زانکۆی سەلاحەدین - هەولێر
Salahaddin University-Erbil

The Zagreb indices of the Commuting Graphs of Dihedral Groups

Research Project

Submitted to the department of (Mathematics) in partial fulfillment of the requirements for the degree of BSc. in (Mathematics

By:

Muhamad Yusif Ali

Supervised by:

Assistant Lecturer Bushra N. Abdulgaphur

April-2023

Certification of the supervisors:

I certify that this work was prepared under my supervision at the department of mathematics /college of education/Salahaddin university –Erbil in partial fulfilment of the requirements for the degree of bachelor of philosophy of science in mathematics.

Signature: 

Supervisor: Bushra N. Abdulgaphur

Scientific grade: Lecturer

In view of the available recommendations, forward this work for debate by the examining committee.

Signature:

Name: Dr. Rashad R. Haji

Scientific grade: Assistant Professor

Chairman of the Mathematics Department

Acknowledgement

My great thanks to God for giving me power and patience to perform this study. All existing word in the world would not be sufficient to express my deep feeling and gratitude to my respectful supervisor assistant lecture **Bushra N. Abdul-Gaphur** for all she has done for me and taught me as my supervisor.

My best gratitude to my family for encouraging me to fulfill my present research studies.

I highly appreciate the support which was given by Professor Dr. **Rashad Rashid** the head of Mathematics Department of the College of Education.

Finally, I would like to state my thanks to all my teachers for helping me during difficult times.

Abstract

Throughout this work all groups are *presumed* to be finite and non-abelian. The aim of this work is to compute and calculate the Zagreb indices of the graphs of the dihedral group $D_{2n} =< \mathrm{r}, \mathrm{s} \mid r^n = \mathrm{s}^2 = 1, \mathrm{s}^{-1}\mathrm{rs} = \mathrm{r}^{-1} >$ For $n \geq 3$ and consider D_{2n} . Let r be a rotation clockwise by 360∘/n and let s and s^{-1} be any two adjacent reflections of a regular n-gon. $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$.

Table of Contents

Introduction

The dihedral group is the group of symmetries of a regular polygon. In abstract algebra, it's a classic example of a finite, non-abelian group. Dihedral groups are also a great example of how transformations can be thought of as elements in a group. Here, a transformation that involves picking up the polygon, and flipping or rotating it around, and then placing it back so that it lines up with the starting position. These transformations form a group. After all, since each transformation returns the group to its original position, so does the composition of any two transformations. It is a non-abelian group (i.e. non-commutative) because a flip followed by a rotation is different than a rotation followed by a flip.

Let r be a rotation clockwise by 360∘/n and let s and s^{-1} be any two adjacent reflections of a regular n-gon. The dihedral group $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$

The Zagreb indices of a graph, denoted by M_1 (Γ) and M_2 (Γ), and defined by

M_1 (Γ)=$\sum_{v\in(v(\Gamma)}$ $(\deg(v))^2$ and M_2 (Γ)= $\sum_{uv\in E(\Gamma)}$ (deg u)×(deg v) (I.Gustman and N.Trinjajstic, 1972),

Suppose that Γ is a simple graph, which is undirected and contains no multiple edges or loops. We denote the set of vertices of Γ by V(Γ) and the set of edges of Γ by E(Γ). We write uv ∈ E(Γ) if u and v form an edge in Γ. The size of the vertex-set of Γ is denoted by |V(Γ)| and the number of edges of Γ is denoted by |E(Γ)|. The degree of a vertex v in Γ, denoted by deg(v), is defined as the number of edges incident to v. The distance between any pair of vertices u and v in Γ, denoted by d(u, v), is the shortest u−v path in Γ. For a vertex v in Γ. A graph Γ is called complete if every pair of vertices in Γ are adjacent

This thesis is divided into two chapters. The first chapter contains some basic definitions, which we need in the present work and some 0examples to illustrating the definitions. The second chapter, includes the definition of Zagreb indices, we draw the graph of some groups of commuting dihedral group and determined the degree of each vertex of this graphs. Finally, we calculate Zagreb indices of each graph commuting dihedral group. Note that all figures are drawn via website http://graphonline.ru/en/

Chapter One

Background Materials

In this chapter some basic definitions that we need in our work and examples to illustrating this definition.

Definition 1.1: (Marlo Anderson and Todd Feil, 2015)

A group G consists of a set G together with a binary operation $*$ for which the following properties are satisfied:

i. $(x * y) * z = x * (y * z)$ for all elements x, y and z of G (the Associative Law).
ii. there exists an element e of G (known as the identity element of G) such that
iii. $e * x = x = x * e$, for all elements x of .
iv. for each element x of G there exists an element x' of G (known as the inverse of x) such that $x * x' = e = x' * x$ (where e is the identity element of G) .

Example:

1- $(Z, +)$, $(R, +)$, $(R - \{0\}, .)$ and $(Mn\times(R), +)$ are groups.

Definition1.2: (Burton, 1980) Prime numbers are numbers greater than 1, they only have two factors 1 and the numbers cannot be divide by any number other than 1.

Example: The number 2,3,5,7,11,13,17,… are prime numbers

Definition 1.3: (Behzad & CHartrand, 1979) A graphs is a finite non empty set of objects called vertices (the singular word is vertex) together with a (Possibly empty) set of un order pairs of distinct vertices of called edges.

Definition1.4: Order and Size of a Graph. (Naduvath, 2017)

The order of a graph G, denoted by V(G), is the number of its vertices and the size of G, denoted by E(G), is the number of its edges. A graph with p-vertices and q-edges is called a (p,q)-graph.

Definition1.5 : Self-loop. (Naduvath, 2017) An edge of a graph that joins a node to itself is called loop or a self-loop. That is, a loop is an edge uv, where u = v

Definition1.6: Parallel Edges. (Naduvath, 2017) The edges connecting the same pair of vertices are called multiple edges or parallel edges.

Definition 1.7: Simple Graphs and Multigraphs. (Naduvath, 2017)

A graph G which does not have loops or parallel edges is called a simple graph. A graph which is not simple is generally called a multigraph.

Definition 1.8: Degree of a vertex. (Naduvath, 2017)

The number of edges incident on a vertex v, with self-loops counted twice, is called the degree of the vertex v and is denoted by degG(v) or deg(v) or simply d(v).

Chapter Two

The Zagreb indices

This chapter, includes the definition of Zagreb indices, we draw the graph of some groups of commuting dihedral group and determined the degree of each vertex of this graphs, and we calculate Zagreb indices of each graph commuting dihedral group.

Definition: (I.Gustman and N.Trinjajstic, 1972)

The Zagreb indices of a graph Γ, denoted by M_1 (Γ) and M_2 (Γ), and defined by M_1 (Γ)=$\sum_{v\in(v(\Gamma)}$ $(\deg(v))^2$ and M_2 (Γ)= $\sum_{uv\in E(\Gamma)}$ (deg u)×(deg v)

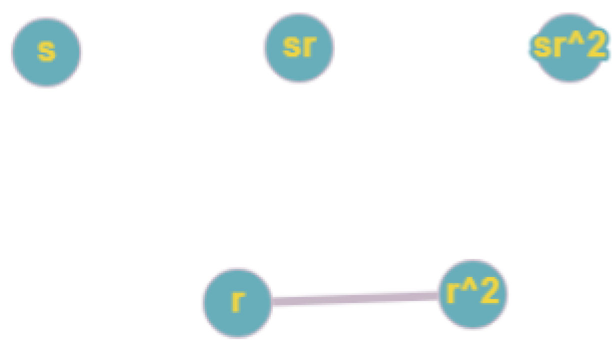
Compute the zero divisor of some ring of integers modulo n, n from 6 to 30, n is note prime.

1) If n=3 then $D_{2n} = D_6 = \{1, r, r^2, s, sr, sr^2\}$

The center of $D_6 = \{1\}$

The set of vertex of $D_6 = \{r, r^2, s, sr, sr^2\}$

The graph of D_6



The degree of the vertex of $\Gamma(D_6)$ are

$d(s) = d(sr) = d(sr^2) = 0,$ $d(r) = d(r^2) = 1$

The Zagreb indices of the graph of D_6

$1 - M_1\ (\Gamma(D_6)) = \sum_{v \in (v(\Gamma(D_6))}\ (\deg(v))^2$

$= (0)^2 + (0)^2 + (0)^2 + (1)^2 + (1)^2$
$= 2$

$2 - M_2\ (\Gamma(D_6) = \sum_{uv \in E(\Gamma(D_6))}\ (\deg u) \times (\deg v)$

$= d(r).\,d(r^2)$

$= (1 \times 1)$

$=1$

2) If n=4 then $D_{2n} = D_8 = \{1, r, r^2, r^3, s, sr, sr^2, sr^3\}$

The center of $D_8 = \{1, r^2\}$

The set of vertex of $D_8 = \{r, r^3, s, sr, sr^2, sr^3\}$

The graph of D_6

The degree of the vertex of $\Gamma(D_8)$ are

$d(s) = d(sr) = d(sr^2) = d(sr^3) = 1, \qquad d(r) = d(r^3) = 1$

The Zagreb indices of the graph of D_8

$1 - M_1\ (\Gamma(D_8)) = \sum_{v \in (v(\Gamma(D_8))}\ (\deg(v))^2$

$= (1)^2 + (1)^2 + (1)^2 + (1)^2 + (1)^2 + (1)^2$
$= 6$

$2 - M_2\ (\Gamma(D_8) = \sum_{uv \in E(\Gamma(D_8))}\ (\deg u) \times (\deg v)$

$= d(r).\mathrm{d}(\mathrm{r}^2) + \mathrm{d}(\mathrm{s})\mathrm{d}(sr^2) + \mathrm{d}(\mathrm{sr})\mathrm{d}(sr^3)$

$= (1 \times 1) + (1 \times 1) + (1 \times 1)$

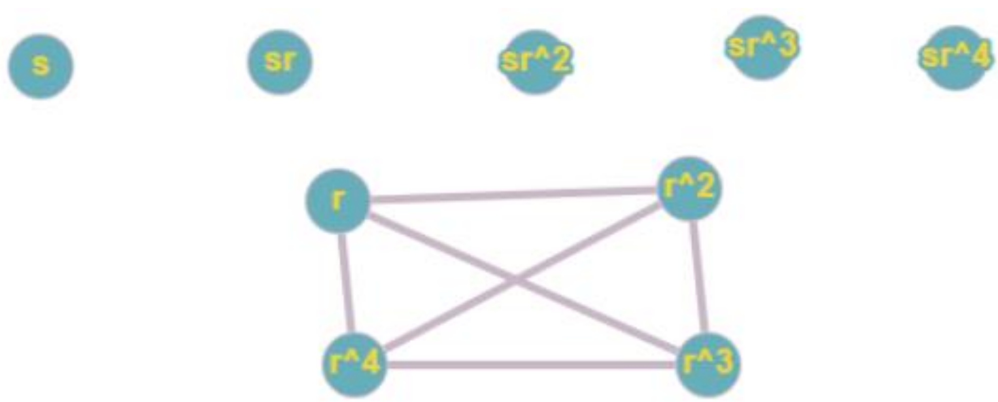
$=3$

3) If n=5 then $D_{2n} = D_{10} = \{1, r, r^2, r^3, r^4, s, sr, sr^2, sr^3, sr^4\}$

The center of $D_{10} = \{1\}$

The set of vertex of $D_{10} = \{r, r^2, r^3, r^4, s, sr, sr^2, sr^3, sr^4\}$

The graph of D_{10}



The degree of the vertex of $\Gamma(D_{10})$ are

$\mathrm{d}(\mathrm{s}) = \mathrm{d}(\mathrm{sr}) = \mathrm{d}(sr^2) = \mathrm{d}(sr^3) = \mathrm{d}(sr^4) = 0,$

$\mathrm{d}(\mathrm{r}) = \mathrm{d}(\mathrm{r}^2) = \mathrm{d}(r^3) = \mathrm{d}(r^4) = 3$

The Zagreb indices of the graph of D_{10}

$1 - M_1\ (\Gamma(D_{10})) = \sum_{v \in (v(\Gamma(D_{10}))} (\deg(v))^2$

$= (3)^2 + (3)^2 + (3)^2 + (3)^2$
$= 9+9+9+9$
$= 36$

$2 - M_2\ (\Gamma(D_{10}) = \sum_{uv \in E(\Gamma(D_{10}))} (\deg u) \times (\deg v)$

$= d(r).d(r^2) + d(r)d(r^3) + d(r)d(r^4) + d(r^2)d(r^3) + d(r^2)d(r^4) + d(r^3)d(r^4)$

$= (3 \times 3) + (3 \times 3) + (3 \times 3) + (3 \times 3) + (3 \times 3) + (3 \times 3)$

$= 9+9+9+9+9+9$

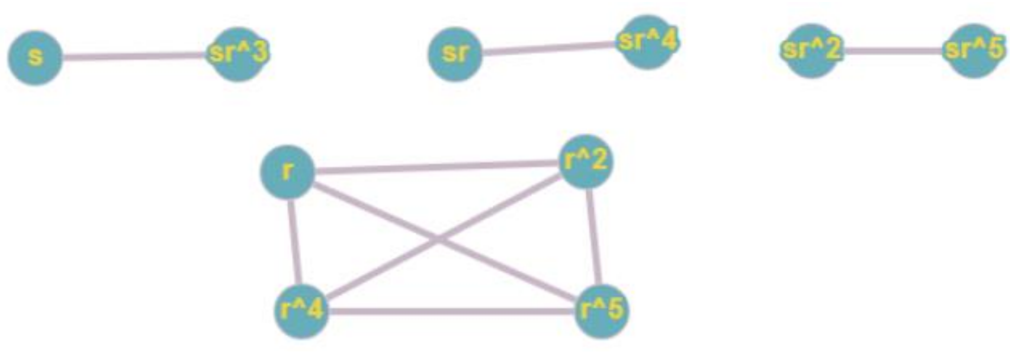
$= 54$

4) If n=6 then $D_{2n} = D_{12} = \{1, r, r^2, r^3, r^4, r^5, s, sr, sr^2, sr^3, sr^4, sr^5\}$

The center of $D_{12} = \{1, , r^3\}$

The set of vertex of $D_{12} = \{r, r^2, r^4, r^5, s, sr, sr^2, sr^3, sr^4, sr^5\}$

The graph of D_{12}



The degree of the vertex of $\Gamma(D_{12})$ are

$d(s) = d(sr) = d(sr^2) = d(sr^3) = d(sr^4) = d(sr^5) = 1,$

$d(r) = d(r^2) = d(r^4) = d(r^5) = 3$

The Zagreb indices of the graph of D_{12}

$1 - M_1\ (\Gamma(D_{12})) = \sum_{v \in (v(\Gamma(D_{12}))} (\deg(v))^2$

$= (3)^2 + (3)^2 + (3)^2 + (3)^2 + (1)^2 + (1)^2 + (1)^2 + (1)^2 + (1)^2 + (1)^2$
= 9+9+9+9+1+1+1+1+1+1
= 42

$2 - M_2\ (\Gamma(D_{12}) = \sum_{uv \in E(\Gamma(D_{12}))} (\deg u) \times (\deg v)$

$= d(r).d(r^2) + d(r)d(r^3) + d(r)d(r^4) + d(r^2)d(r^3) + d(r^2)d(r^4) + d(r^3)d(r^4)$

$= (3 \times 3) + (3 \times 3) + (3 \times 3) + (3 \times 3) + (3 \times 3) + (3 \times 3)$

= 9+9+9+9+9+9

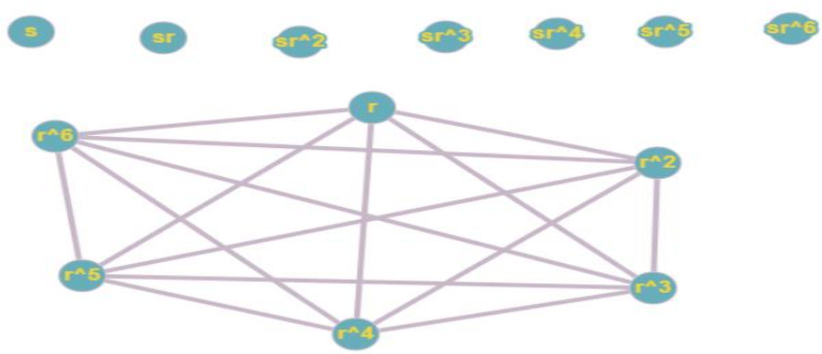
= 54

4) If n=7 then $D_{2n} = D_{14} = \{1, r, r^2, r^3, r^4, r^5, r^6, s, sr, sr^2, sr^3, sr^4, sr^5, sr^6\}$

The center of $D_{14} = \{1\}$

The set of vertex of $D_{14} = \{r, r^2, r^3, r^4, r^5, r^6, s, sr, sr^2, sr^3, sr^4, sr^5, sr^6\}$

The graph of D_{14}



The degree of the vertex of $\Gamma(D_{14})$ are

$d(s) = d(sr) = d(sr^2) = d(sr^3) = d(sr^4) = d(sr^5) = d(sr^6) = 0,$

$d(r) = d(r^2) = d(r^3) = d(r^4) = d(r^5) = d(r^6) = 5$

The Zagreb indices of the graph of D_{14}

$1 - \mathrm{M}_1\ (\Gamma(D_{14})) = \sum_{\mathrm{v} \in (\mathrm{v}(\Gamma(D_{14}))}\ (\deg(\mathrm{v}))^2$

$= (5)^2 + (5)^2 + (5)^2 + (5)^2 + (5)^2 + (5)^2$
= 25+25+25+25+25+25
= 150

$2 - \mathrm{M}_2\ (\Gamma(D_{14}) = \sum_{\mathrm{uv} \in \mathrm{E}(\Gamma(D_{14}))}\ (\deg \mathrm{u}) \times (\deg \mathrm{v})$

$= (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5)$

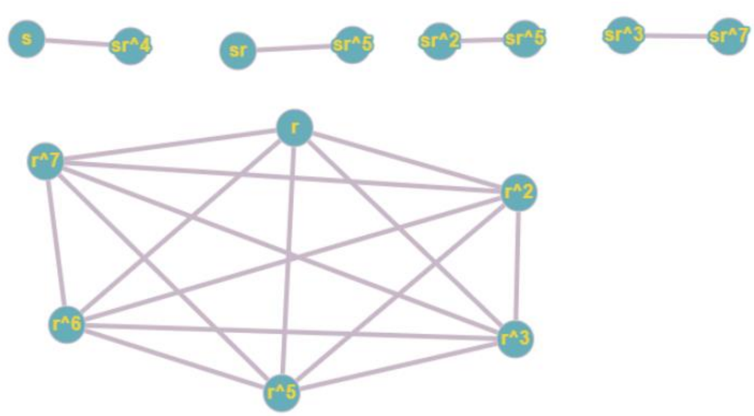
$= 375$

5)n=8 then $D_{2n} = D_{16} = \{1, r, r^2, r^3, r^4, r^5, r^6, r^7, s, sr, sr^2, sr^3, sr^4, sr^5, sr^6, sr^7\}$

The center of $D_{16} = \{1, r^4\}$

The set of vertex of $D_{16} = \{r, r^2, r^3, r^5, r^6, r^7, s, sr, sr^2, sr^3, sr^4, sr^5, sr^6, sr^7\}$

The graph of D_{16}



The degree of the vertex of $\Gamma(D_{16})$ are

$\mathrm{d}(\mathrm{s}) = \mathrm{d}(\mathrm{sr}) = \mathrm{d}(sr^2) = \mathrm{d}(sr^3) = \mathrm{d}(sr^4) = \mathrm{d}(sr^5) = \mathrm{d}(sr^6) = \mathrm{d}(sr^7) = 1,$

$\mathrm{d}(\mathrm{r}) = \mathrm{d}(\mathrm{r}^2) = \mathrm{d}(r^3) = \mathrm{d}(r^5) = \mathrm{d}(r^6) = \mathrm{d}(r^7) = 5$

The Zagreb indices of the graph of D_{16}

$1 - M_1\ (\Gamma(D_{16})) = \sum_{v\in(v(\Gamma(D_{16}))} (\deg(v))^2$

$= 1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+5^2+5^2+5^2+5^2+5^2+5^2$

$=158$

$2 - M_2\ (\Gamma(D_{16}) = \sum_{uv\in E(\Gamma(D_{16}))} (\deg u)\times(\deg v)$

$= (1\times1)+ (1\times1)+ (1\times1)+ (1\times1)+ (5\times5)+ (5\times5)+ (5\times5)+ (5\times5)+ (5\times5)+ (5\times5)+ (5\times5)+ (5\times5)+ (5\times5)+ (5\times5)+ (5\times5)+ (5\times5)+ (5\times5)+ (5\times5)+ (5\times5)$

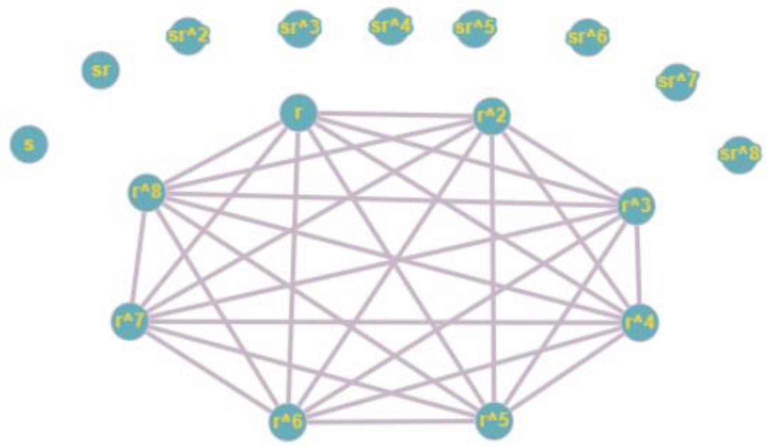
$= 379$

6) n=9 then D_{2n}=D_{18} $=\{1,r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8\}$

The center of D_{14}={1}

The set of vertex of D_{18} $= \{r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8\}$

The graph of D_{18}



The degree of the vertex of $\Gamma(D_{18})$ are $d(s) = d(sr) = d(sr^2) = d(sr^3) = d(sr^4) = d(sr^5) = d(sr^6) = d(sr^7) = d(sr^8) = 0,$ $d(r) = d(r^2) = d(r^3) = d(r^4) = d(r^5) = d(r^6) = d(r^7) = d(r^8) = 7$

The Zagreb indices of the graph of D_{18}

$1 - M_1 (\Gamma(D_{18})) = \sum_{v\in(v(\Gamma(D_{18}))} (deg(v))^2$

$= (7)^2 + (7)^2 + (7)^2 + (7)^2 + (7)^2 + (7)^2 + (7)^2 + (7)^2$

$= 49+49+49+49+49+49+49+49$

$= 392$

$2 - M_2 (\Gamma(D_{18}) = \sum_{uv\in E(\Gamma(D_{18}))} (deg\ u)\times(deg\ v)$

$$= (7\times7) + (7\times7)$$

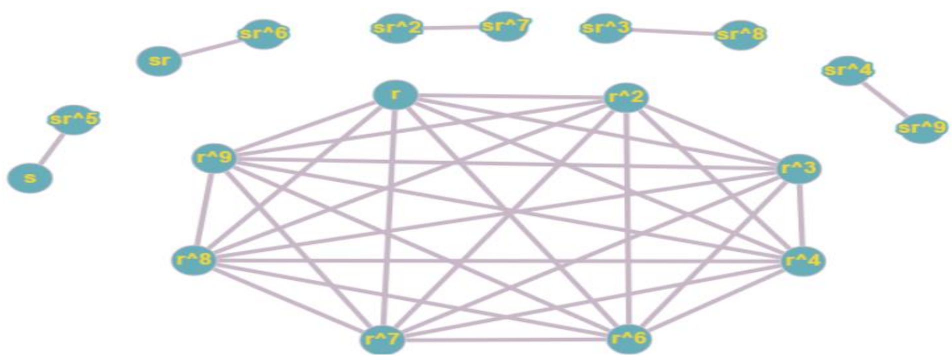
$= 1372$

7) n=10 then D_{2n}=D_{20} $=\{1,r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^9,s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9\}$

The center of D_{20}=$\{1,r^5\}$

The set of vertex of D_{20} $= \{r,r^2,r^3,r^4,r^6,r^7,r^8,r^9,s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9\}$

The graph of D_{20}



The degree of the vertex of $\Gamma(D_{20})$ are $d(s) = d(sr) = d(sr^2) = d(sr^3) = d(sr^4) = d(sr^5) = d(sr^6) = d(sr^7) = d(sr^8) = d(sr^9) = 1,\ d(r) = d(r^2) = d(r^3) = d(r^4) = d(r^6) = d(r^7) = d(r^8) = d(r^9) = 7$

The Zagreb indices of the graph of D_{20}

$1 - \mathrm{M}_1 \ (\Gamma(D_{20})) = \sum_{\mathrm{v} \in (\mathrm{v}(\Gamma(D_{20}))} (\deg(\mathrm{v}))^2$

$= 1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+7^2+7^2+7^2+7^2+7^2+7^2+7^2+7^2$

$=49+49+49+49+49+49+49+49+10$

$= 402$

$2 - \mathrm{M}_2 \ (\Gamma(D_{20}) = \sum_{\mathrm{uv} \in \mathrm{E}(\Gamma(D_{20}))} (\deg \mathrm{u}) \times (\deg \mathrm{v})$

$= (1\times1)+ (1\times1)+ (1\times1)+ (1\times1)+ (1\times1)+ (7\times7)$

$=1377$

8) n=11 then

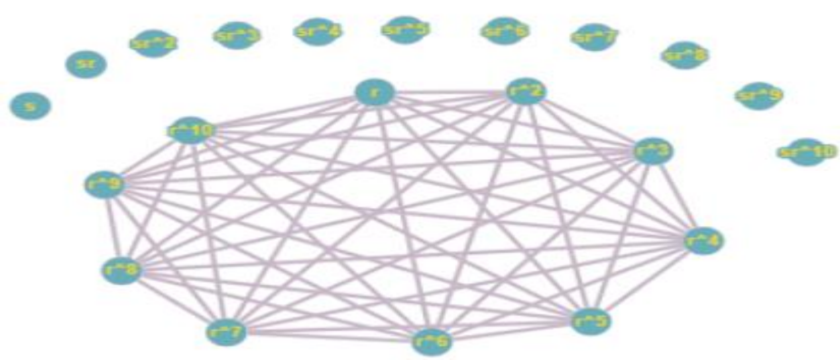
$D_{2n}=D_{22} =\{1,r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^9,r^{10},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}\}$

The center of $D_{22}=\{1\}$

The set of vertex of

$D_{22} = \{r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^9,r^{10},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}\}$

The graph of D_{22}



The degree of the vertex of $\Gamma(D_{22})$ are

$\mathrm{d}(s) = \mathrm{d}(\mathrm{sr}) = \mathrm{d}(sr^2) = \mathrm{d}(sr^3) = \mathrm{d}(sr^4) = \mathrm{d}(sr^5) = \mathrm{d}(sr^6) = \mathrm{d}(sr^7)$

$= \mathrm{d}(sr^8) = \mathrm{d}(sr^9) = \mathrm{d}(sr^{10}) = 0,$

$\mathrm{d}(\mathrm{r}) = \mathrm{d}(\mathrm{r}^2) = \mathrm{d}(r^3) = \mathrm{d}(r^4) = \mathrm{d}(r^5) = \mathrm{d}(r^6) = \mathrm{d}(r^7) = \mathrm{d}(r^8)$

$= \mathrm{d}(r^9) = \mathrm{d}(r^{10}) = 9$

The Zagreb indices of the graph of D_{22}

$1 - \mathrm{M}_1\ (\Gamma(D_{22})) = \sum_{\mathrm{v}\in(\mathrm{v}(\Gamma(D_{22}))}\ (\deg(\mathrm{v}))^2$

$= 9^2+9^2+9^2+9^2+9^2+9^2+9^2+9^2+9^2+9^2$

=81+81+81+81+81+81+81+81+ 81 + 81
=810

$2 - \mathrm{M}_2\ (\Gamma(D_{22}) = \sum_{\mathrm{uv}\in\mathrm{E}(\Gamma(D_{22}))}\ (\deg \mathrm{u})\times(\deg \mathrm{v})$

= (9×9)+ (9×9)

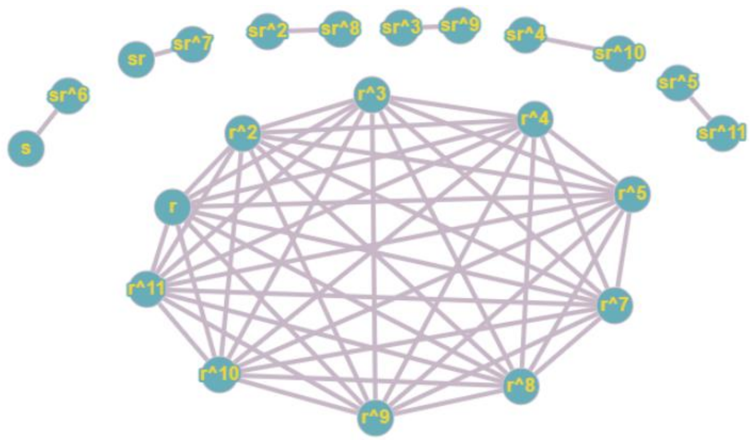
=3645

9)n=12 then D_{2n}=D_{24} $=\{1,r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^9,r^{10},r^{11},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}, sr^{11}\}$

The center of D_{24}=$\{1,r^6\}$

The set of vertex of D_{24} $=\{r,r^2,r^3,r^4,r^5,r^7,r^8,r^9,r^{10},r^{11},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}, sr^{11}\}$

The graph of D_{24}



The degree of the vertex of $\Gamma(D_{24})$ are

$$\mathrm{d}(s) = \mathrm{d}(\mathrm{sr}) = \mathrm{d}(sr^2) = \mathrm{d}(sr^3) = \mathrm{d}(sr^4) = \mathrm{d}(sr^5) = \mathrm{d}(sr^6) = \mathrm{d}(sr^7)$$
$$= \mathrm{d}(sr^8) = \mathrm{d}(sr^9) = \mathrm{d}(sr^{10}) = \mathrm{d}(sr^{11}) = 1,$$
$$\mathrm{d}(\mathrm{r}) = \mathrm{d}(\mathrm{r}^2) = \mathrm{d}(r^3) = \mathrm{d}(r^4) = \mathrm{d}(r^5) = \mathrm{d}(r^7) = \mathrm{d}(r^8) = \mathrm{d}(r^9)$$
$$= \mathrm{d}(r^{10}) = \mathrm{d}(r^{11}) = 9$$

The Zagreb indices of the graph of D_{24}

$1 - \mathrm{M}_1\ (\Gamma(D_{24})) = \sum_{\mathrm{v}\in(\mathrm{v}(\Gamma(D_{24}))}\ (\deg(\mathrm{v}))^2$

$=1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+9^2+9^2+9^2+9^2+9^2+9^2+9^2+9^2+9^2+9^2$

$=81+81+81+81+81+81+81+81+81+81+12$

$=822$

$2 - \mathrm{M}_2\ (\Gamma(D_{24}) = \sum_{\mathrm{uv}\in \mathrm{E}(\Gamma(D_{24}))}\ (\deg \mathrm{u})\times(\deg \mathrm{v})$

$= (1\times1)+ (1\times1)+ (1\times1)+ (1\times1)+ (1\times1)+ (1\times1)+ (9\times9)+$

(9×9)+ (9×9)

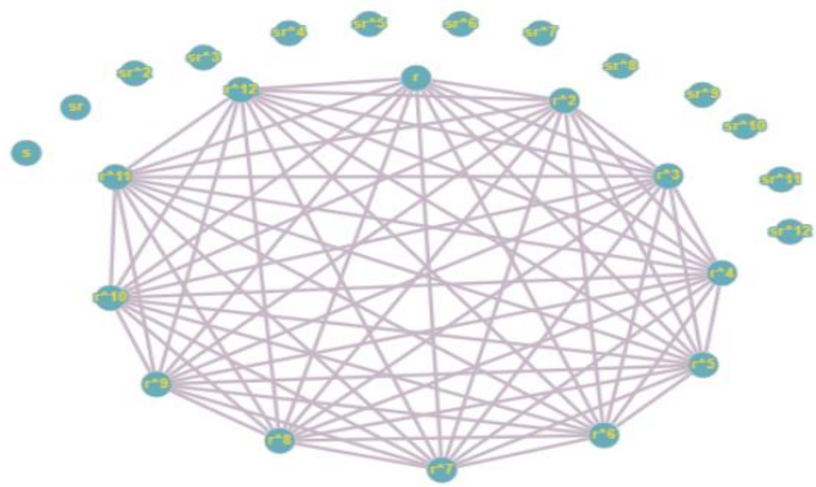
=3651

10) n=13 then D_{2n}=D_{26} ={1,r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^9,r^{10},r^{11},r^{12},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}, sr^{11},sr^{12}}

The center of D_{26}={1}

The set of vertex of D_{24} = {r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^9,r^{10},r^{11},r^{12},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}, sr^{11},sr^{12}}

The graph of D_{26}



The degree of the vertex of $\Gamma(D_{26})$ are

$$\mathrm{d}(s) = \mathrm{d}(\mathrm{sr}) = \mathrm{d}(sr^2) = \mathrm{d}(sr^3) = \mathrm{d}(sr^4) = \mathrm{d}(sr^5) = \mathrm{d}(sr^6) = \mathrm{d}(sr^7)$$
$$= \mathrm{d}(sr^8) = \mathrm{d}(sr^9) = \mathrm{d}(sr^{10}) = \mathrm{d}(sr^{11}) = \mathrm{d}(sr^{12}) = 0,$$
$$\mathrm{d}(\mathrm{r}) = \mathrm{d}(\mathrm{r}^2) = \mathrm{d}(r^3) = \mathrm{d}(r^4) = \mathrm{d}(r^5) = \mathrm{d}(r^6) = \mathrm{d}(r^7) = \mathrm{d}(r^8)$$
$$= \mathrm{d}(r^9) = \mathrm{d}(r^{10}) = \mathrm{d}(r^{11}) = \mathrm{d}(r^{12}) = 11$$

The Zagreb indices of the graph of D_{26}

$$1 - \mathrm{M}_1\ (\Gamma(D_{26})) = \sum_{\mathrm{v}\in(\mathrm{v}(\Gamma(D_{26}))}\ (\deg(\mathrm{v}))^2$$

$=11^2+11^2+11^2+11^2+11^2+11^2+11^2+11^2+11^2+11^2+11^2+11^2$

= 121+121+121+121+121+121+ 121+121+ 121+121

=1452

$2-\mathrm{M}_2\ (\Gamma(D_{26})=\sum_{uv\in E(\Gamma(D_{26}))}$ (deg u)×(deg v)

=(11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11×11)+ (11×11)+ (11×11)+ (11×11) + (11 ×11)+ (11 ×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11×11)+ (11×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11×11)+ (11×11)+ (11×11)+ (11×11) +(11×11)+ (11×11)+ (11×11)+ 11×11)+ (11×11)+ (11×11)+ (11×11) + 11×11)+ (11×11)+ (11×11)+ (11×11) + (11×11)+ (11×11)+ (11×11) +(11×11)+ (11×11)+ (11×11)

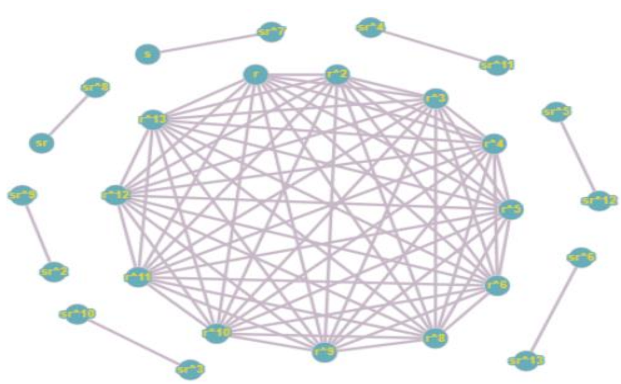
=7986

11) n=14 then D_{2n}=D_{28} $=\{1,r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^9,r^{10},r^{11},r^{12},r^{13},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}, sr^{11},sr^{12},sr^{13}\}$

The center of $D_{28}=\{1,r^7\}$

The set of vertex of $D_{24}=\{r,r^2,r^3,r^4,r^5,r^6,r^8,r^9,r^{10},r^{11},r^{12},r^{13},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}, sr^{11},sr^{12},sr^{13}\}$

The graph of D_{28}



The degree of the vertex of $\Gamma(D_{28})$ are

$$\mathrm{d(s)} = \mathrm{d(sr)} = \mathrm{d}(sr^2) = \mathrm{d}(sr^3) = \mathrm{d}(sr^4) = \mathrm{d}(sr^5) = \mathrm{d}(sr^6) = \mathrm{d}(sr^7)$$
$$= \mathrm{d}(sr^8) = \mathrm{d}(sr^9) = \mathrm{d}(sr^{10}) = \mathrm{d}(sr^{11}) = \mathrm{d}(sr^{12}) = \mathrm{d}(sr^{13}) = 1,$$
$$\mathrm{d(r)} = \mathrm{d(r^2)} = \mathrm{d}(r^3) = \mathrm{d}(r^4) = \mathrm{d}(r^5) = \mathrm{d}(r^6) = \mathrm{d}(r^8) = \mathrm{d}(r^9)$$
$$= \mathrm{d}(r^{10}) = \mathrm{d}(r^{11}) = \mathrm{d}(r^{12}) = \mathrm{d}(r^{13}) = 13$$

The Zagreb indices of the graph of D_{28}

$1 - \mathrm{M}_1\ (\Gamma(D_{28})) = \sum_{\mathrm{v}\in(\mathrm{v}(\Gamma(D_{28}))}\ (\deg(\mathrm{v}))^2$

$=1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+1^2+11^2+11^2+11^2+11^2+11^2+11^2+11^2+11^2+11^2+11^2+11^2+11^2$

$= 121+121+121+121+121+121+ 121+121+ 121+121 +14$

$=1466$

$2 - \mathrm{M}_2\ (\Gamma(D_{28})= \sum_{\mathrm{uv}\in\mathrm{E}(\Gamma(D_{28}))}\ (\deg \mathrm{u})\times(\deg \mathrm{v})$

=(1×1)+ (1×1)+ (1×1)+ (1×1)+ (1×1)+ (1×1)+ (1×1)+ (11×11)+
(11 ×11)+ (11 ×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11×11)+
(11×11)+ (11×11)+ (11×11) + (11 ×11)+ (11 ×11)+ (11×11)+ (11 ×11)+
(11 ×11)+ (11×11)+ (11×11)+ (11×11)+ (11×11)+ (11 ×11)+ (11 ×11)+
(11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11×11)+ (11×11)+ (11 ×11)+
(11 ×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11×11)+ (11 ×11)+
(11 ×11)+ (11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11 ×11)+ (11 ×11)+
(11×11)+ (11 ×11)+ (11 ×11)+ (11×11)+ (11×11)+ (11×11)+ (11×11)+ (11×11)
+(11×11)+ (11×11)+ (11×11)+ 11×11)+ (11×11)+ (11×11)+ (11×11) +
11×11)+ (11×11)+ (11×11)+ (11×11) + (11×11)+ (11×11)+ (11×11) +(11×11)+
(11×11)+ (11×11)

=7993

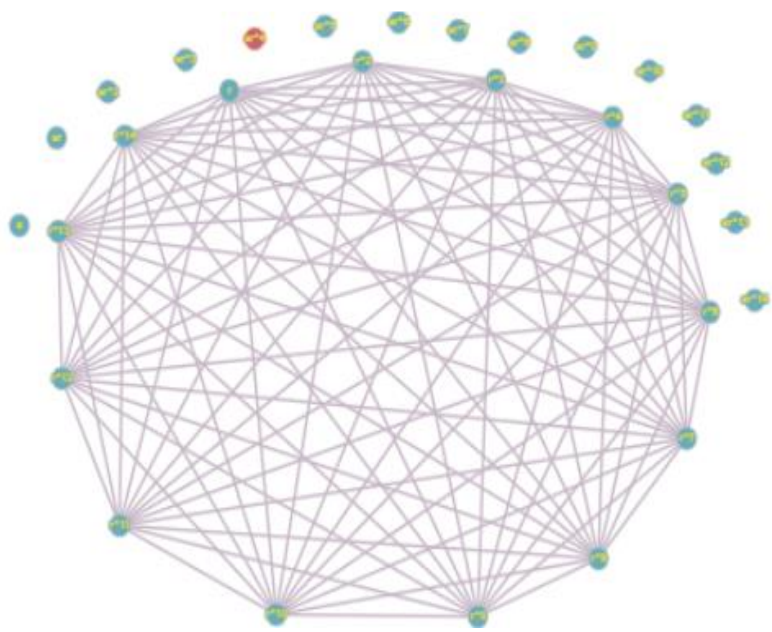
12) If n=15 , then

D_{2n}=D_{30} ={1,r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^9,r^{10},r^{11},r^{12},r^{13},r^{14},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}, sr^{11},sr^{12},sr^{13} ,sr^{14}}

The center of D_{30}={1}

The set of vertex of
D_{24} ={r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^9,r^{10},r^{11},r^{12},r^{13},r^{14},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}, sr^{11},sr^{12},sr^{13} ,sr^{14}}

The graph of D_{30}



The degree of the vertex of $\Gamma(D_{30})$ are

$$\begin{aligned} \mathrm{d}(s) = \mathrm{d}(sr) = \mathrm{d}(sr^2) = \mathrm{d}(sr^3) = \mathrm{d}(sr^4) = \mathrm{d}(sr^5) = \mathrm{d}(sr^6) = \mathrm{d}(sr^7) \\ = \mathrm{d}(sr^8) = \mathrm{d}(sr^9) = \mathrm{d}(sr^{10}) = \mathrm{d}(sr^{11}) = \mathrm{d}(sr^{12}) = \mathrm{d}(sr^{13}) \\ = \mathrm{d}(sr^{14}) = 0, \\ \mathrm{d}(r) = \mathrm{d}(r^2) = \mathrm{d}(r^3) = \mathrm{d}(r^4) = \mathrm{d}(r^5) = \mathrm{d}(r^6) = \mathrm{d}(r^7) = \mathrm{d}(r^8) \\ = \mathrm{d}(r^9) = \mathrm{d}(r^{10}) = \mathrm{d}(r^{11}) = \mathrm{d}(r^{12}) = \mathrm{d}(r^{13}) = \mathrm{d}(r^{14}) = 13 \end{aligned}$$

The Zagreb indices of the graph of D_{30}

$1 - M_1\ (\Gamma(D_{30})) = \sum_{v \in (v(\Gamma(D_{30}))}\ (\deg(v))^2$

$=13^2+13^2+13^2+13^2+13^2+13^2+13^2+13^2+13^2+13^2+13^2+13^2+13^2+13^2$

$= 169+169+169+169+169+169+ 169+169+169+169+169+169 +169+169$

$=2366$

$2 - M_2\ (\Gamma(D_{30}) = \sum_{uv \in E(\Gamma(D_{30}))}\ (\deg u)\times(\deg v)$

$= (13 \times 13)+ (13\times 13)+ (13\times 13)+ (13\times 13)+ (13\times 13)+ (13\times 13) + (13 \times 13)+$
$(13 \times 13)+ (13\times 13)+ (13 \times 13)+ (13 \times 13)+ (13\times 13)+ (13\times 13)+ (13 \times 13)+$
$(13\times 13)+ (13\times 13)+ (13\times 13)+ (13\times 13)+ (13\times 13) + (13 \times 13)+ (13 \times 13)+$
$(13\times 13)+ (13 \times 13)+ (13 \times 13)+ (13\times 13)+ (13 \times 13)+ (13\times 13)+ (13\times 13)+$
$(13\times 13)+ (13\times 13)+ (13\times 13) + (13 \times 13)+ (13 \times 13)+ (13\times 13)+ (13 \times 13)+$
$(13 \times 13)+ (13 \times 13)+ (13\times 13)+ (13\times 13)+ (13 \times 13)+ (13\times 13)+ (13\times 13) +$
$(13 \times 13)+ (13 \times 13)+ (13\times 13)+ (13 \times 13)+ (13 \times 13)+ (13\times 13)+ (13\times 13)+$
$(13\times 13)+ (13\times 13)+ (13\times 13) + (13 \times 13)+ (13 \times 13)+ (13\times 13)+ (13\times 13)+$
$(13\times 13)+ (13\times 13)+ (13\times 13)+ (13\times 13) + (13 \times 13)+ (13 \times 13)+ (13\times 13)+$
$(13\times 13)+ (13\times 13)+ (13\times 13)+ (13\times 13) + (13 \times 13)+ (13 \times 13)+ (13\times 13)+$
$(13\times 13)+ (13\times 13)+ (13\times 13) + (13\times 13)+ (13 \times 13)+ (13\times 13)+ (13\times 13)+ (13\times 13)$
$+ (13 \times 13)+ (13 \times 13)+ (13\times 13)+ (13\times 13) + (13 \times 13)+ (13 \times 13)+ (13\times 13)+$
$(13 \times 13)+ (13 \times 13)+ (13\times 13)+ (113 \times 13)+ (13\times 13)+ (13 \times 13)$

$=15379$

For all vertex a in the set $\{s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}, sr^{11},sr^{12},sr^{13} ,sr^{14}\}$

$D(a)=0$

For all vertex b in the set $\{r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^{9,}r^{10},r^{11},r^{12},r^{13},r^{14}\ \}$

$D(b)=13$

13) n=16, then

$D_{2n}=D_{32} =\{1,r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^{9,}r^{10},r^{11},r^{12},r^{13},r^{14},r^{15},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,s$
$r^8,sr^9,sr^{10}, sr^{11},sr^{12},sr^{13} ,sr^{14},sr^{15}\}$

The center of D_{32}={1,r^8}

The set of vertex of D_{32} ={r,r^2,r^3,r^4,r^5,r^6,r^7,$r^{9,}$$r^{10}$,$r^{11}$,$r^{12}$,$r^{13}$,$r^{14}$,$r^{15}$,s,sr,$sr^2$,$sr^3$,$sr^4$,$sr^5$,$sr^6$,$sr^7$,$sr^8$,$sr^9$,$sr^{10}$, sr^{11},sr^{12},sr^{13} ,sr^{14},sr^{15}}

The degree of the vertex of $\Gamma(D_{32})$ are

$$\begin{aligned} \mathrm{d}(s) = \mathrm{d}(\mathrm{sr}) = \mathrm{d}(sr^2) = \mathrm{d}(sr^3) = \mathrm{d}(sr^4) = \mathrm{d}(sr^5) = \mathrm{d}(sr^6) = \mathrm{d}(sr^7) \\ = \mathrm{d}(sr^8) = \mathrm{d}(sr^9) = \mathrm{d}(sr^{10}) = \mathrm{d}(sr^{11}) = \mathrm{d}(sr^{12}) = \mathrm{d}(sr^{13}) \\ = \mathrm{d}(sr^{14}) = \mathrm{d}(sr^{15}) = 1, \\ \mathrm{d}(\mathrm{r}) = \mathrm{d}(\mathrm{r}^2) = \mathrm{d}(r^3) = \mathrm{d}(r^4) = \mathrm{d}(r^5) = \mathrm{d}(r^6) = \mathrm{d}(r^7) = \mathrm{d}(r^9) \\ = \mathrm{d}(r^{10}) = \mathrm{d}(r^{11}) = \mathrm{d}(r^{12}) = \mathrm{d}(r^{13}) = \mathrm{d}(r^{14}) = \mathrm{d}(r^{15}) = 15 \end{aligned}$$

The Zagreb indices of the graph of D_{32}

$1 - \mathrm{M}_1$ $(\Gamma(D_{32}))$ =$\sum_{\mathrm{v}\in(\mathrm{v}(\Gamma(D_{32}))}$ $(\deg(\mathrm{v}))^2$

=$16(1)^2$+14($15^{2)}$

=16+14(225)

= 3166

$2 - \mathrm{M}_2$ $(\Gamma(D_{32})$= $\sum_{\mathrm{uv}\in\mathrm{E}(\Gamma(D_{32}))}$ (deg u)×(deg v)

= 8(1×1)+14(15 ×15) +14(15 ×15) +14(15 ×15) +14(15 ×15) +14(15 ×15) +14(15 ×15) +14(15 ×15) +14(15 ×15) +14(15 ×15) +14(15 ×15) +14(15 ×15) +14(15 ×15)

= 8+14(14(15 ×15))

=8+196 (225)

=8+44100

=4418

14) n=17 , then

D_{2n}=D_{34} ={1,r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^9,r^{10},r^{11},r^{12},r^{13},r^{14},r^{15},r^{16},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}, sr^{11},sr^{12},sr^{13} ,sr^{14},sr^{15},sr^{16}}

The center of D_{34}={1}

The set of vertex of D_{34} ={r,r^2,r^3,r^4,r^5,r^6,r^7,r^9,r^{10},r^{11},r^{12},r^{13},r^{14},r^{15},r^{16},s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}, sr^{11},sr^{12},sr^{13} ,sr^{14} ,sr^{15},sr^{16}}

The degree of the vertex of $\Gamma(D_{34})$ are

$$\begin{aligned} \mathrm{d}(s) = \mathrm{d}(\mathrm{sr}) = \mathrm{d}(sr^2) = \mathrm{d}(sr^3) = \mathrm{d}(sr^4) = \mathrm{d}(sr^5) = \mathrm{d}(sr^6) = \mathrm{d}(sr^7) \\ = \mathrm{d}(sr^8) = \mathrm{d}(sr^9) = \mathrm{d}(sr^{10}) = \mathrm{d}(sr^{11}) = \mathrm{d}(sr^{12}) = \mathrm{d}(sr^{13}) \\ = \mathrm{d}(sr^{14}) = \mathrm{d}(sr^{15}) = \mathrm{d}(sr^{16}) = 0, \\ \mathrm{d}(\mathrm{r}) = \mathrm{d}(\mathrm{r}^2) = \mathrm{d}(r^3) = \mathrm{d}(r^4) = \mathrm{d}(r^5) = \mathrm{d}(r^6) = \mathrm{d}(r^7) = \mathrm{d}(r^8) \\ = \mathrm{d}(r^9) = \mathrm{d}(r^{10}) = \mathrm{d}(r^{11}) = \mathrm{d}(r^{12}) = \mathrm{d}(r^{13}) = \mathrm{d}(r^{14}) = \mathrm{d}(r^{15}) \\ == \mathrm{d}(r^{16}) = 15 \end{aligned}$$

The Zagreb indices of the graph of D_{34}

$1 - \mathrm{M}_1 (\Gamma(D_{34})) = \sum_{\mathrm{v} \in (\mathrm{v}(\Gamma(D_{34}))} (\deg(\mathrm{v}))^2$

=16($15^{2)}$
=16(225)
= 3600

$2 - \mathrm{M}_2 (\Gamma(D_{34}) = \sum_{\mathrm{uv} \in \mathrm{E}(\Gamma(D_{34}))} (\deg \mathrm{u}) \times (\deg \mathrm{v})$

=15×(15× 15)+14×(15× 15)+13×(15× 15)+12×(15× 15)+11×(15× 15)+10×(15× 15)+9×(15× 15)+8×(15× 15)+7×(15× 15)+6×(15× 15)+5×(15× 15)+4×(15× 15)+3×(15× 15)+2×(15× 15)+ (15× 15)=(15× 15)(15+14+13+12+11+10+9+8+7+6+5+4+3+2+1)=27000

15) n=18 , then

D_{2n}=D_{36} ={1,r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,$r^{9,}$$r^{10}$,$r^{11}$,$r^{12}$,$r^{13}$,$r^{14}$,$r^{15}$,$r^{16}$,$r^{17}$,s,sr,$sr^2$,$sr^3$,$sr^4$,$sr^5$,$sr^6$,$sr^7$,$sr^8$,$sr^9$,$sr^{10}$, sr^{11},sr^{12},sr^{13} ,sr^{14},sr^{15},sr^{16},sr^{17}}

The center of D_{36}={1,r^9}

The set of vertex of D_{36} ={r,r^2,r^3,r^4,r^5,r^6,r^7,r^8,r^{10},r^{11},r^{12},r^{13},r^{14},r^{15},r^{16},r^{17} s,sr,sr^2,sr^3,sr^4,sr^5,sr^6,sr^7,sr^8,sr^9,sr^{10}, sr^{11},sr^{12},sr^{13} ,sr^{14} ,sr^{15},sr^{16},sr^{17}}

The degree of the vertex of $\Gamma(D_{36})$ are

$$\begin{aligned}
\mathrm{d}(s) = \mathrm{d}(\mathrm{sr}) = \mathrm{d}(sr^2) = \mathrm{d}(sr^3) = \mathrm{d}(sr^4) = \mathrm{d}(sr^5) = \mathrm{d}(sr^6) = \mathrm{d}(sr^7) \\
= \mathrm{d}(sr^8) = \mathrm{d}(sr^9) = \mathrm{d}(sr^{10}) = \mathrm{d}(sr^{11}) = \mathrm{d}(sr^{12}) = \mathrm{d}(sr^{13}) \\
= \mathrm{d}(sr^{14}) = \mathrm{d}(sr^{15}) = \mathrm{d}(sr^{16}) = \mathrm{d}(sr^{17}) = 1, \\
\mathrm{d}(\mathrm{r}) = \mathrm{d}(\mathrm{r}^2) = \mathrm{d}(r^3) = \mathrm{d}(r^4) = \mathrm{d}(r^5) = \mathrm{d}(r^6) = \mathrm{d}(r^7) = \mathrm{d}(r^8) \\
= \mathrm{d}(r^{10}) = \mathrm{d}(r^{11}) = \mathrm{d}(r^{12}) = \mathrm{d}(r^{13}) = \mathrm{d}(r^{14}) = \mathrm{d}(r^{15}) = \\
= \mathrm{d}(r^{16}) + \mathrm{d}(r^{17}) = 15
\end{aligned}$$

The Zagreb indices of the graph of D_{36}

$1 - \mathrm{M}_1$ $(\Gamma(D_{36}))$ =$\sum_{\mathrm{v}\in(\mathrm{v}(\Gamma(D_{36}))}$ $(\deg(\mathrm{v}))^2$

=18$(1)^2$+16($15^{2)}$

=14(225)

= 3618

$2 - \mathrm{M}_2$ $(\Gamma(D_{32})$= $\sum_{\mathrm{uv}\in\mathrm{E}(\Gamma(D_{32}))}$ (deg u)×(deg v)

=9×(1× 1)+15×(15× 15)+14×(15× 15)+13×(15× 15)+12×(15× 15)+11×(15× 15)+10×(15× 15)+9×(15× 15)+8×(15× 15)+7×(15× 15)+6×(15× 15)+5×(15× 15)+4×(15× 15)+3×(15× 15)+2×(15× 15)+ (15× 15)

= 9+27000

=27009

References

Behzad M and Chartrand G, 1979. *Graph and Diagraphs.* California : Wodsworth.

Burton, D. M., 1980. *Elemerntary Number Theory*. London: Allyn and Bacon, Inc.

I. Gutman and Trinajstic, n.d. s.l.:s.n.

I.Gustman and N.Trinjajstic, 1972. Graph theory and milecular orbitals. *Total pi-electron energy of altennant hydrocarbons, Chem. Phys. Lett*, 11, Volume 17, pp. 535-538.

Marlo Anderson and Todd Feil, 2015. *A First Course in Abstract Algebra rings, groups and fields.* Third Edition ed. s.l.:s.n.

Naduvath, S., 2017. *Lecture Notes On Graph Theory.* s.l.:Centre for Studies in Discrete Mathematics.

پوختە

بە درێژایی ئەم کارە هەموو گروپەکان بە شێوەیەکی کۆتایی و بێ ئابێلان دادەنرێن.

ئامانجی ئەم کارە هەژمارکردن و هەژمارکردنی پێوەرەکانی زاگرۆبە بۆ گرافەکانی گروپی دیهیدرال
D2n =<r،s | rn = s2 = 1 - s-1 rs = r-1 >

n≥3 بیر لە D2n بکەرەوە r /بە ئاڕاستەی کاتژمێر بە ٣٦٠ بێت و ڕێگە بە

S^{-1} و S بدەن ببنە دوو ڕەنگدانەوەی تەنیشتی n-gon .ئاسایی

$$D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}.$$