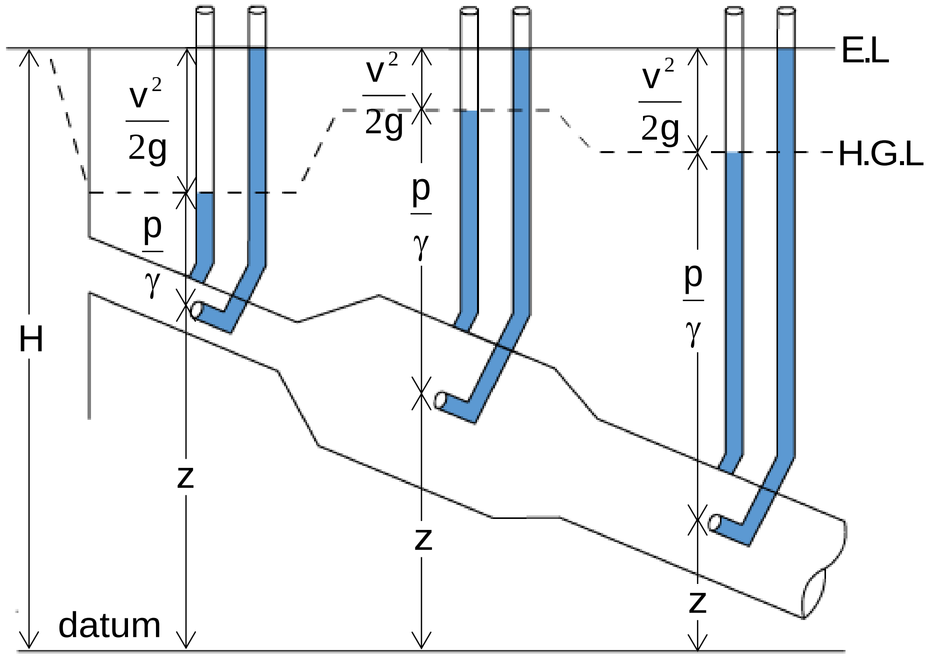
The Energy Line & the Hydraulic Grade Line



The sum of the pressure head, the potential head, and the velocity head can be measured by a Pitot static tube. Connecting the readings of these tubes produce the energy line (E.L.) which represent the total energy possessed by the fluid. While the hydraulic grade line (H.G.L) represents the sum of the static pressure and potential pressure heads.

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = H$$

Example

Fourteen cubic meters per second of water flow in a rectangular open channel 6 m wide and 2.4 m deep. After passing through a transition structure into a trapezoidal canal of 1.5 m base width with sides sloping at 30°, the velocity is 2m/s. Calculate the depth (h) of water in the canal.

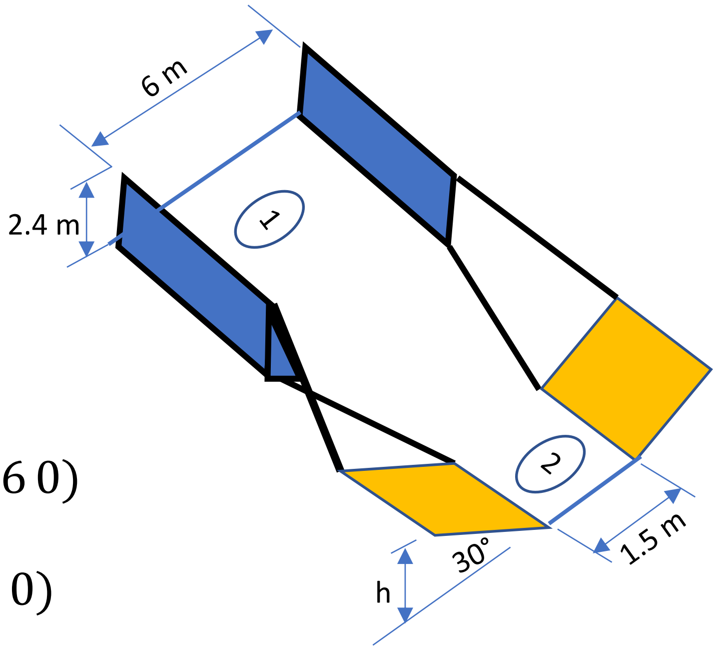
Solution

$$A_2 = \frac{Q}{V_2} = \frac{14}{2} = 7m^2$$

$$A_2 = h(\frac{b_1+b_2}{2}) \quad \dots (1)$$

$$b_2 = 1.5 + 2x \quad \dots (2)$$

$$x = h\, tan\, 6\, 0 \quad \dots (3)$$



substitute Eqs. 2 & 3 in 1

$$A_2 = \frac{h}{2}(b_1 + 1.5 + 2h\, tan\, 6\, 0)$$

$$7 = \frac{h}{2}(b_1 + 1.5 + 2h\, tan\, 6\, 0)$$

$$1.732h^2 + 1.5h + (-7) = 0$$

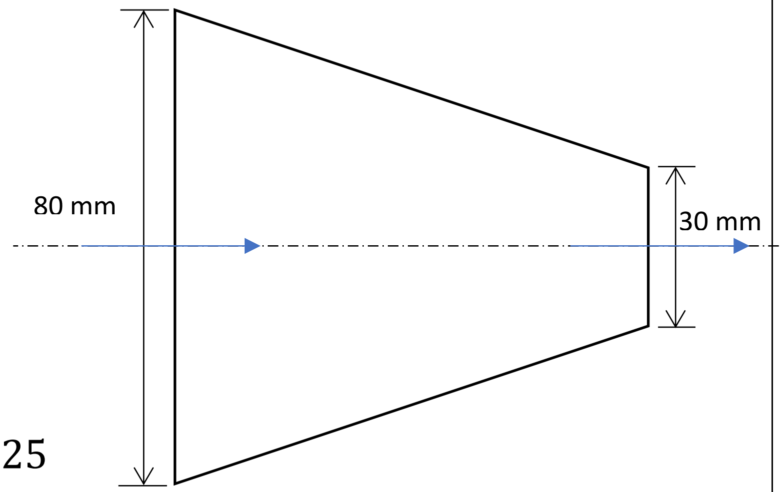
$$h = \frac{-1.5 \mp \sqrt{(1.5)^2 - 4(1.732)(-7)}}{2(1.732)}$$

$$h = \frac{-1.5 \mp 7.123}{3.464} = 1.623m$$

Example

A nozzle with a base diameter of 80 mm and with 30 mm diameter tip discharges 10 L/s. Derive an expression for the fluid velocity along the axis of the nozzle. Measure the distance (x) along the axis from the plane of the larger diameter.

Solution



$$\frac{z}{y} = \frac{s}{L}$$

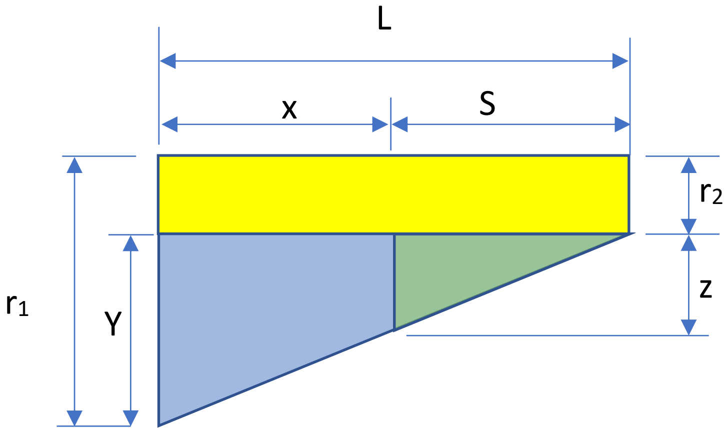
$$\frac{z}{y} = \frac{L - x}{L}$$

$$\frac{z}{y} = 1 - \frac{x}{L}$$

but

$$y = r_1 - r_2 = 0.04 - 0.015 = 0.025$$

$$z = r - r_2 = r - 0.015$$



$$\frac{r - 0.015}{0.025} = 1 - \frac{x}{L}$$

$$r - 0.015 = 0.025 - 0.025\frac{x}{L}$$

$$r = 0.04 - 0.025\frac{x}{L}$$

$$d = 0.08 - 0.05\frac{x}{L}$$

$$V = \frac{Q}{A} = \frac{Q * 4}{\pi * d^2}$$

$$V = \frac{0.01 * 4}{\pi\left(0.08 - 0.05\frac{x}{L}\right)^2}$$

$$V = \frac{4}{\pi\left(0.8 - 0.5\frac{x}{L}\right)^2}$$

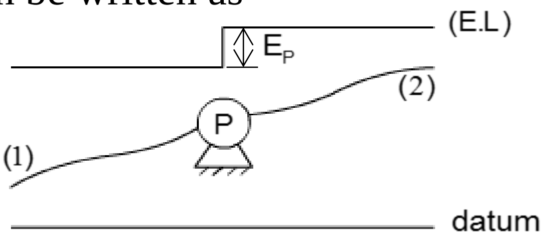
$$V = \frac{1.273}{\left(0.8 - 0.5\frac{x}{L}\right)^2}$$

Flow with Energy Exchange

Energy may be added to the flow by a pump, or it may be extracted from the flow by a turbine. Energy added by a pump per unit weight of fluid is denoted by E_p. Energy extracted per unit weight of fluid by a turbine is denoted by E_T.

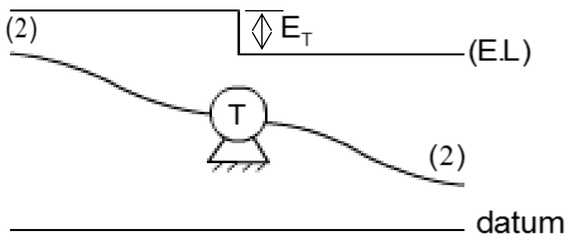
For a system that comprise a pump, B.E can be written as

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + E_P = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$



For a system that comprise a Turbine, B.E can be written as

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + E_T$$



The power required to drive a pump, or the power extracted from a turbine can be computed as follows.

$$P_{pump} = \frac{\rho g Q E_P}{\eta_p}$$

$$P_{Turbine} = \frac{\rho g Q E_T}{\eta_T}$$

$$\eta_p = Pump\ efficiency$$

$$\eta_T = Turbine\ efficiency$$