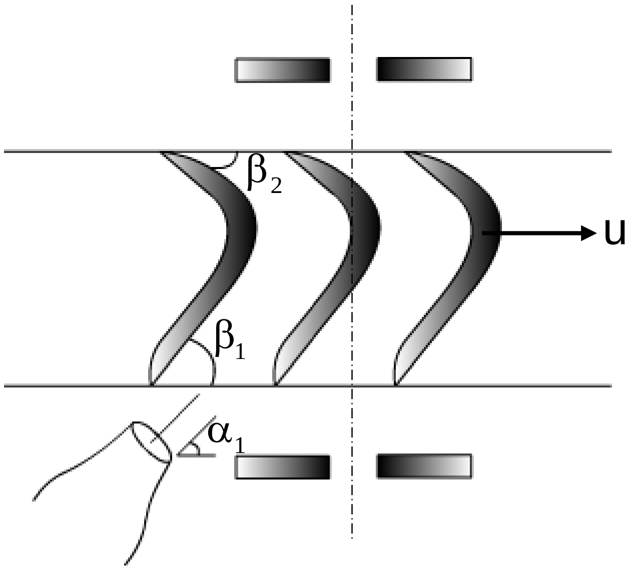
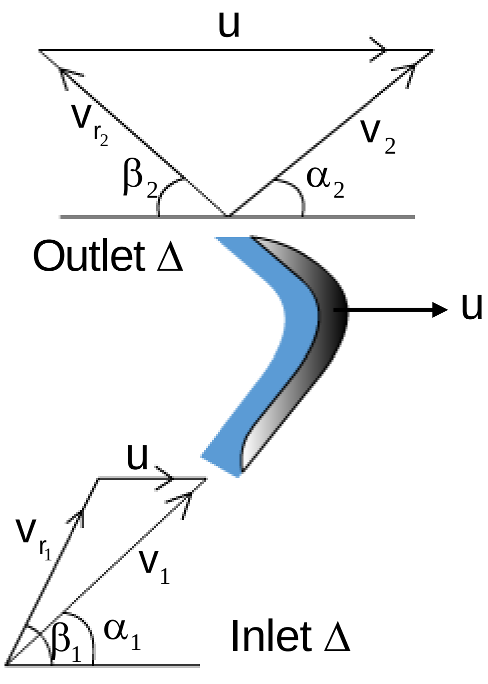
5. Force on Series of Blades (Blade Cascade)

The flow relative to the blade enters and leaves tangent to the profile of the blade. Constructing velocity triangles at inlet & outlet and applying momentum equation the force components in the x & y directions on the blade system can be obtained.





<u>X – direction</u>

$-F_x = \rho Q (V_{2x} - V_{1x})$

$Q = A_1 V_1 \qquad (a\ Series\ of\ blades)$

$-F_x = \rho A_1 V_1 (V_2 \cos\alpha_2 - V_1 \cos\alpha_1)$

<u>Y – direction</u>

$F_y = \rho A_1 V_1 (V_2 \sin\alpha_2 - V_1 \sin\alpha_1)$

Power developed

$P = F_x * u$

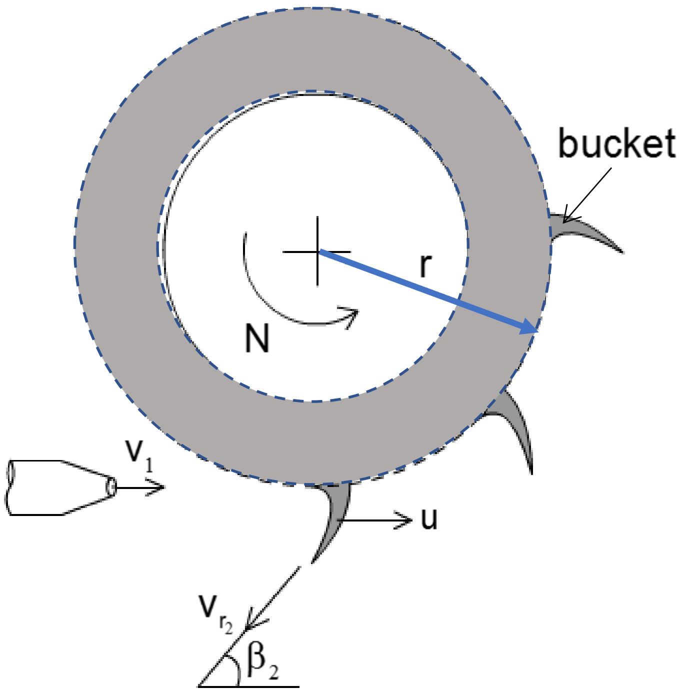
The power developed can also be calculate from.

$$P = \rho g Q \left(\frac{V_1^2 - V_2^2}{2g}\right)$$

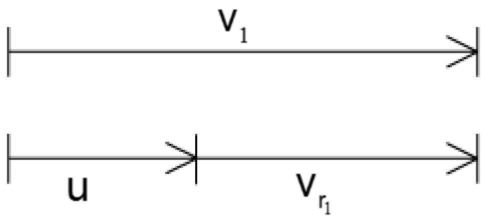
6. The Pelton Wheel (Impulse Turbine)

The velocity of the bucket

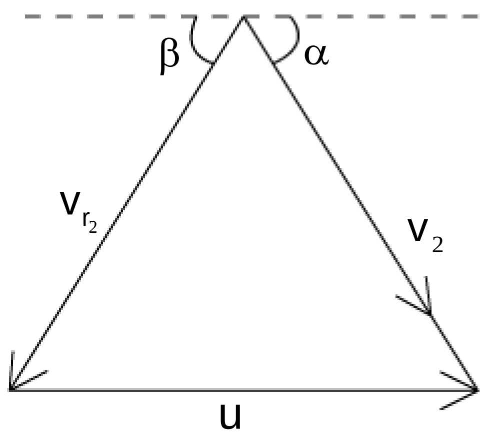
$$u = \omega . r = \frac{2\pi N}{60} . r$$



Velocity triangle at inlet



Velocity triangle at outlet



<u>**X – direction**</u>

$$-F_X = \rho Q (V_{2x} - V_{1x})$$

$$-F_X = \rho A_1 V_1 (V_2 \cos \alpha - V_1)$$

or

$$F_X = \rho A_1 V_1 (V_1 - V_2 \cos \alpha)$$

<u>From the velocity diagram at outlet.</u>

$$V_2 \cos \alpha = u - V_{r_2} \cos \beta$$

but $V_{r_2} = V_{r_1} = V_1 - u$

$$V_2 \cos\alpha = u - (V_1 - u)\cos\beta$$

hence

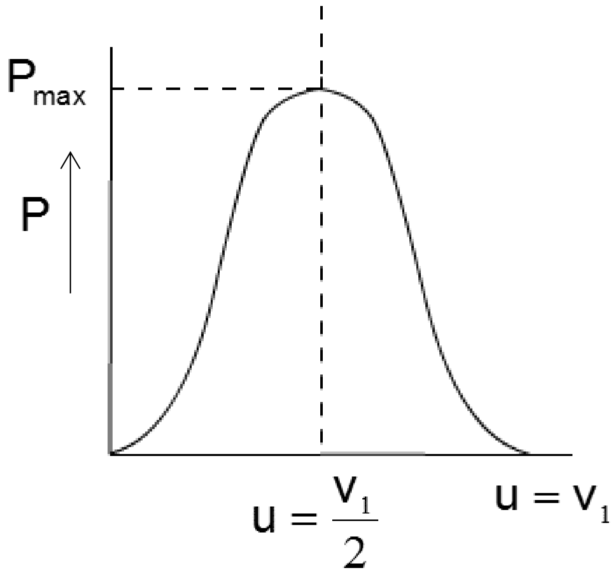
$$F_x = \rho A_1 V_1((V_1 - u) + (V_1 - u)\cos\beta)$$

$$F_x = \rho A_1 V_1 (V_1 - u)(1 + \cos\beta)$$

power developed

$$P = F_x * u$$

$$P = \rho A_1 V_1 (V_1 - u)(1 + \cos\beta) * u$$



For max power

$\frac{dp}{du} = 0$

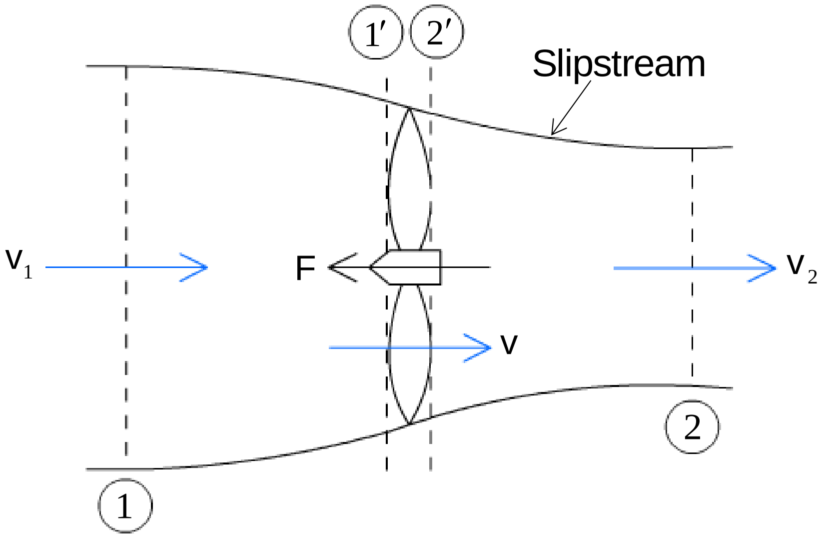
$$\frac{dp}{du} = \rho A_1 V_1 (1 + \cos\beta)(V_1 - 2u) = 0$$

or

$$V_1 - 2u = 0$$

$$u = \frac{1}{2} V_1$$

7. The Screw Propeller

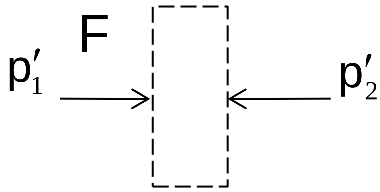


Consider a bulk of fluid within a slipstream accelerated by the screw propeller. The propeller moves forward with a velocity V_1, thus the fluid a head of the propeller at a section where it is not influenced by the propeller moves relative to the propeller with a velocity V_1 as shown. The fluid through the propeller moves with a velocity V. The propulsive force (Thrust) on the disk formed by the propeller blades is

$$F = \rho Q(V_2 - V_1)$$

The force F on the propeller can also be calculate from.

$$F = (p_2' - p_1')A$$



Thus

$$(p_2' - p_1')A = \rho Q(V_2 - V_1)$$

$$p_2' - p_1' = \frac{\rho Q}{A}(V_2 - V_1)$$

$$p_2' - p_1' = \rho V(V_2 - V_1) \quad \dots\dots\dots\dots\dots\dots (1)$$

Applying B.E. between sections 1 and 1´

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_1'}{\rho g} + \frac{V^2}{2g} \quad \dots\dots\dots\dots\dots\dots (2)$$

Applying B.E. between sections 2 and 2´

$$\frac{p_2}{\rho g}+\frac{V_2^2}{2g}=\frac{p_2^{'}}{\rho g}+\frac{V^2}{2g} \qquad \dots\dots\dots\dots\dots..(3)$$

Eq. (3) – Eq. (2)

$$\frac{p_2-p_1}{\rho g}+\frac{V_2^2-V_1^2}{2g}=\frac{p_2^{'}-p_1^{'}}{\rho g}$$

$p_2=p_1$ (Pressure of surrounding)

$$p_2^{'}-p_1^{'}=\frac{\rho}{2}(V_2^2-V_1^2)$$

Substituting in (1)

$$\frac{\rho}{2}(V_2^2-V_1^2)=\rho V(V_2-V_1)$$

$$\frac{(V_2+V_1)(V_2-V_1)}{2}=V(V_2-V_1)$$

$$V=\frac{(V_2+V_1)}{2}$$

The Useful power derived from the propeller.

$$P_{out}=F*V_1$$

$$=\rho Q(V_2-V_1)V_1$$

B.E. between (1) & (2)

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+E_{prop}=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}$$

$$E_{prop}=\frac{V_2^2-V_1^2}{2g}$$

$$Power\ input=\rho g Q.E_{prop}$$

$$P_{in} = \rho g A V \frac{V_2^2 - V_1^2}{2g}$$

$$P_{in} = \rho A V \left[\frac{(V_2 - V_1)(V_2 + V_1)}{2}\right]$$

But

$$F = \rho A V (V_2 - V_1)$$

Therefore

$$P_{in} = F * V$$

and

$$\eta_{prop} = \frac{P_{out}}{P_{in}} = \frac{V_1}{V}$$