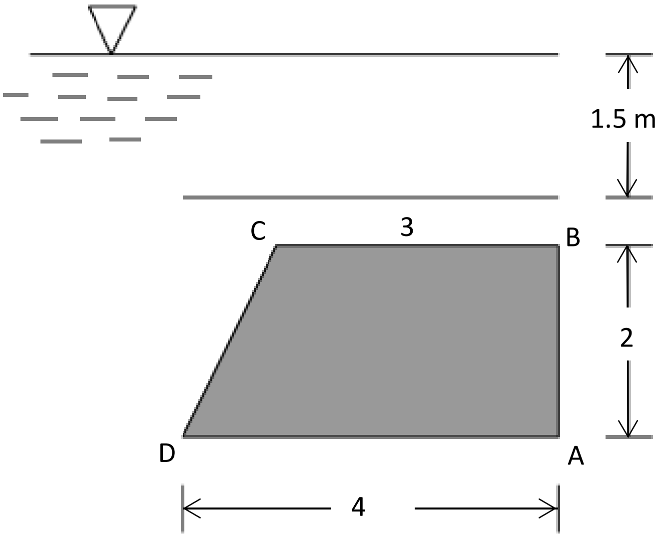
$$F_L = \rho g . h_c . A$$

$$F_L = \rho g . h . Lw$$

Example

Calculate the resultant force acting on one side of the horizontally immersed plane surface shown below and find the center of pressure.

Solution



$$F_{\Delta} = \rho g . h_c . A$$

$$F_{\Delta} = 9810 * 1.5 * \left[\frac{1*2}{2}\right] = 14.71\ kN$$

$$F_{\boxdot} = \rho g . h_c . A$$

$$F_{\boxdot} = 9810 * 1.5 * 3 * 2 = 88.29 kN$$

$$F_T = F_{\Delta} + F_{\boxdot}$$

$$F_T = 14.71 + 88.29 = 103 kN$$

$$M_{AB} = F_T * x_{cp}$$

$$F_T * x_{cp} = [F * x_{cp}]_{\Delta} + [F * x_{cp}]_{\boxdot}$$

$$103 * x_{cp} = \left[14.71 * \left(3 + \frac{1}{3}\right)\right] + [88.29 * 1.5]$$

$$x_{cp} = 1.7617\ m$$

$$M_{AD} = F_T * y_{cp}$$

$$F_T * y_{cp} = [F * y_{cp}]_{\Delta} + [F * y_{cp}]_{\boxdot}$$

$$103 * y_{cp} = \left[14.71 * \left(\frac{2}{3}\right)\right] + [88.29 * 1]$$

$$y_{cp} = 0.95\ m$$