زانكۆی سەلاحەدین- هەولێر

Salahaddin-University-Erbil

The Zagreb indices of Zero-divisor Graph of the Ring of Integers Modulo n

Research Project

Submitted to the department of (Mathematics) in partial fulfillment of the requirements for the degree of BSc. in (Mathematics)

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Certification of the Supervisor

I certify that this work was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University- Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

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Date: / /2023

In view of available recommendations, I forward this word for debate by the examining committee.

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Finally, I would like to state my thanks to all my teachers for helping me during difficult times.

Abstract

Let Z_n be the ring of integers modulo n. The purpose of this work is to study some zero-divisor graph of Z_n. If n is a prime number, then Z_n has no zero-divisors; so $\Gamma(Z_n)$ is the null graph. Hence in this work, we only consider the case that n is a composite. We calculate the Zagreb indices of $\Gamma(Z_n)$. Note that all figures are drawn via website http://graphonline.ru/en/

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Introduction

Let Z_n be the ring of integers modulo n and $Z(Z_n)$ the set of nonzero zero divisors of Z_n. The zero-divisor graph of Z_n, denoted by $\Gamma(Z_n)$, is the simple graph with vertex set $Z(Z_n)$, and for distinct a , b ∈ $Z(Z_n)$, a and b are adjacent if and only if a b = 0. Clearly, $\Gamma(Z_n)$ is the null graph if and only if Z_n is an integral domain, the vertex and edge-sets of the graph Z_n are represented by $V(Z_n)$ and $E(Z_n)$, respectively

The Zagreb indices of zero-divisor graph of the ring of integers modulo n of a simple undirected graph, denoted by M_1 (Γ) and M_2 (Γ), and defined by

$M_1(\Gamma)=\sum_{v\in(v(\Gamma)}(\deg(v))^2$ and $M_2(\Gamma)=\sum_{uv\in E(\Gamma)}(\deg u)\times(\deg v)$ **Invalid source specified.**, Where deg(v) denote the degree of a vertex v in a graph of Z_n.

This thesis is divided into two chapters. The first chapter contains some basic definitions, which we need in the present work and some examples to illustrating the definitions.

The second chapter, includes the definition of Zagreb indices and we find the zero divisors of the ring of integers modulo n, where n is composite, also we draw the graph of some rings and determined the degree of each vertices of this graphs. Finally we calculate Zagreb indices of each graphs of a ring of integers modulo n, n from 6 to 30.

CHAPTER ONE

Background Materials

In this chapter some basic definitions that we need in our work and examples to illustrating this definition.

Definition 1.1: Invalid source specified.

A group G consists of a set G together with a binary operation $*$ for which the following properties are satisfied:

(I) $(x * y) * z = x * (y * z)$ for all elements x, y and z of G (the Associative Law) .

(II) there exists an element e of G (known as the identity element of G) such that

(III) $e * x = x = x * e$, for all elements x of G .

(IV) for each element x of G there exists an element x' of G (known as the inverse of x) such that $x * x' = e = x' * x$ (where e is the identity element of G) .

Example:

1- $(Z, +)$, $(R, +)$, $(R - \{0\}, .)$ and $(Mn{\times}n(R), +)$ are groups.

Definition 1.2: Invalid source specified.

A ring R is a nonempty set together with two binary operation + and .(called addition and multiplication defined on R) if satisfying the following axioms:

I. $(R, +)$ is an abelian group,

II. $(R, .)$ is semi-group,

III. the distributive law hold in R:

$for\ all\ a, b, c \in R, a.(b + c) = a . b + a. c$ and $(a + b). c = a. c + b$.

Example: (Z , ⊕ , ʘ) is a ring,

Definition1.3: Invalid source specified.

A nonzero element a in a ring R is called a zero divisor if there exists $b \in R$ such that $b \neq 0$ and $ab = 0$. In particular, a is a left divisor of zero and b is a right divisor of zero .

Example: In the ring Z_6={0,1,2,3,4,5}

Since $2.3 = 6 = 0 \quad 3.4 = 12 = 0$, then 2,3,4 are zero divisors of Z_6

Definition1.4: Invalid source specified. Prime numbers are numbers greater than 1, they only have two factors 1 and the numbers cannot be divide by any number other than 1.

Example: The number 2,3,5,7,11,13,17,… are prime numbers

Definition 1.5: (Behzad & CHartrand, 1979) A graphs is a finite non empty set of objects called vertices (the singular word is vertex) together with a (Possibly empty) set of un order pairs of distinct vertices of called edges.

Definition1.6: Order and Size of a Graph. **Invalid source specified.**

The order of a graph G, denoted by V(G), is the number of its vertices and the size of G, denoted by E(G), is the number of its edges. A graph with p-vertices and q-edges is called a (p,q)-graph.

Definition1.7 (Self-loop). **Invalid source specified.** An edge of a graph that joins a node to itself is called loop or a self-loop. That is, a loop is an edge u v, where u = v

Definition1.8 (Parallel Edges). **Invalid source specified.** The edges connecting the same pair of vertices are called multiple edges or parallel edges.

Definition 1.9 (Simple Graphs and Multigraphs). **Invalid source specified.**

A graph G which does not have loops or parallel edges is called a simple graph. A graph which is not simple is generally called a multigraph.

Definition 1.10 (Degree of a vertex). Invalid source specified.

The number of edges incident on a vertex v, with self-loops counted twice, is called the degree of the vertex v and is denoted by degG(v) or deg(v) or simply d(v).

CHAPTER TWO

The Zagreb indices

This chapter includes the definition of Zagreb indices and we find the zero divisors of the ring of integers modulo n, where n is composite, also we draw the graph of some rings and determined the degree of each vertices of this graphs. Finally we calculate Zagreb indices of each graphs of a ring of integers modulo n, n from 6 to 30.

Definition: Invalid source specified.

The Zagreb indices of a graph Γ, denoted by M_1 (Γ) and M_2 (Γ), and defined by M_1 (Γ)=$\sum_{v\in(v(\Gamma)}(\deg(v))^2$ and M_2 (Γ)= $\sum_{uv\in E(\Gamma)}$ (deg u)×(deg v)

Compute the zero divisor of some ring of integers modulo n, n from 6 to 30, n is note prime.

1) Z_6={0,1,2,3,4,5}

Since $2.3 = 6 = 0 \quad 3.4 = 12 = 0$

The set of zero divisors of Z_6 is $Z(Z_6)$ = { 2,3,4 } = $V(Z_6)$, where $V(Z_6)$ the set of vertex of the graph of Z_6.

The set of edges of $\Gamma(Z_6)$ is $E(Z_6)$={(2,3),(3,4)

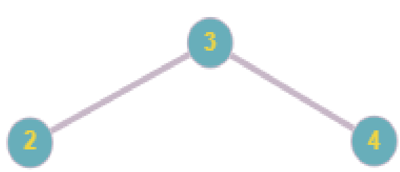

Figure 2.1 The graph of the ring Z_6

The degree of the vertex of Z_6 are d(2)=1 d(3)=2 d(4)=1.

The Zagreb indices of the graph of Z_6

1. $M_1\ (\Gamma(Z_6))=\sum_{v\in(v(\Gamma(Z_6))}\ (\deg(v))^2$

   $M_1\ (\Gamma(Z_6))=(1)^2+(2)^2+(1)^2$

   $= 6$

2. $M_2\ (\Gamma(Z_6)=\sum_{uv\in E(\Gamma(Z_6)}\ (\deg u)\times(\deg v)$

   $= d(2).d(3) + d(3).d(4)$

   $= (1\times 2) + (2\times 3)$

   $= 2 + 6$

   $= 8$

2) Z_8={0,1,2,3,4,5,6,7}

Since 2.4=8=0 and 6.4=24=0

The set of zero divisors of Z_8 is $Z(Z_8)$ = { 2,4,6 } = $V(Z_8)$, where $V(Z_8)$ the set of vertex of the graph of Z_8.

The set of edges of Z_8 is $E(Z_8)$={(2,4),(4,6)}

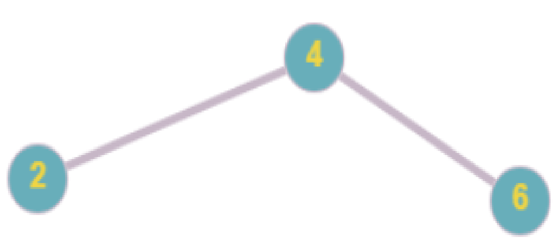

Figure 2.2 The graph of the ring Z_8

The degree of the vertex of $\Gamma(Z_8)$

$$d(2) = 1, \quad d(4) = 2, \quad d(6) = 1$$

The Zagreb indices of the graph of Z_8

$$1 - M_1\ (\Gamma(Z_8)) = \sum_{v\in(v(\Gamma(Z_8))} (\deg(v))^2$$

$$M_1\ (\Gamma(Z_8)) = (1)^2 + (2)^2 + (1)^2$$
$$= 6$$

$$2 - M_2\ (\Gamma(Z_8) = \sum_{uv\in E(\Gamma(Z_8)} (\deg u)\times(\deg v)$$

$$= d(2).d(4) + d(4).d(6)$$
$$= (1 \times 2) + (2 \times 1)$$
$$= 4$$

3) $Z_9=\{0,1,2,3,4,5,6,7,8\}$

Since 3.6=18=0

The set of zero divisors of Z_9 is $Z(Z_9) = \{ 3,6 \} = V(Z_9)$, where $V(Z_9)$ the set of vertex of the graph of Z_9.

The set of edges of Z_9 is $E(Z_9)=\{(3,6)\}$

Figure 2.3 The graph of the ring Z_9

The degree of the vertex of $\Gamma(Z_9)$ are d(3)=1 d(6)=1

The Zagreb indices of the graph of Z_9

$1 - M_1\ (\Gamma(Z_9))=\sum_{v\in(v(\Gamma(Z_9))}\ (\deg(v))^2$

$M_1\ (\Gamma(Z_9))=(1)^2 + (1)^2$

$= 2$

$2 - M_2\ (\Gamma(Z_9)= \sum_{uv\in E(\Gamma(Z_9)}\ (\deg u)\times(\deg v)$

$= d(3).d(6)$

$= (1 \times 1)$

$= 1$

4) Z_{10}={0,1,2,3,4,5,6,7,8,9}

Since $2.5 = 10 = 0$, $4.5 = 20 = 0$, $6.5 = 30 = 0$, $8.5 = 40 = 0$

The set of zero divisors of Z_{10} is $Z(Z_{10})$ = { 2,4,5,6,8 } = $V(Z_{10})$, where $V(Z_{10})$ the set of vertex of the graph of Z_{10}.

The set of edges of Z_{10} is $E(Z_{10})$={(2,5),(4,5),(6,5),(8,5)}3,6)}

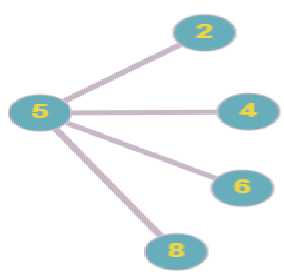

Figure 2.4 The graph of the ring Z_{10}

The degree of the vertex of $\Gamma(Z_{10})$ are d(2)=1, d(4)=1, d(5)=4, d(6)=1, d(8)=1.

The Zagreb indices of the graph of Z_{10}

$1 - M_1\ (\Gamma(Z_{10}))=\sum_{v\in(v(\Gamma(Z_{10}))}\ (\deg(v))^2$

$= (1)^2 + (1)^2 + (1)^2 + (1)^2 + (4)^2$

$= 1 + 1 + 1 + 1 + 16$

$= 4 + 16$

$= 20$

$2 - M_2\ (\Gamma(Z_{10}) = \sum_{uv \in E(\Gamma(Z_{10})}$ (deg u)×(deg v)

$= (1 \times 4) + (4 \times 1) + (4 \times 1) + (4 \times 1)$

$= 4 + 4 + 4 + 4$

$= 16$

5) $Z_{12}=\{0,1,2,3,4,5,6,7,8,9,10,11\}$

Since 2.6=12=0 3.4=12=0 8.3=24=0 4.6=24=0

9.4=36=0 6.10=60=0 6.8=48=0 8.9=72=0

The set of zero divisors of Z_{12} is $Z(Z_{12}) = \{2,3,4,6,8,9,10\} = V(Z_{12})$, where $V(Z_{12})$ the set of vertex of the graph of Z_{12}.

The set of edges of Z_{12} is $E(Z_{12})=\{(2,6),(3,4),(3,8),(4,9),(6,8),(6,10),(4,6),(8,9)\}$

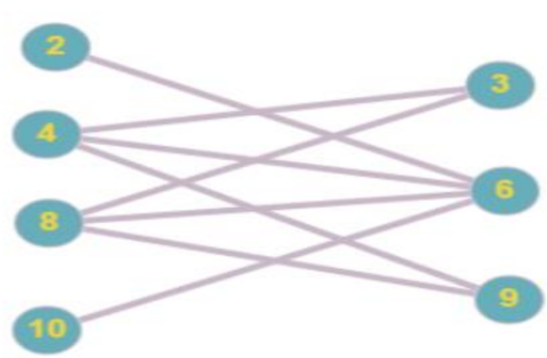

Figure 2.5 The graph of the ring Z_{12}

The degree of the vertex of $\Gamma(Z_{12})$ are

d(2)=1 d(3)=2 d(4)=3 d(6)=4 d(8)=3 d(9)=2 d(10)=1

The Zagreb indices of the graph of Z_{12}

1 − M_1 $(\Gamma(Z_{12}))$ $=\sum_{v\in(v(\Gamma(Z_{12}))}$ $(\deg(v))^2$

$= (1)^2 + (2)^2 + (3)^2 + (4)^2 + (3)^2 + (2)^2 + (1)^2$

$= 1 + 4 + 9 + 16 + 9 + 4 + 1$

$= 44$

2 − M_2 $(\Gamma(Z_{12})= \sum_{uv\in E(\Gamma(Z_{12})}$ $(\deg u)\times(\deg v)$

$= (1 \times 4) + (3\times2) + (2\times3) + (4\times3) + (4\times1) + (4\times3) + (3\times2) + (2\times3)$

$= 4 + 6 + 6 + 12 + 4 + 12 + 6 + 6$

$= 56$

6) $Z_{14}=\{0,1,2,3,4,5,6,7,8,9,10,11,12,13\}$

Since 2.7=14=0 4.7=28=0 6.7=42=0 8.7=56=0 10.7=70=0 12.7=84=0

The set of zero divisors of Z_{14} is $Z(Z_{14}) = \{2,4,6,7,8,10,12\} = V(Z_{14})$, where

$V(Z_{14})$ the set of vertex of the graph of Z_{14}.

The set of edges of Z_{14} is $E(Z_{14}) = \{(2,7),(4,7),(6,7),(8,7),(10,7),(12,7)\}$

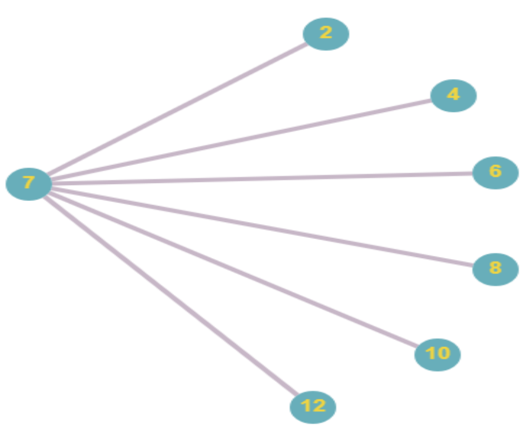

Figure 2.6 The graph of the ring Z_{14}

The degree of the vertex of $\Gamma(Z_{14})$ are

$$d(2) = d(4) = d(6) = d(8) = d(10) = d(12) = 1, \qquad d(7) = 6$$

The Zagreb indices of the graph of Z_{14}

$$1 - M_1\ (\Gamma(Z_{14})) = \sum_{v \in (v(\Gamma(Z_{14}))} (\deg(v))^2$$

$= (1)^2 + (1)^2 + (1)^2 + (1)^2 + (1)^2 + (1)^2 + (6)^2$

$= 1 + 1 + 1 + 1 + 1 + 1 + 36$

$= 6 + 36$

$= 42$

$$2 - M_2\ (\Gamma(Z_{14}) = \sum_{uv \in E(\Gamma(Z_{14})} (\deg u) \times (\deg v)$$

$= (1 \times 6) + (6 \times 1) + (6 \times 1) + (6 \times 1) + (6 \times 1) + (1 \times 6)$

$= 6 + 6 + 6 + 6 + 6 + 6$

$= 36$

7) $Z_{15}=\{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$

Since $3.5=15=0 \quad 6.5=30=0 \quad 10.6=60=0 \quad 12.5=60=0$

$9.5=45=0 \quad 10.12=120=0 \quad 9.10=90=0 \quad 3.10=30=0$

The set of zero divisors of Z_{15} is $Z(Z_{15}) = \{3,5,6,9,10,12\} = V(Z_{15})$, where $V(Z_{15})$ the set of vertex of the graph of Z_{15}.

The set of edges of Z_{15} is

$E(Z_{15}) = \{(3,5),(3,10),(5,12),(5,9),(5,6),(6,10),(9,10),(10,12)\}$

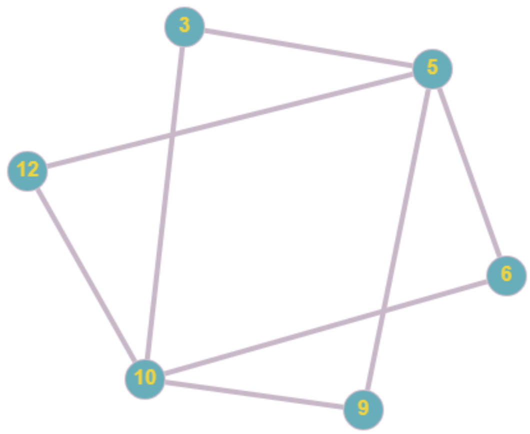

Figure 2.7 The graph of the ring Z_{15}

The degree of the vertex of $\Gamma(Z_{15})$ are

d(3)=2 d(5)=4 d(6)=2 d(9)=2 d(10)=4 d(12)=2

The Zagreb indices of the graph of Z_{15}

$$1 - M_1(\Gamma(Z_{15})) = \sum_{v\in(v(\Gamma(Z_{15}))} (\deg(v))^2$$

$$= (2)^2 + (4)^2 + (2)^2 + (2)^2 + (2)^2 + (4)^2$$

$$= 4 + 16 + 4 + 4 + 4 + 16$$

$$= 48$$

$$2 - M_2(\Gamma(Z_{15}) = \sum_{uv\in E(\Gamma(Z_{15})} (\deg u)\times(\deg v)$$

$= d(3).d(5) + d(3).d(10) + d(5).d(6) + d(5).d(9) + d(5).d(12) + d(6).d(10) + d(9).d(10) + d(10).d(12)$

$$= 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8$$

$$= 64$$

8) $Z_{16}=\{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$

Since 2.8=16=0 4.8=24=0 6.8=48=0

$10.8=80=0 \quad 12.4=48=0 \quad 8.12=96=0 \quad 14.8=112=0$

The set of zero divisors of Z_{16} is $Z(Z_{16}) = \{2,4,6,8,10,12,14\}$

$= V(Z_{16})$, where $V(Z_{16})$ the set of vertex of the graph of Z_{16}.

The set of edges of Z_{16} is $E(Z_{16}) =\{(2,8),(4,8),(8,6),(8,10),(8,14),(8,12),(12,4)$

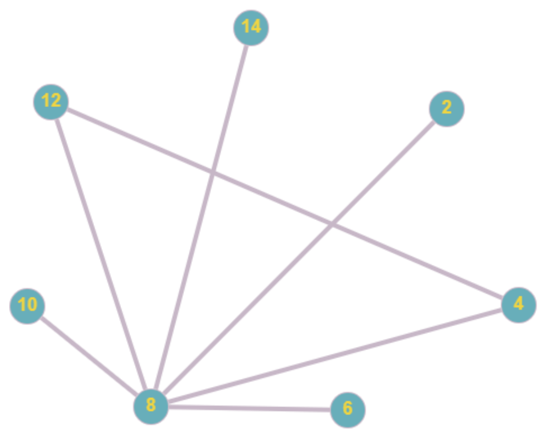

Figure 2.8 The graph of the ring Z_{16}

The degree of the vertex of $\Gamma(Z_{16})$ are

d(2)=1 d(4)=2 d(6)=1 d(8)=6 d(10)=1 d(12)=2 d(14)=1

The Zagreb indices of the graph of Z_{16}

$1 - M_1 (\Gamma(Z_{16})) = \sum_{v\in(v(\Gamma(Z_{16}))} (\deg(v))^2$

$= (1)^2 + (2)^2 + (1)^2 + (6)^2 + (1)^2 + (2)^2 + (1)^2$

$= 1 + 4 + 1 + 36 + 1 + 4 + 1$

$= 48$

$2 - M_2 (\Gamma(Z_{16}) = \sum_{uv\in E(\Gamma(Z_{16})} (\deg u)\times(\deg v)$

$= d(2)d(8) + d(4)d(8) + d(6)(8) + d(10)d(8) + d(12)d(8) + d(14)d(8) + d(4)d(12)$

= 6 + 6 + 6 + 6+ 6+ 6+ 6+ 6

= 52

9) Z_{18}={0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17}

Since 2.9=18=0 3.6=18=0 12.3=36=0 4.9=36=0 6.9=54=0

6.12=72=0 6.15=90=0 9.10=90=0 8.9=72=0 9.12=108=0

9.14=126=0 9.16=144=0 12.15=180=0

The set of zero divisors of Z_{18} is $Z(Z_{18})$ = {2,3,4,6,8,9,10,12,14,15,16}

= $V(Z_{18})$, where $V(Z_{18})$ the set of vertex of the graph of Z_{18}.

The set of edges of Z_{18} is $E(Z_{18})$

={(2,9),(3,6),(4,9),(9,6),(6,12),(6,15),(9,10),(9,12),(9,14),(12,3),(8,9),(15,10),(16,9)
,(12,15)}

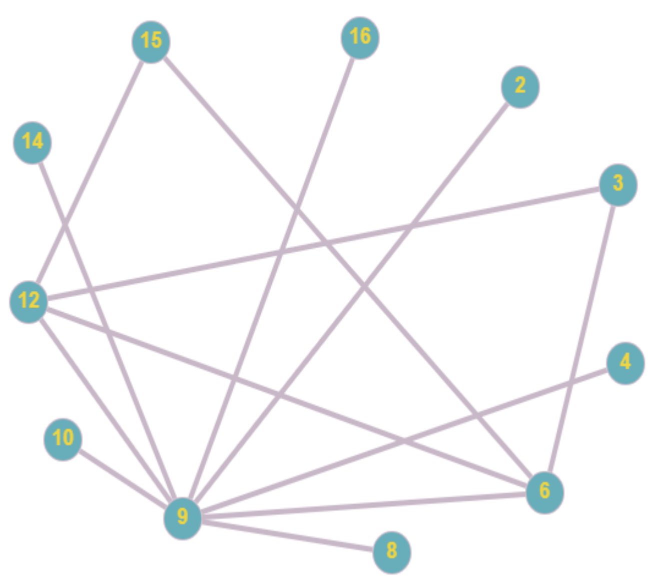

Figure 2.9 The graph of the ring Z_{18}

The degree of the vertex of $\Gamma(Z_{18})$ are

$$d(2) = 1 \;\; d(3) = 2 \;\; d(4) = 1 \;\; d(6) = 4 \;\; d(8) = 1$$

$$d(9) = 5 \;\; d(10) = 1 \;\; d(12) = 4 \; d(14) = 1 \; d(15) = 9 \; d(16) = 1$$

The Zagreb indices of the graph of Z_{18}

$1 - M_1\ (\Gamma(Z_{18})) = \sum_{v \in (v(\Gamma(Z_{18}))}\ (\deg(v))^2$

$= 6(1)^2 + (2)^2 + 2\,(4)^2 + (2)^2 + (9)^2$

$= 6 + 4 + 32 + 4 + 81$

$= 127$

$2 - M_2\ (\Gamma(Z_{18}) = \sum_{uv \in E(\Gamma(Z_{18})}$ (deg u)×(deg v)

$$= d(2).d(9) + d(3).d(6) + d(3).d(12) + d(4).d(9) + d(6).d(9) + d(6).d(12) + d(6).d(15) + d(8).d(9) + d(9).d(10) + d(9).d(12) + d(9).d(14) + d(9).d(16) + d(12).d(15)$$

$$= (1).(8) + (2).(4) + (2).(4) + (1).(8) + (4).(8) + (4).(4) + (4).(8) + (1).(8) + (1).(8) + (8).(4) + (1).(8) + (8).(1) + (2).(4)$$

$= 184$

10) Z_{20}={0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19}

Since 2.10=20=0 4.5=20=0 4.10=40=0 4.15=60=0 10.14=140=0

10.16=160=0 10.28=180=0 5.8=40=0 5.16=80=0 6.10=160=0

12.15=180=0 16.15=240=0 8.10=80=0 8.15=120=0 10.12=120=0

The set of zero divisors of Z_{20} is $Z(Z_{20})$ {2,4,5,6,8,10,12,14,15,16,18}

= $V(Z_{20})$, where $V(Z_{20})$ the set of vertex of the graph of Z_{20}.

The set of edges of Z_{20} is $E(Z_{20})$ ={(2,10),(4,5),(4,10),(4,15),(10,14),(10,16),(10,18),(5,8),(5,12),(5,16),(6,10),(12,15),(16,15),(8,10),(8,15),(10,12)},

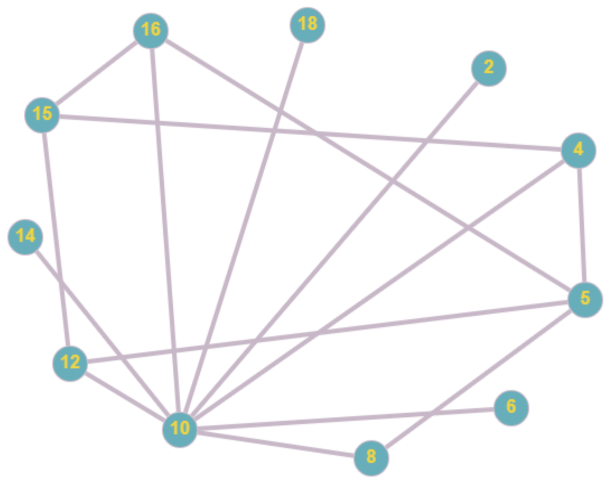

Figure 2.10 The graph of the ring Z_{20}

The degree of the vertex of $\Gamma(Z_{20})$ are

d(2)=1 d(4)=3 d(5)=4 d(6)=1 d(8)=2 d(10)=8

d(12)=3 d(14)=1 d(15)=3 d(16)=3 d(18)=1

The Zagreb indices of the graph of Z_{20}

$1 - M_1\ (\Gamma(Z_{20})) = \sum_{v\in(v(\Gamma(Z_{20}))}\ (\deg(v))^2$

$= 4(1)^2 + (2)^2 + 4\ (3)^2 + (4)^2 + (8)^2$

$= 4 + 4 + 36 + 16 + 64$

$= 124$

$2 - M_2\ (\Gamma(Z_{20}) = \sum_{uv\in E(\Gamma(Z_{20})}\ (\deg u)\times(\deg v)$

$$= d(2).d(10) + d(4).d(10) + d(4).d(15) + d(4).d(5)$$
$$+ \quad d(16).d(5) + d(5).d(12) + d(8).d(5)$$
$$+ d(6).d(10) + d(8).d(10) + d(10).d(12)$$
$$+ d(15).d(12) + d(10).d(14) + d(16).d(15)$$
$$+ d(16).d(10) + d(18).(10)$$

$$= (1).(8) + (3).(4) + (3).(8) + (3).(3) + (4).(2) + (4).(3) + (4).(3)$$
$$+ (1).(8) + (2).(8) + (3).(8) + (8).(1) + (3).(8)$$
$$+ (3).(3) + (3).(3)$$

= 8 + 12 + 24 + 9 + 8 + 12 + 12 + 8 + 16 + 24 + 8 + 24 + 9 +9

= 191

11) Z_{21}={0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20}

Since 3.7=21=0 3.14=42=0 6.7=42=0 6.14=84=07.9=63=0 7.12=84=0 7.15=105=0 14.18=252=0 7.18=126=0 9.14=126=0 12.14=168=0 14.15=210=0

The set of zero divisors of Z_{21} is Z(Z_{21})= {3,6,7,9,12,14,15,18}

V(Z_{21}), where V(Z_{21}) the set of vertex of the graph of Z_{21}.

The set of edges of Z_{21} is

E(Z_{21})={(3,7),(3,14),(6,7),(6,14),(7,9),(7,12),(7,15),(14,18),(7,18),(9,14),(12,14),(14,15)}

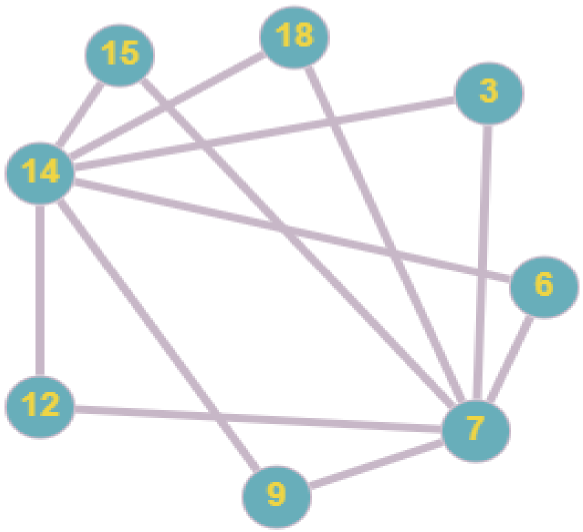

Figure 2.11 The graph of the ring Z_{21}

The degree of the vertex of $\Gamma(Z_{21})$ are

d(3)=2 d(6)=2 d(7)=6 d(9)=2 d(12)=2 d(14)=6 d(15)=2 d(18)=2

The Zagreb indices of the graph of Z_{21}

$1 - M_1\ (\Gamma(Z_{21})) = \sum_{v\in(v(\Gamma(Z_{21}))}\ (\deg(v))^2$

$= 6(2)^2 + 2\ (6)^2$

$= 24 + 72$

$= 96$

$2 - M_2\ (\Gamma(Z_{21}) = \sum_{uv\in E(\Gamma(Z_{21})}\ (\deg u)\times(\deg v)$

$$= d(3).d(7) + d(3).d(14) + d(6).d(7) + d(14).d(6) + d(7).d(9) + d(7).d(12) + d(7).d(15) + d(7).d(18) + d(9).d(14) + d(14).d(12) + d(15).d(14) + d(18).d(14)$$

$= (2).(6) + (2).(6) + (2).(6) + (2).(6) + (6).(2) + (2).(6) + (2).(6) + (2).(6) + (2).(6) + (2).(6) + (2).(6) + (2).(6)$

$$= 12.(2).(6)$$

$$= 12.12$$

$$= 144$$

12) Z_{22}={0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21}

Since 2.11=22=0 4.11=44=0 6.11=66=0 8.11=88=0 1O.11=110=0

12.11=132=0 14.11=154=0 16.11=176=0 18.11=198=0 20.11=220=0

The set of zero divisors of Z_{22} is $Z(Z_{22})$= ={2,4,6,8,10,11,12,14,16,18,20}

= $V(Z_{22})$, where $V(Z_{22})$ the set of vertex of the graph of Z_{22}.

The set of edges of Z_{22} is

$E(Z_{22})$={(2,11),(4,11),(6,11),(8,11),(10,11),(12,11),(14,11),(16,11),(18,11),(20,11)}.

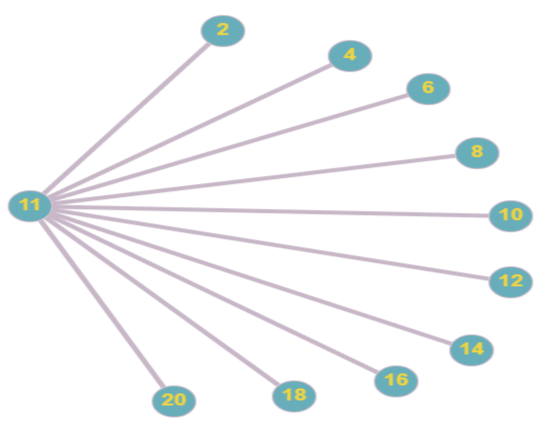

Figure 2.12 The graph of the ring Z_{22}

The degree of the vertex of $\Gamma(Z_{22})$ are d(2)=1 d(4)=1 d(6)=1 d(8)=1 d(10)=1 d(11)=10

d(12)=1 d(14)=1 d(16)=1 d(18)=1 d(20)=1

The Zagreb indices of the graph of Z_{22}

$1 - M_1 (\Gamma(Z_{22})) = \sum_{v \in (v(\Gamma(Z_{22}))} (\deg(v))^2$

$= 10(1)^2 + (10)^2$

$= 10 + 100$

$= 110$

$2 - M_2 (\Gamma(Z_{22})) = \sum_{uv \in E(\Gamma(Z_{22})} (\deg u) \times (\deg v)$

$= 10.10$

$=100$

13) Z_{24}={0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23}

Since 2.12=24=0 3.8=24=0 4.6=24=0 4.12=48=0 4.18=72=0 6.8=48=0
6.12=72=0 6.16=0 6.20=120=0 6.24=144=0 12.20=240=0

12.22=264=0 6.12=72=0 6.20=120=0 6.24=144=0 8.9=72=0

8.12=96=0 8.15=120=0 15.16=240=0 16.18=288=0 16.21=336=0

8.21=168=0 9.16=144=0 10.12=120=0 12.14=168=0 12.16=192=0

12.18=216=0 18.20=360=0

The set of zero divisors of Z_{24} is $Z(Z_{24})$={2,3,4,6,8,9,10,12,14,15,16,18,20,21,22}

= $V(Z_{24})$, where $V(Z_{24})$ the set of vertex of the graph of Z_{24}.

The set of edges of Z_{24} is $E(Z_{24})$={(2,12),(3,8),(4,6),(4,12),(4,18),(8,6),(12,20),(12,22),(6,12),(6,20),(8,9),(8,12),(8.15),(16,15),(16,18),(16,21),(8,21),(9,16),(12,10),(12,14),(12,16),(18,12),(18,20)}

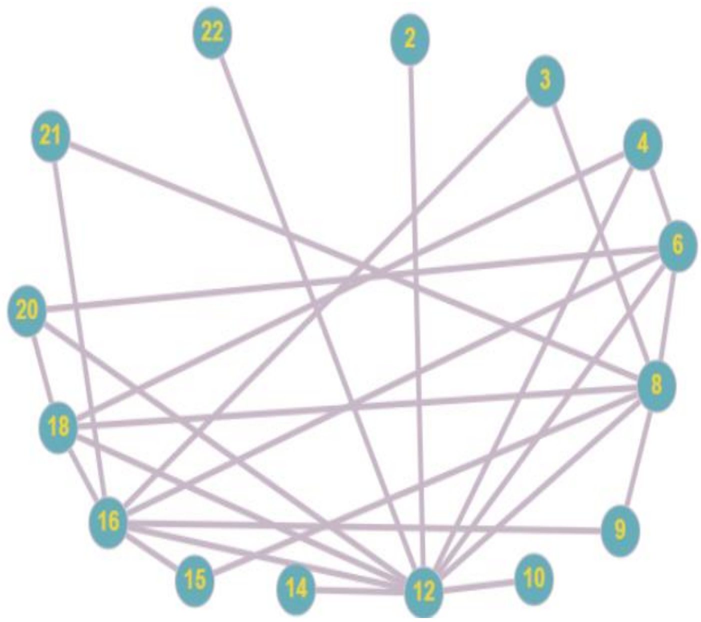

Figure 2.13 The graph of ring Z_{24}

The degree of the vertex of $\Gamma(Z_{24})$ are

d(2)=1 d(3)=2 d(4)=3 d(6)=5 d(8)=7 d(9)=2 d(10)=1 d(12)=10 d(14)=1 d(15)=2 d(16)=5 d(18)=4 d(20)=3 d(21)=2 d(22)=1

The Zagreb indices of the graph of Z_{24}

$1 - M_1(\Gamma(Z_{24})) = \sum_{v\in(v(\Gamma(Z_{24})))} (\deg(v))^2$

$$= 4(1)^2 + 4(2)^2 + 2(3)^2 + 4^2 + 2(5)^2 + (7)^2 + 10^2$$
$$= 4 + 16 + 18 + 16 + 50 + 49 + 100$$

$= 253$

$2 - M_2\ (\Gamma(Z_{24}) = \sum_{uv \in E(\Gamma(Z_{24})}$ (deg u)×(deg v)

= d(2).d(12) + d(3).d(8) + d(3).d(16) + d(4).d(6) + d(4).d(12) + d().d()

= (1).(10) + (2).(7)+ (2).(7) + (3).(5) + (3).(10)

=

14) Z_{25}={0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24}

Since 10.5=50=0 5.15=75=0 20.5=100=0

10.15=150=0 10.20=200=0 15.20=300=0

The set of zero divisors of Z_{25} is $Z(Z_{25})$= {5,10,15,20}

= $V(Z_{25})$, where $V(Z_{25})$ the set of vertex of the graph of Z_{25}.

The set of edges of Z_{25} is

$E(Z_{25})$={(5,10)(5,15),(5,20),(10,15),(10,20),(15,20)}

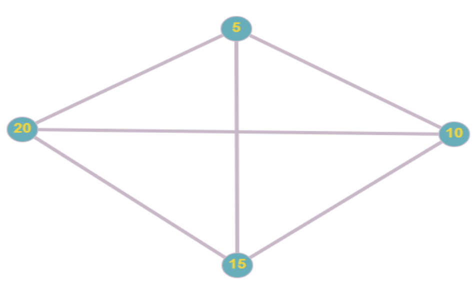

Figure 2.14 The graph of the ring Z_{25}

The degree of the vertex of $\Gamma(Z_{25})$ are

d(5)=3 d(10)=3 d(15)=3 d(20)=3

The Zagreb indices of the graph of Z_{25}

$1 - M_1\ (\Gamma(Z_{25})) = \sum_{v\in(v(\Gamma(Z_{25}))}\ (\deg(v))^2$

$= 4(3)^2$

$= 36$

$2 - M_2\ (\Gamma(Z_{25}) = \sum_{uv\in E(\Gamma(Z_{25})}$ (deg u)×(deg v)

$= d(5).d(10) + d(5).d(15) + d(5).d(20) + d(10).d(15) + d(10).d(20) + d(15).d(20)$

= (3).(3) + (3).(3) + (3).(3) + (3).(3) + (3).(3) + (3).(3)

= 54

15) Z_{26}={0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25}

Since 2.13=26=0 4.13=52=0 6.13=78=0 8.13=104=0 10.13=130=0 14.13=182=0 16.13=208=0 18.13=234=0 20.13=260=0 22.13=286=0 24.13=312=0

The set of zero divisors of Z_{26} is Z(Z_{26})= {2,4,6,8,10,12,13,14,16,18,20,22,24}

= V(Z_{26}), where V(Z_{26}) the set of vertex of the graph of Z_{26}.

The set of edges of Z_{26} is

E(Z_{26})={(13,2)(13,4),(13,6),(13,8),(13,10),(13,12),(13,14),(13,16),(13,18),(13,20),(13,22),(13,24)}

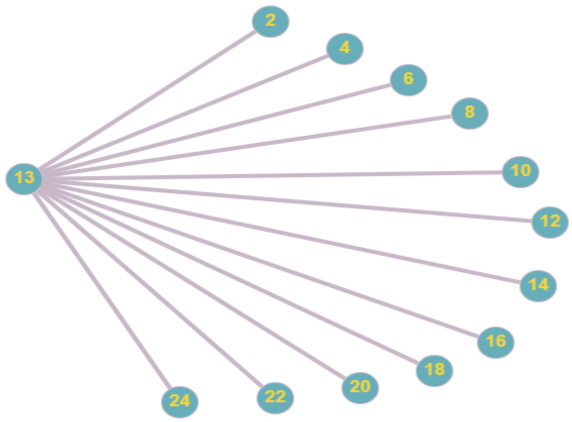

Figure 2.15 The graph of the ring Z_{26}

The degree of the vertex of $\Gamma(Z_{26})$ are

$d(13)=11$,$d(x)=1\ \forall\ x \in V(Z_{26} / 13)$

The Zagreb indices of the graph of Z_{26}

$1 - M_1\ (\Gamma(Z_{26})) = \sum_{v\in(v(\Gamma(Z_{26}))}\ (\deg(v))^2$

$= 12(1)^2 + (12)^2$

$= 12 + 144$

$= 156$

$2 - M_2\ (\Gamma(Z_{26}) = \sum_{uv\in E(\Gamma(Z_{26})}\ (\deg u)\times(\deg v)$

$= 12.12$

$= 144$

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پوختە

باسەکە(Z_n) ببێتە ئەڵقەی ژمارەیی، ئامانجی ئەم کارە ئەوەیە کەکار لەسەر هەندێک گرافی سفری بکەین لە(Z_n) ، ئەگەر(n) ژمارەیەکی سەرەتایی بێت ئەوا(Z_n) دابەشکەری سفری نییە، کەواتە$\Gamma(Z_n)$ گرافێکی بێ وێنەیە ، لەبەرئەوە لەم کارەدا تەنها ئەو حاڵەتە ڕەچاو دەکەین کە n ناوێتەیە

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