زانكۆی سەلاحەدین- هەولێر

Salahaddin University-Erbil

The Zagreb indices of Zero-divisor Graph of the Ring $C(a, n)$ and Relation between Idempotent and Nilpotent Elements

Research Project

Submitted to the department of (Mathematics) in partial fulfillment of the requirements for the degree of BSc. in MATHEMATICS.

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Certification of the Supervisor

I certify that this work was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University- Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

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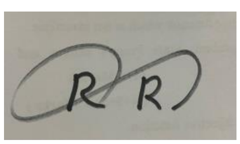
Signature: 

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Abstract

Let $C(a,n) = \{0, a, 2a, 3a, \dots, (t-1)a\}\, mod\, n$ be a commutative ring modulo n, where a, n are integers such that $a \neq 0\ and\ n > 1$and $d = gcd(a,n)$, $b = a|d$, $t = n|d$. The purpose of this work is to study the relation between the nilpotent and idempotent elements in some case $a = p$ and n =p^2, pq and pqr. Also we calculate the Zagreb indices of a zero-divisor graph of the Ring $C(a,n)$, Note that all figures are drawn via website http://graphonline.ru/en/

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INTRODUCTION

Let $C(a,n) = \{0, a, 2a, 3a, \dots, (t-1)a\}\ mod\ n$ be a commutative ring modulo n, where a, n are integers such that $a \neq 0\ and\ n > 1$and $d = gcd(a,n)$, $b = a|d$, $t = n|d$. Z($C(a,n)$) the set of nonzero zero divisors of $C(a,n)$. The zero-divisor graph of $C(a,n)$, denoted by Γ($C(a,n)$), is the simple graph with vertex set Z($C(a,n)$), and for distinct a , b ∈ Z($C(a,n)$), a and b are adjacent if and only if a b = 0.

The Zagreb indices of zero-divisor graph of the ring of integers modulo n of a simple undirected graph, denoted by M_1 (Γ) and M_2 (Γ), and defined by

M_1 (Γ)=$\sum_{v\in(v(\Gamma)}$ $(\deg(v))^2$ and M_2 (Γ)= $\sum_{uv\in E(\Gamma)}$ (deg u)×(deg v) (I.Gustman and N.Trinjajstic 1972), Where deg(v) denote the degree of a vertex v in a graph of Γ.

This thesis is divided into two chapters. The first chapter contains some basic definitions, which we need in the present work and some examples to illustrating the definitions.

The second chapter, consists two sections, in the first section we compare the relation between nilpotent and idempotent elements in the ring $C(a,n)$ in case $a = p$ and n =p^2 , $pq\ and\ pqr$. The second section includes the definition of Zagreb indices of a graph of the Ring $C(a,n)$, we find the zero divisors of the ring $C(a,n)$, where a=p and n =p^2, also we draw the graph of some rings . Finally we calculate Zagreb indices of the ring $a = p$ and $n = p^2$, in this case all graphs are complete graphs and determined the Zagreb indices of the ring $a = p$ and $n = pqr$, in this case the graphs are not complete.

CHAPTER ONE

LITERATURE REVIEW

In this chapter some basic definitions that we need in our work and examples to illustrating this definition.

Definition 1.1: (Marlo Anderson and Todd Feil 2015)

A ring R is a nonempty set together with two binary operation + and .(called addition and multiplication defined on R) if satisfying the following axioms:

I. $(R, +)$ is an abelian group,

II. $(R, .)$ is semi-group,

III. the distributive law hold in R:

$for\ all\ a, b, c \in R, a.(b + c) = a \,.\, b + a.\, c$ and $(a + b).\, c = a.\, c + b$.

Example: (Z , ⨁ , ʘ) is a ring,

Definition1.2: (Marlo Anderson and Todd Feil 2015)

A nonzero element a in a ring R is called a zero divisor if there exists $b \in R$ such that $b \neq 0$ and $ab = 0$. In particular, a is a left divisor of zero and b is a right divisor of zero .

Example: In the ring Z_6={0,1,2,3,4,5}

Since $2.3 = 6 = 0 \quad 3.4 = 12 = 0$, then 2,3,4 are zero divisors of Z_6

Definition1.3: (Burton 1980) Prime numbers are numbers greater than 1, they only have two factors 1 and the numbers cannot be divide by any number other than 1.

Example: The number 2,3,5,7,11,13,17,… are prime numbers

Definition 1.4: (Behzad & CHartrand, 1979) A graphs is a finite non empty set of objects called vertices (the singular word is vertex) together with a (Possibly empty) set of un order pairs of distinct vertices of called edges.

Definition1.5: (Naduvath 2017): The order of a graph G, denoted by V(G), is the number of its vertices and the size of G, denoted by E(G), is the number of its edges. A graph with p-vertices and q-edges is called a (p,q)-graph.

Definition1.6 : (Naduvath 2017) An edge of a graph that joins a node to itself is called loop or a self-loop. That is, a loop is an edge uv, where u = v

Definition1.7: (Naduvath 2017) The edges connecting the same pair of vertices are called multiple edges or parallel edges.

Definition 1.8: (Naduvath 2017):A graph G which does not have loops or parallel edges is called a simple graph. A graph which is not simple is generally called a multigraph.

Definition 1.9: (Naduvath 2017): The number of edges incident on a vertex v, with self-loops counted twice, is called the degree of the vertex v and is denoted by degG(v) or deg(v) or simply d(v).

Definition 1.10: (Burton 1980) An integer b said to be divisible by an integer $a \neq 0$, in symbols $a|$b, if there exists some integer c such that $b = ac$.

We write $a \nmid b$ to indicate that b is not divisible by a .

Definition 1.11: (Marlo Anderson and Todd Feil 2015) An element $x \in R$ is a nilpotent if $x^n = 0$, and the element $x \in R$ is idempotent if $x^2 = x$

CHAPTER TWO

This chapter includes two sections, in the first section we compare the relation between nilpotent and idempotent elements in the ring $C(a,n)$ in case $a = p$ and n =p^2 , pq and pqr. The second section includes the definition of Zagreb indices of a graph of the Ring $C(a,n)$ we find the zero divisors of the ring $C(a,n)$, where $a = p$ and $n = p^2$, also we draw the graph of some rings. Finally we calculate Zagreb indices this rings, and determined the Zagreb indices of the ring $a = p$ and $n = pqr$, in this case the graphs are not complete.

2.1 The compare between the nilpotent and idempotent elements in The ring C(a,n)

Case 1. If $\boldsymbol{a = p \ and \, n = p^2}$

1) In the ring $C(3,9) =, gcd(3,9) = 3 = d \ \ and \ \ t = \frac{9}{3} = 3$

$C(3,9) = \{\ 0,3,6\} mod\ 9$

The set of nilpotent={0,3,6}

The set of idempotent={0}

Then $\{0\} \subset \{0,3,6\}$

2) $In\ the\ ring\ C(5,25), gcd(5,25) = 5 = d\ and\ \ t = \frac{25}{5} = 5$

$C(5,25) = \{0,5,10,15,20\}\ mod\ 25$

The set of nilpotent={0,5,10,15,20}

The set of idempotent={0}

Then $\{0\} \subset \{0,5,10,15,20\}$

3) $In\ the\ ring\ C(7,49), gcd(7,49) = 7 = d\ and\ t = \frac{49}{7} = 7$

$C(7,49) = \{0,7,14,21,28,35,42\}\ mod 49$

The set of nilpotent={0,7,14,21,28,35,42}

The set of idempotent={0}

4) $In\ thr\ ring\ C(11,121), gcd(11,121) = 11 = d\ and\ t = \frac{121}{11} = 11$

$C(11,121) = \{0,11,22,33,44,55,66,77,88,99,110\}\ mod 121$

The set of nilpotent={0,11,22,33,44,55,66,77,88,99,110 }

The set of idempotent={0}

5) $In\ the\ ring\ C(13,169), gcd(13,169) = 13 = d\ and\ t = \frac{169}{13} = 13$

$C(7,49) = \{0,13,26,39,52,65,78,91,104,117,130,143,156\}\ mod 169$

The set of nilpotent={ 0,13,26,39,52,65,78,91,104,117,130,143,156 }

The set of idempotent={0}

Note that: In this case the set of idempotent of each rings is equal to zero.

Case 2. If $\boldsymbol{a = p\ and\ n = pq}$

a) If p=2

1) $In\ the\ ring\ C(2,6), \text{gcd}(2,6) = 2 = d\ and\ t = \frac{6}{2} = 3$

$C(2,6) = \{0,2,4\} mod\ 6$

The set of nilpotent={0}

The set of idempotent={0}

2) $In\ the\ ring\ C(2,10),\ \text{gcd}(2,10) = 2 = d\ and\ t = \frac{10}{2} = 5$

$C(2,10) = \{0,2,4,6,8\} mod 10$

The set of nilpotent={0}

The set of idempotent={0,6}

3) $In\ the\ ring\ C(2,14), gc\,d(2,14) = 2 = d\ and\ t = \frac{14}{2} = 7$

$C(2,10) = \{0,2,4,6,8,10,12\} mod 14$

The set of nilpotent={0}

The set of idempotent={0,8}

4) $In\ the\ ring\ C(2,22), gc\,d(2,22) = 2 = d\ and\ t = \frac{22}{2} = 7$

$C(2,22) = \{0,2,4,6,8,10,12,14,16,18,20\}mod22$
The set of nilpotent={0,6,8,10,12,14,16,18,20}
The set of idempotent={0,12}

b) If p=3

5) $In\ the\ ring\ C(3,15), \text{gcd}(3,15) = 3 = d\ and\ t = \frac{15}{3} = 5$

$C(3,15) = \{0,3,6,9,12\}mod15$
The set of nilpotent={0}
The set of idempotent={0,6}

6) $In\ the\ ring\ C(3,21), \text{gcd}(3,21) = 3 = d\ and\ t = \frac{21}{3} = 7$

$C(3,21) = \{0,3,6,9,12,15,18\}mod21$
The set of nilpotent ={0,9,12,15,18}
The set of idempotent={0,15}

7) In the ring $C(3,33), \text{gcd}(3,33) = 3 = d\ \ and\ t = \frac{33}{3} = 11$

$C(3,33) = \{0,3,6,9,12,15,18,21,24,27,30\}mod33$
The set of nilpotent={0,6,9,12,15,18,21,24,27,30}
The set of idempotent={0,12}

8) $In\ the\ ring\ C(3,39), \text{gcd}(3,39) = 3 = d\ and\ t = \frac{39}{3} = 13$

$C(3,39) = \{0,3,6,9,12,15,18,21,24,27,30,33,36\}mod39$
The set of nilpotent={0,3,6,9,12,15,18,21,24,27,30,36}
The set of idempotent={0,27}

c) If p=5

9) $in\ the\ ring\ C(5,10), gcd(5,10) = 5 = d\ and\ t = \frac{10}{5} = 2$

$C(5,10) = \{0,5\}mod10$
$the\ set\ of\ nilpotent = \{0\}$
$the\ set\ of\ idempotent = \{0,5\}$

10) In the ring $C(5,15), gc\,d(5,15) = 5 = d\ and\ t = \frac{15}{5} = 3$

$C(5,15) = \{0,5,10\}mod15$
$the\ set\ of nilpotent = \{0\}$

$The\ set\ of\ idempotent = \{0,10\}$

11) $In\ the\ ring\ C(5,35), \text{gcd}(5,35) = 5 = d\ and\ t = \frac{35}{5} = 7$

$C(5,35) = \{0,5,10,15,20,25,30\} mod 35$
The set of nilpotent={0,5,10,15,20,25,30}
The set of idempotent={0,15}

12) In the ring C, $\gcd(5,55) = 5 = d$ and $t = \frac{35}{5} = 11$
$C(5,55) = \{0,5,10,15,20,25,30,35,40,45,50\} mod 55$
The set of nilpotent={0,5,10,15,20,25,30,35,40,45,50}
The set of idempotent={0,45}

13) $In\ the\ ring\ C(5,65), \gcd(5,65) = 5 = d\ \ and\ t = \frac{65}{5} = 13$
$C(5,35) = \{0,5,10,15,20,25,30,35,40,45,50,55,60\} mod 65$
The set of nilpotent={0,5,10,15,20,25,30,35,40,45,50,55,60}
The set of idempotent={0,40}

Note that: In this case if q $>$ 5 the set of idempotent is a subset of the set of nilpotent.

Case 3. If $a = p$ and $n = pqr$

1) $in\ the\ ring\ C(2,30), \gcd(2,30) = 2 = d\ \ and\ t = \frac{30}{2} = 15$
$C(2,30) = \{0,2,4,6,8,10,12,14,16,18,20,22,24,26,28\} mod 30$
The set of nilpotent={0,6,8,10,12,14,16,18,20}
The set of idempotent={0,6,10,16}
2) $In\ the\ ring\ C(3,30), \gcd(3,30) = 3 = d\ \ and\ t = \frac{30}{3} = 10$
$C(3,30) = \{0,3,6,9,12,15,18,21,24,27\} mod 30$
The set of nilpotent= {0,6,9,12,15,18,21,24,27}
The set of idempotent={0,6,15,21}
3) $In\ the\ ring\ C(5,30), \gcd(3,30) = 5 = d\ \ and\ t = \frac{30}{5} = 6$
$C(5,30) = \{0,5,10,15,20,25\} mod 30$
The set of nilpotent={0,5,10,15,20,25 }
The set of idempotent={0,10,15,25}
4) $in\ the\ ring\ C(3,105), gcd(3,105) = 3 = d\ and\ t = \frac{105}{3} = 35$
$C(3,105) =$
$\left\{\begin{matrix} 0,3,6,9,12,15,18,21,24,27,30,33,36,39,42,45,48,51,54,57,60,63,66,69, \\ 72,75,78,81,84,87,90,93,96,99,102 \end{matrix}\right\} mod 105$
The set of nilpotent=
$\left\{\begin{matrix} 0,3,6,12,15,18,21,24,27,30,33,42,45,48,51,54,57,60,66,69,75,81,84,87, \\ 90,93,96,99,102 \end{matrix}\right\}$

The set of idempotent={0,15,21}

5) $In\ the\ ring\ C(5{,}105),\ \gcd(5{,}105) = 5 = d\ \ and\ t = \frac{105}{5} = 21$

$C(5{,}105)$
$= \{0{,}5{,}10{,}15{,}20{,}25{,}30{,}35{,}40{,}45{,}50{,}55{,}60{,}65{,}70{,}75{,}80{,}85{,}90{,}95{,}100\} mod 105$

6) $in\ the\ ring\ C(7{,}105), gcd(7{,}105) = 7 = d\ and\ t = \frac{105}{7} = 15$

$C(7{,}105) = \{0{,}7{,}14{,}21{,}27{,}35{,}42{,}49{,}56{,}63{,}70{,}77{,}84{,}91{,}98\} mod 105$

The set of nilpotent={0,7,14,21,35,42,49,56,63,70,77,84,91,98}

The set of idempotent={0,21,70,91}

Note that: In this section the set of idempotent is a subset of the set of nilpotent.

2.2 The Zagreb indices of a graph of the Ring C(a,n)

Case 1. The Zagreb indices of a graph of the Ring C(a,n) , where $a = p$ and $n = p^2$, p is prime

Definition: (I.Gustman and N.Trinjajstic 1972)

The Zagreb indices of a graph Γ, denoted by M_1 (Γ) and M_2 (Γ), and defined by M_1 (Γ)=$\sum_{v\in(v(\Gamma)}$ $(\deg(v))^2$ and M_2 (Γ)= $\sum_{uv\in E(\Gamma)}$ (deg u)×(deg v) (I.Gustman and N.Trinjajstic 1972).

In this section we use some rings C(a,n), determined the zero divisors, draw the graph of this rings, also we calculate Zagreb indices of this graphs.

1) C(3,9)= {0,3,6} $mod\ \ 9$

The set of zero divisors of C(3,9) is { 3,6 } = V(C(3,9)), where V(C(3,9)) the set of vertex of the graph of C(3,9).

Figure 2. 1 The graph of the ring C(3,9).

The degree of the vertex of C(3,9) are $d(3) = 1$, $d(6) = 1$ $d(3,6) = 1$

The Zagreb indices of the graph of C(3,9)

$1 - M_1$ $(\Gamma(C(3,9)))=\sum_{v\in(v(\Gamma(C(3,9)))}$ $(\deg(v))^2$

$=(1)^2 + (1)^2$

$= 2$

$2 - M_2$ $(\Gamma(C(3,9))= \sum_{uv\in E(\Gamma(C(3,9))}$ $(\deg u)\times(\deg v)$

$= d(3).d(6)$

$= (1 \times 1)$

$= 1$

2) C(5,25)= $\{0,5,10,15,20\} mod 25$

The set of zero divisors of C(5,25) is $=\{5,10,15,20\}$

= V(C(5,25)) where V(C(5,25))the set of vertex of the graph of C(5,25).

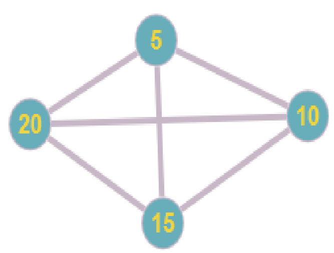

Figure 2. 2 The graph of the ring C(5,25).

The degree of the vertex of C(5,25) are
$d(5) = 3$, $d(10) = 3$, $d(15) = 3$, $d(20) = 3$,

The Zagreb indices of the graph of C(5,25)

$1 - M_1$ $(\Gamma(C(5,25)))=\sum_{v\in(v(\Gamma(C(5,25)))}$ $(\deg(v))^2$

$=(3)^2 + (3)^2 + (3)^2 + (3)^2$

= 9+9+9+9

=36

$2 - M_2$ $(\Gamma(C(5,25)) = \sum_{uv\in E(\Gamma(C(5,25))}$ (deg u)×(deg v)

$$= d(5).d(10) + d(5).d(15) + d(5).d(20) + d(10).d(15) + d(10).d(20) + d(15).d(20)$$
$$= 3.3 + 3.3 + 3.3 + 3.3 + 3.3 + 3.3 = 54$$

3) $C(7,49) = \{0,7,14,21,28,35,42\}\ mod 49$

The set of zero divisors of C(7,49) is $=\{7,14,21,28,35,42\}$

= V(C(7,49)) where V(C(7,49))the set of vertex of the graph of C(7,49).

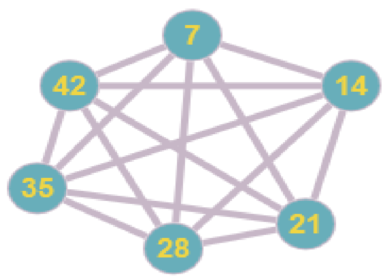

Figure 2. 3The graph of the ring C(7,49).

The degree of the vertex of C(7,49) are
$d(7) = d(14) = d(21) = d(28) = d(35) = d(42) = 5$

The Zagreb indices of the graph of C(7,49)

$1 - M_1$ $(\Gamma(C(7,49)))=\sum_{v\in(v(\Gamma(C(7,49)))}$ $(\deg(v))^2$

$$=(5)^2 + (5)^2 + (5)^2 + (5)^2 + (5)^2 + (5)^2$$

$$= 25+25+25+25+25+25$$

$$=150$$

$2 - M_2$ $(\Gamma(C(7,49)) = \sum_{uv\in E(\Gamma(C(7,49))}$ (deg u)×(deg v)

$$= d(7).d(14) + d(7).d(21) + d(7).d(28) + d(7).d(35) + d(7).d(42) + d(14).d(21) + d(14).d(28) + d(14).d(35) + d(14).d(42) + d(21).d(28) + d(21).d(35) + d(21).d(42) + d(28).d(35) + d(28).d(42) + d(35).d(42)$$
$$= 5.5 + 5.5 + 5.5 + 5.5 + 5.5 + 5.5 + 5.5 + 5.5 + 5.5 + 5.5 + 5.5 + 5.5 + 5.5 + 5.5 + 5.5$$

$= 15\ (25)$
$= 375$

4) $C(11{,}121) = \{0{,}11{,}22{,}33{,}44{,}55{,}66{,}77{,}88{,}99{,}110\}\ mod 121$

The set of zero divisors of C(11,121) is $=\{11{,}22{,}33{,}44{,}55{,}66{,}77{,}88{,}99{,}110\}$

= V(C(11,121)) where V(C(11,121))the set of vertex of the graph of C(11,121).

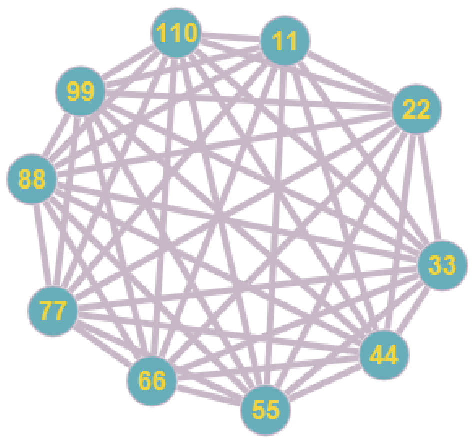

Figure 2. 4 The graph of the ring C(11,121).

The degree of the vertex of C(11,121) are

$$d(11) = d(22) = d(33) = d(44) = d(55) = d(66) = d(77) = d(88) = d(99) = d(110) = 9$$

The Zagreb indices of the graph of C(11,121)

$1 - M_1\ (\Gamma(C(11{,}121)))=\sum_{v\in(v(\Gamma(C(11,121)))}\ (\deg(v))^2$

$$=(9)^2 + (9)^2 + (9)^2 + (9)^2 + (9)^2 + (9)^2 + (9)^2 + (9)^2 + (9)^2 + (9)^2$$

$$= 10(81)$$

$$= 81$$

$2 - M_2\ (\Gamma(C(11{,}121))= \sum_{uv\in E(\Gamma(C(11,121))}\ (\deg u)\times(\deg v)$

$$
\begin{aligned}
&= d(11).d(22) + d(11).d(33) + d(11).d(44) + d(11).d(55) \\
&\quad + d(11).d(66) + d(11).d(77) + d(11).d(88) + d(11).d(99) \\
&\quad + d(11).d(110) + d(22).d(33) + d(22).d(44) + d(22).d(55) \\
&\quad + d(22).d(66) + d(22).d(77) + d(22).d(88) + d(22).d(99) \\
&\quad + d(22).d(110) + d(33).d(44) + d(33).d(55) + d(33).d(66) \\
&\quad + d(33).d(77) + d(33).d(88) + d(33).d(99) \\
&\quad + d(33).d(110) + d(44).d(55) + d(44).d(66) \\
&\quad + d(44).d(77) + d(44).d(88) + d(44).d(99) + d(44).d(110) \\
&\quad + d(55).d(66) + d(55).d(77) + d(55).d(88) + d(55).d(99) \\
&\quad + d(55).d(110) + d(66).d(77) + d(66).d(88) \\
&\quad + d(66).d(99) + d(66).d(110) + d(77).d(88) + d(77).d(99) \\
&\quad + d(77).d(110) + d(88).d(99) + d(88).d(110) \\
&\quad + d(99).d(110) \\
&\quad = 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 \\
&\quad + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 \\
&\quad + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 \\
&\quad + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 + 9.9 = 3645
\end{aligned}
$$

5) $C(13,169) = \{0,13,26,39,52,65,78,91,104,117,130,143,156\}\, mod169$

The set of zero divisors of C(13,169) is $=\{11,22,33,44,55,66,77,88,99,110\}$

= V(C(13,169)) where V(C(13,169))the set of vertex of the graph of C(13,169).

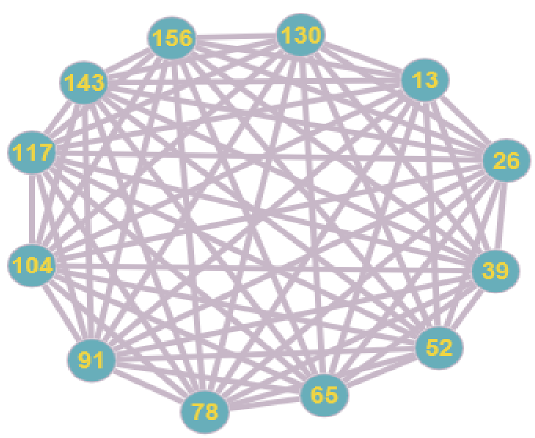

Figure 2. 5 The graph of the ring C(13,169)

The degree of the vertex of C(13,169) are

$$
\begin{aligned}
&d(13) = d(26) = d(39) = d(52) = d(65) = d(78) = d(91) = d(104) \\
&\quad = d(117) = d(130) = d(143) = d(156) = 10
\end{aligned}
$$

The Zagreb indices of the graph of C(13,169)

$1 - \mathrm{M}_1$ $(\Gamma(\mathrm{C}(13,169)))=\sum_{\mathrm{v}\in(\mathrm{v}(\Gamma(\mathrm{C}(13,169)))} (\deg(\mathrm{v}))^2$

$$= d(13)^2 + d(26)^2 + d(39)^2 + d(52)^2 + d(65)^2 + d(78)^2 + d(91)^2 + d(104)^2 + d(117)^2 + d(130)^2 + d(143)^2 + d(156)^2 = 1200$$

$=(10)^2 + (10)^2 + (10)^2 + (10)^2 + (10)^2 + (10)^2 + (10)^2 + (10)^2 + (10)^2 + (10)^2 + (10)^2 + (10)^2$

= 12(100)

= 1200

$2 - \mathrm{M}_2$ $(\Gamma(\mathrm{C}(13,169))= \sum_{\mathrm{uv}\in\mathrm{E}(\Gamma(\mathrm{C}(13,169))} (\deg \mathrm{u})\times(\deg \mathrm{v})$

$$
\begin{aligned}
M_2 = {} & d(13).d(26) + d(13).d(39) + d(13).d(52) + d(13).d(65) \\
& + d(13).d(78) + d(13).d(91) + d(13).d(104) \\
& + d(13).d(117) + d(13).d(130) + d(13).d(143) \\
& + d(13).d(156) + d(26).d(39) + d(26).d(52) + d(26).d(65) \\
& + d(26).d(78) + d(26).d(91) + d(26).d(104) \\
& + d(26).d(117) + d(26).d(130) + d(26).d(143) \\
& + d(26).d(156) + d(39).d(52) + d(39).d(65) \\
& + d(39).d(78) + d(39).d(91) + d(39).d(104) \\
& + d(39).d(117) + d(39).d(130) + d(39).d(143) \\
& + d(39).d(156) + d(52).d(65) + d(52).d(78) + d(52).d(91) \\
& + d(52).d(104) + d(52).d(117) + d(52).d(130) \\
& + d(52).d(143) + d(52).d(156) + d(65).d(78) \\
& + d(65).d(91) + d(65).d(104) + d(65).d(117) \\
& + d(65).d(130) + d(65).d(143) + d(65).d(156) \\
& + d(78).d(91) + d(78).d(104) + d(78).d(117) \\
& + d(78).d(130) + d(78).d(143) + d(78).d(156) \\
& + d(91).d(104) + d(91).d(117) + d(91).d(130) \\
& + d(91).d(143) + d(91).d(156) + d(104).d(117) \\
& + d(104).d(130) + d(104).d(143) + d(104).d(156) \\
& + d(117).d(130) + d(117).d(143) + d(117).d(156) \\
& + d(130).d(143) + d(130).d(156) + d(143).d(156) = 6600
\end{aligned}
$$

Note that: In this section in the ring C(p,p^2)

1) every element is zero divisor expect zero element.

2) Every graph is a complete graph**.**

Case 2. The Zagreb indices of a graph of the Ring $\boldsymbol{C(a,n)}$, where $\boldsymbol{a=p}$ and $\boldsymbol{n=pqr}$**, p is prime**

1) C(2,30)= {0,2,4,6,8,10,12,14,16,18,20,22,24,26,28} $mod 30$

The set of zero divisors of C(2,30) is ={6,10,12,18,20,24}

= V(C(2,30)) where V(C(2,30)))the set of vertex of the graph of C(2,30).

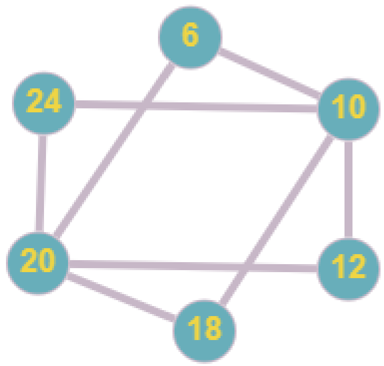

Figure 2. 6 The graph of the ring C(2,30).

The degree of the vertex of C(2,30) are

$$d(6) = 2, d(10) = 4, d(12) = 2, d(18) = 2, d(20) = 4, d(24) = 2$$

The Zagreb indices of the graph of C(2,30)

$$M_1 = d(6)^2 + d(10)^2 + d(12)^2 + d(18)^2 + d(20)^2 + d(24)^2$$

$$= 2^2 + 4^2 + 2^2 + 2^2 + 4^2 + 2^2$$

$$= 48$$

$$M_2 = d(6).d(10) + d(6).d(20) + d(10).d(12) + d(10).d(18) + d(10).d(24) + d(12).d(20) + d(18).d(20) + d(20).d(24)$$

$$= 2.4 + 2.4 + 4.2 + 4.2 + 4.2 + 2.4 + 2.4 + 4.2$$

$$= 64$$

2) $C(3,30)$= {0,3,6,9,12,15,18,21,24,27} $mod 30$

The set of zero divisors of C(3,30) is ={6,12,15,18,24}

= V(C(3,30)) where V(C(3,30)))the set of vertex of the graph of C(3,30).

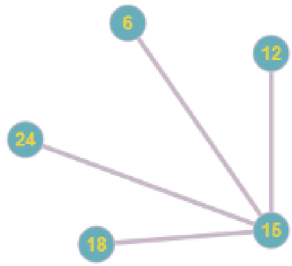

Figure 2. 7 The graph of the ring C(3,30).

The degree of the vertex of C(3,30) are

$$d(6) = \mathrm{d}(12) = \mathrm{d}(18) = \mathrm{d}(24) = 1, \mathrm{d}(15) = 4$$

The Zagreb indices of the graph of C(2,30)

$$M_1 = d(6)^2 + \mathrm{d}(12)^2 + \mathrm{d}(18)^2 + \mathrm{d}(24)^2 + \mathrm{d}(15)^2 = 20$$

$$M_2 = d(6).d(16) + d(12).d(15) + d(15).d(18) + d(15).d(24) = 16$$

3) $\boldsymbol{C(5,30)} = \{0,5,10,15,20,25\}\, mod30$

The set of zero divisors of C(5,30) is ={10,15,20}

= V(C(5,30)) where V(C(5,30)))the set of vertex of the graph of C(5,30).

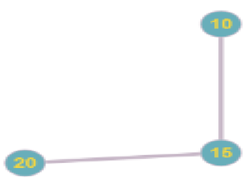

Figure 2. 8 The graph of the ring C(5,30).

The degree of the vertex of C(5,30) are

$$d(10) = 1, d(15) = 2, d(20) = 1$$

The Zagreb indices of the graph of C(5,30)

$$M_1 = d(10)^2 + d(15)^{2)} + d(20)^2 = 6$$

$$M_2 = d(10).d(15) + d(15).d(20) = 4$$

4) $C(3,105) =$

$$\left\{\begin{matrix} 0,3,6,9,12,15,18,21,24,27,30,33,36,39,42,45,48,51,54,57,60,63,66,69, \\ 72,75,78,81,84,87,90,93,96,99,102 \end{matrix}\right\} mod 105$$

The set of zero divisors of C(3,105) is ={15,21,30,42,45,60,63,75,84,90}

= V(C(3,105)) where V(C(3,105)))the set of vertex of the graph of C(3,105).

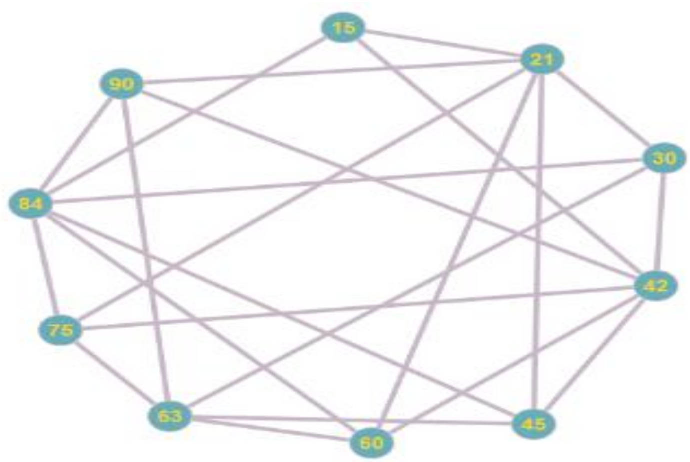

Figure 2. 9 The graph of the ring C(3,105).

The degree of the vertex of C(3,105) are

$$(15) = d(30) = d(45) = d(60) = d(75) = d(90) = 4, d(21) = d(42) = d(63) = d(84) = 6$$

The Zagreb indices of the graph of C(3,105)

$$M_1 = d(15)^2 + d(30)^2 + d(45)^2 + d(60)^2 + d(75)^2 + d(90)^2 + d(21)^2 + d(42)^2 + d(63)^2 + d(84)^2 = 240$$

$$M_2 = d(15).d(21) + d(15).d(42) + d(15).d(63) + d(15).d(84) + d(21).d(30) + d(21).d(45) + d(21).d(60) + d(21).d(75) + d(21).d(90) + d(30).d(42) + d(30).d(63) + d(30).d(84) + d(42).d(45) + d(42).d(60) + d(42).d(75) + d(42).d(90) + d(45).d(63) + d(45).d(84) + d(60).d(63) + d(60).d(84) + d(63).d(75) + d(63).d(90) + d(75).d(84) + d(84).d(90) = 24.(4.6) = 576$$

5)

C(5,105)=

$\{0,5,10,15,20,25,30,35,40,45,50,55,60,65,70,75,80,85,90,95,100\}mod105$

The set of zero divisors of C(5,105) is $=\{15,30,36,45,60,70,75,90\}$

= V(C(5,105)) where V(C(5,105)))the set of vertex of the graph of C(5,105).

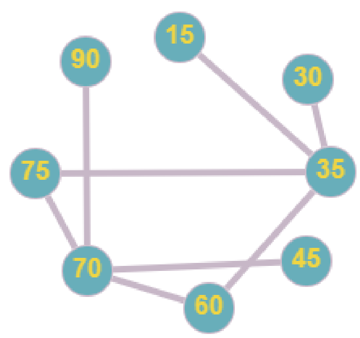

Figure 2. 10 The graph of the ring C(5,105).

The degree of the vertex of C(5,105) are

$d(15) = d(30) = d(45) = d(90) = 1, d(35) = d(70) = 4, d(60) = d(75) = 2$

The Zagreb indices of the graph of C(5,105)

$$M_1 = d(15)^2 + d(30)^2 + d(35)^2 + d(45)^2 + d(60)^2 + d(70)^2 + d(75)^2 + d(90)^2 = 44$$

$$M_2 = d(15).d(35) + d(30).d(35) + d(35).d(60) + d(35).d(75) + d(45).d(70) + d(60).d(70) + d(70).d(75) + d(70).d(90) = 48$$

6) C(7,105)= $\{0,7,14,21,27,35,42,49,56,63,70,77,84,91,98\}mod105$

The set of zero divisors of C(7,105) is $=\{21,36,42,63,70,84\}$

= V(C(7,105)) where V(C(7,105)))the set of vertex of the graph of C(7,105).

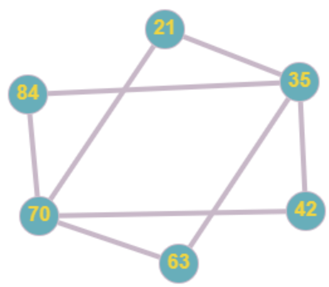

Figure 2. 11 The graph of the ring C(7,105)

The degree of the vertex of C(7,105) are

$$d(21) = 2, d(35) = 4, d(42) = 2, d(63) = 2,, d(70) = 4, d(84) = 2$$

The Zagreb indices of the graph of C(7,105)

$$M_1 = d(21)^2 + d(35)^2 + d(42)^2 + d(63)^2 + d(70)^2 + d(84)^2 = 48$$

$$M_2 = d(21).d(35) + d(21).d(70) + d(35).d(42) + d(35).d(63) + d(35).d(84) + d(42).d(70) + d(63).d(70) + d(70).d(84) = 64$$

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پوختە

ئەڵقەی ئاڵوگۆڕکراو بە پێوەری n $C(a,n)=\{0,a,2a,3a,\dots,(t-1)a\}mod\ n$

کاتێک a,n ژمارەیەکی تەواون وەک$0 \neq a$ و $n > 1$ وە

$$d = gcd(a,n)\ , t = n|d\ , b = a|d$$

ئیشەکامان لێکۆڵینەوەیە لە پەیوەندی نێوان **nilpotent** و **idempotent** لە هەندێک حاڵەتدا $a = p$ وە $n = p^2, p.q, p.q.r$ هەروەها پێوەرەکانی زاگرۆب لە گرافێکی دابەشکەری سفری ئەڵقەی$C(a,n)$ هەژمار دەکەین

تێبینی: هەموو گرافەکان لە ڕێگەی ئەم ماڵپەڕەوە کێشراون