Chapter Three

Determinants and Inverses

Definition: Let A be a square matrix. The determinant of A is a function which assign for A to the number of the field F. And denoted by det(A), f(A)=|A|.

Determinants of 1×1 matrices

$f([a]_{1\times1}) = |a| = a$

For example A=$[-5]$ then $|-5| = -5$

Determinants of 2×2 matrices

The value of the determinant of a 2 × 2 matrix $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{21} \end{pmatrix}$, can be given as

det(A)=$|A| = a_{11}a_{21} - a_{12}a_{21}$

Let us take an example to understand this very clearly,

Example 1: The matrix is given by, $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$

Find the value of |A|.

det(A)=$|A| = 3 \cdot 4 - 2 \cdot 1 = 10.$

Example 2: The matrix is given by, $A = \begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix}$

Find the value of |A|.

det(A)=$|A| = 3 \cdot 3 - (-1) \cdot 4 = 13.$

Determinants of 3×3 matrices

The first Way

The value of the determinant of a 3 × 3 matrix A=$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}_{3\times3}$, can be given as

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \bullet \det\begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \bullet \det\begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \bullet \det\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

Example 1: The matrix is given by, $A = \begin{pmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{pmatrix}$

Find the value of |A|.

det(A)=$|A| = 2 \cdot \begin{vmatrix} 0 & -1 \\ 4 & 5 \end{vmatrix} - (-3) \cdot \begin{vmatrix} 2 & -1 \\ 1 & 5 \end{vmatrix} + 1 \begin{vmatrix} 2 & 0 \\ 1 & 4 \end{vmatrix}$

$= 2 \ .(4) \ + \ 3 \ . \ 11 + 1 \ . \ 8$

$= 8 + 33 + 8$

$= 49$

Example 2: The matrix is given by, $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & 3 \\ 1 & 5 & 4 \end{pmatrix}$

Find the value of |A|.

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 3 \\ 1 & 5 & 4 \end{bmatrix} = \begin{bmatrix} \boxed{1} & -2 & 3 \\ 2 & 0 & 3 \\ 1 & 5 & 4 \end{bmatrix} - \begin{bmatrix} 1 & \boxed{-2} & 3 \\ 2 & 0 & 3 \\ 1 & 5 & 4 \end{bmatrix} + \begin{bmatrix} 1 & -2 & \boxed{3} \\ 2 & 0 & 3 \\ 1 & 5 & 4 \end{bmatrix}$$

$$= \boxed{1} \times \begin{vmatrix} 0 & 3 \\ 5 & 4 \end{vmatrix} - \boxed{-2} \times \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + \boxed{3} \times \begin{vmatrix} 2 & 0 \\ 1 & 5 \end{vmatrix}$$

$$= 1 \times (0 - 15) + 2 \times (8 - 3) + 3 \times (10 - 0)$$

$$= 1(-15) + 2(5) + 3(10)$$

$$= -15 + 10 + 30$$

$$= 25$$

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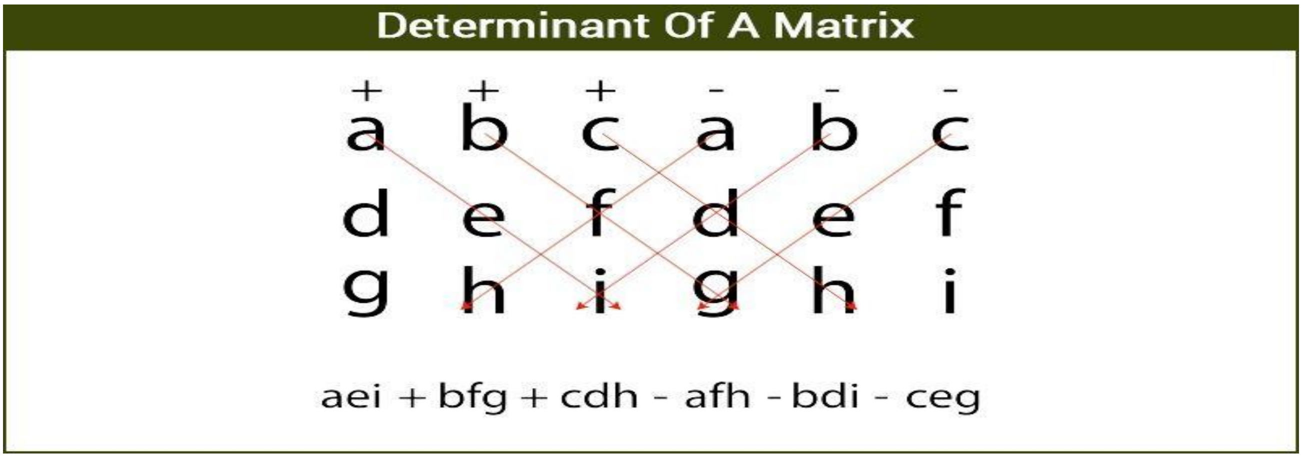
The second Way



For example A=$\begin{pmatrix} 1 & 2 & -1 \\ 3 & -2 & 0 \\ 5 & 0 & -5 \end{pmatrix}_{3\times 3}$ then find det(A)

$$\begin{array}{c} \quad + \quad + \quad + \quad - \quad - \quad - \\ \text{det(A)}=|A| = \left|\begin{matrix} 1 & 2 & -1 \\ 3 & -2 & 0 \\ 5 & 0 & -5 \end{matrix}\right| \begin{matrix} 1 & 2 & -1 \\ 3 & -2 & 0 \\ 5 & 0 & -5 \end{matrix} \end{array}$$

$= (1)(-2)(-5) + (2)(0)(5) + (-1)(3)(0) - (1)(0)(0) - (2)(3)(-5) -$
$(-1)(-2)(5) = 10 + 0 + 0 - 0 + 30 - 10 = 30$

Compute the determinant for the following:

1. $\begin{vmatrix} 4 & -1 \\ 6 & 2 \end{vmatrix}$
2. $\begin{vmatrix} 10 & 6 \\ 4 & 5 \end{vmatrix}$

Evaluate the determinant for the following:

1. $\begin{vmatrix} 6 & 4 & 2 \\ 3 & -7 & 1 \\ 5 & 5 & 3 \end{vmatrix}$
2. $\begin{vmatrix} 3 & 6 & 4 \\ 9 & 2 & 7 \\ 6 & 5 & -1 \end{vmatrix}$

1- Find the det(A), where A= $\begin{pmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 5 & -3 & 6 \end{pmatrix}$

2- Evaluate $|B|$, where $B = \begin{pmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{pmatrix}$

Cofactor Expansion

Let A=(a_{ij}) be an n *by* n matrix and C=(c_{ij}) *be an* $(n-1) \times (n-1)$ sub matrix of A obtained by deleting the i th row and j th column of A.

And the cofactor A_{ij} of a_{ij} is defined by $A_{ij} = (-1)^{i+j} \; |C_{ij}|$. Then

(Expansion of $|A|$ about the ith row) $|A| = a_{i1}A_{i1} + a_{i2}A_{i2} + \cdots + a_{in}A_{in}$

(Expansion of $|A|$ about the jth column) $|A| = a_{1j}A_{1j} + a_{2j}A_{2j} + \cdots + a_{nj}A_{nj}$

Example 1: Let A = $\begin{pmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{pmatrix}$. Evaluate det(A) by cofactor expansion along the first row of A.

Solution: $|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$a_{11} = 3, a_{12} = 1, a_{13} = 0$

$A_{ij} = (-1)^{i+j} \; |C_{ij}|,$

$C_{11} = \begin{pmatrix} -4 & 3 \\ 4 & -2 \end{pmatrix}, \qquad C_{12} = \begin{pmatrix} -2 & 3 \\ 5 & -2 \end{pmatrix}, \qquad C_{13} = \begin{pmatrix} -2 & -4 \\ 5 & 4 \end{pmatrix}$

$A_{11} = (-1)^{1+1} \begin{vmatrix} -4 & 3 \\ 4 & -2 \end{vmatrix} = (-1)^2\big((-4)(-2) - (3)(4)\big) = -4$

$A_{12} = (-1)^{1+2} \begin{vmatrix} -2 & 3 \\ 5 & -2 \end{vmatrix} = (-1)^3\big((-2)(-2) - (3)(5)\big) = (-1)(4 - 15) = 11$

$A_{13} = (-1)^{1+3} \begin{vmatrix} -2 & -4 \\ 5 & 4 \end{vmatrix} = (-1)^4\big((-2)(4) — 4\big)(5) = 12$

$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}, \; |A| = (3)(-4) + (1)(11) + (0)(12) = -1.$

Example 2 : Let $= \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 0 & 0 & 1 \\ 3 & 1 & 2 & 1 \\ 0 & 3 & 1 & 0 \end{pmatrix}$. Evaluate det(A) by cofactor expansion

Solution: $|A| = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} + a_{24}A_{24}$

$a_{21} = 2,\ a_{22} = 0,\ a_{23} = 0,\ a_{23} = 1$

, $A_{ij} = (-1)^{i+j}\ |C_{ij}|,\ \ C_{21} = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 3 & 1 & 0 \end{pmatrix}, C_{22} = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix},$

$$C_{23} = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 1 \\ 0 & 3 & 0 \end{pmatrix}, \qquad C_{24} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 0 & 3 & 1 \end{pmatrix}$$

$$\begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 3 & 1 & 0 \end{vmatrix} \begin{matrix} 2 & 3 \\ 1 & 2 \\ 3 & 1 \end{matrix} = 0 + 9 + 1 - 6 - 2 - 0 = 2$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 0 & 3 & 1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ 0 & 3 \end{matrix} = 1 + 0 + 27 - 0 - 6 - 6 = 16$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 3 & 1 & 0 \end{vmatrix} = (-1)^3(2) = -2,\quad A_{23} =?,\quad A_{24} =?.$$

$$A_{24} = (-1)^{2+4} \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 0 & 3 & 1 \end{vmatrix} = (-1)^6(16) = 16$$

$$|A| = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} + a_{24}A_{24}$$
$$= (2)(-2) + (0)A_{22} + (0)A_{23} + (1)(16) = 12$$

Properties of Determinants

1-If a square a matrix A has a row of zero or a column of zeros, then $|A|$= 0

For example $A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ then $|A|$= 0.

2- Let A be a square matrix. Then $|A| = |A^t|$

For example A= $\begin{pmatrix} -4 & 3 \\ 2 & -2 \end{pmatrix}$ then $|A| = (-4)(-2) - (3)(2) = 2 = |A^t|$

3- If A is an $n \times n$ traianglar matrix (upper triangular, lower triangular, or diagonal), then $|A|$ is the product of the entries on the main diagonal of the matrix; that is

$|A|=a_{11}\cdot a_{22}\cdot\cdots\cdot a_{nn}.$

For example i) $A=\begin{pmatrix}1&0&0&0\\2&2&0&0\\3&1&2&0\\0&3&1&3\end{pmatrix}$, $then\ |A|=(1)(2)(2)(3)=12$

ii) A=$\begin{pmatrix}1&3&1\\0&2&1\\0&0&3\end{pmatrix}$ $then\ |A|=6.$

iii) A=$\begin{pmatrix}2&0&0\\0&-4&0\\0&0&3\end{pmatrix}$ $then\ |A|=-24.$

4- Let A be an $n\times n$ matrix. If B is the matrix that results when a single row or single column of A is multiplied by a scalar k, then det(B) = k det(A).

For example $A=\begin{pmatrix}1&2&1\\3&1&0\\2&3&3\end{pmatrix}$ $and\ B=\begin{pmatrix}1&2&1\\3&1&0\\4&6&6\end{pmatrix}$ $then\ |B|=2|A|$

$$|B|=2(-8)=-16$$

5- Let A be an $n\times n$ matrix. If B is the matrix that results when two rows or two columns of A is interchanging , then det(B) = − det(A).

For example A=$\begin{pmatrix}1&2&-1\\3&-2&0\\5&0&-5\end{pmatrix}_{3\times3}$ then det(A)=30

and B=$\begin{pmatrix}1&2&-1\\5&0&-5\\3&-2&0\end{pmatrix}_{3\times3}$ then det(B)=−30

6- If two row(columns) of a matrix A are equal, then det(A)=0.

For example A=$\begin{pmatrix}1&2&2\\3&2&2\\5&0&0\end{pmatrix}_{3\times3}$ then det(A)=0

7- Let A be any $n\times n$ matrix, then det $(kA)=k^n$ det(A).

For example A= $\begin{pmatrix}1&1\\2&-2\end{pmatrix}$ then det(A)=−4

and det(3A)=$\begin{vmatrix}3&3\\6&-6\end{vmatrix}=-36=(3^2)(-4)$

8- If A and B are square matrices of the same size, then det(A.B) = det(A).det(B).

For example $A = \begin{pmatrix} 1 & 1 \\ 2 & -2 \end{pmatrix}$ then det(A)=-4

and $B = \begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix}$ then det(B)=2, $A \cdot B = \begin{pmatrix} 4 & 7 \\ 0 & -2 \end{pmatrix}$ and det($A \cdot B$)=-8

det(A) det(B)=$(2)(-4) = -8 =$ det($A \cdot B$).

Matrix Invers

- **Singular matrix**

A *singular matrix* is a square matrix with a determinant value equal to zero. We cannot find the inverse of a singular matrix. For a singular matrix, |A| = 0.

- **Non-singular Matrix**

A non-singular matrix is a square matrix with a non-zero determinant. To find the inverse of a matrix, the non-singular matrix property must be satisfied. For a non-singular matrix, |A| ≠ 0.

- **Adjoint of a Matrix**

Let A=$[a_{ij}]$ be a square matrix of order n. The adjoint of a matrix A is the transpose of the cofactor matrix of A. It is denoted by adj (A)

Example:

Find the adjoint of the matrix.

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

To find the adjoint of a matrix A, first find the cofactor matrix of the given matrix. Then find the transpose of the cofactor matrix.

Cofactor of $3 = A_{11} = \begin{vmatrix} -2 & 0 \\ 2 & -1 \end{vmatrix} = 2$

Cofactor of $1 = A_{12} = -\begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} = 2$

Cofactor of $-1 = A_{13} = \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} = 6$

Cofactor of $2 = A_{21} = -\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1$

Cofactor of $-2 = A_{22} = \begin{vmatrix} 3 & -1 \\ 1 & -1 \end{vmatrix} = -2$

Cofactor of $0 = A_{23} = -\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = -5$

Cofactor of $1 = A_{31} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -2$

Cofactor of $2 = A_{32} = -\begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} = -2$

Cofactor of $-1 = A_{33} = \begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix} = -8$

The cofactor matrix of A is $[A_{ij}] = \begin{bmatrix} 2 & 2 & 6 \\ -1 & -2 & -5 \\ -2 & -2 & -8 \end{bmatrix}$

Now find the transpose of A_{ij} .

$$adj\,A = (A_{ij})^T$$
$$= \begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$$

Definition: A square matrix A is invertible (or nonsingular) if there exists a square matrix B such that $A_{n\times n}B_{n\times n} = I_{n\times n} = B_{n\times n}A_{n\times n}$, such that I is identity matrix and B is inverse of matrix A and denoted by $B = A^{-1}$, that is

$A\ A^{-1} = I = A^{-1}A$.

Inverse Matrix Method

The inverse of a matrix can be found using the three different methods. However, any of these three methods will produce the same result.

Method 1:

Let $\mathbf{A}= \begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{bmatrix}$

The inverse of a matrix A is found using the following formula

$$A^{-1} = \begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{bmatrix}^{-1}$$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

For example A= $\begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$, Find A^{-1}

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{(2)(3)-(-2)(-1)} \begin{bmatrix} 3 & -(-2) \\ -(-1) & 2 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$$

Method 2:

Let $\mathbf{A}= \begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{bmatrix}$

The inverse of a matrix A is found using the following formula

$$\mathbf{A}\ A^{-1} = \begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{bmatrix} \begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{bmatrix}^{-1}$$

Example: Find the inverse of the matrix A= $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

Solution: Since $A\,A^{-1} = I$ then suppose that $A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$, A\,A^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a+c & b+d \\ -a+c & -b+d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Then $a + c = 1$ $\qquad b + d = 0$

$\underline{.\quad -a + c = 0}$ $\qquad \underline{-b + d = 1}$

$2c = 1 \Rightarrow c = \frac{1}{2}$ $\qquad 2d = 1 \Rightarrow d = \frac{1}{2}$

$a = \frac{1}{2}$ $\qquad b = -\frac{1}{2} \Rightarrow A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$

Properties

A few important properties of the inverse matrix are listed below.

- If A is nonsingular, then$(A^{-1})^{-1} = A$
- If A and B are nonsingular matrices, then AB is nonsingular. Thus,
- $(AB)^{-1} = B^{-1}A^{-1}$
- If A is nonsingular then $(A^{-1})^{T} = (A^{T})^{-1}$.
- If the inverse of the matrix A exists then it is unique.

Method 3:

One of the most important methods of finding the matrix inverse involves finding the determinants and cofactors of elements of the given matrix. Observe the below steps to understand this method clearly.

- The inverse matrix is also found using the following equation:

$$A^{-1} = \frac{1}{det(A)}\, adj(A).$$

where *adj(A)* refers to the adjoint of a matrix A, *det(A)* refers to the determinant of a matrix A.

1- Find determinant of A.

2- Find the cofactor of each element of A and arrange them in matrix C(A)

3- Find the transpose of C(A) (this new matrix is called *adjoint* of A and denoted by adj(A)), where adj(A)=$(C(A))^t$

4- Using this formula $A^{-1} = \frac{1}{det(A)}\, adj(A)$.

Example: Find the inverse of the matrix A= $\begin{pmatrix} 4 & 3 \\ -3 & -1 \end{pmatrix}$ by using *adjoint*.

Solution: We can check that $det(A) = 5$

Thus the cofactors of A are

$A_{11} = -1$ $A_{12} = 3$

$A_{21} = -3$ $A_{22} = 4$,

, $C(A) = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ -3 & 4 \end{pmatrix}$

$adj(A) = (C(A))^t = \begin{pmatrix} -1 & -3 \\ 3 & 4 \end{pmatrix}$.

$A^{-1} = \frac{1}{det(A)}\, adj(A)$

$= \frac{1}{5}\begin{pmatrix} -1 & -3 \\ 3 & 4 \end{pmatrix}$.

Example. Find the inverse of the matrix $\begin{pmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{pmatrix}$ by using adjoint

Solution: We can check that det(A) = 64. Thus The cofactors of A are

$A_{11} = 12$ $A_{12} = 6$ $A_{13} = -16$

$A_{21} = 4$ $A_{22} = 2$ $A_{23} = 16$

$A_{31} = 12$ $A_{32} = -10$ $A_{33} = 16$

So the matrix C(A)=$\begin{pmatrix} 12 & 6 & -16 \\ 4 & 2 & 16 \\ 12 & -10 & 16 \end{pmatrix}$

and the adjoint of A is $adj(A) = \begin{pmatrix} 12 & 4 & 12 \\ 6 & 2 & -10 \\ -16 & 16 & 16 \end{pmatrix}$

$$A^{-1} = \frac{1}{64}\begin{pmatrix} 12 & 4 & 12 \\ 6 & 2 & -10 \\ -16 & 16 & 16 \end{pmatrix} = \begin{pmatrix} 12/64 & 4/64 & 12/64 \\ 6/64 & 2/64 & -10/64 \\ -16/64 & 16/64 & 16/64 \end{pmatrix}$$

Example: Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$

Exercise: 1- Let $A = \begin{pmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{pmatrix}$ Find the inverse of A.

2- Let $A = \begin{pmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{pmatrix}$ Find all the cofactors.

3-Let $A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{pmatrix}$

, $B = \begin{pmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{pmatrix}$, $a = 4,\ b = -7$

Show that

(a) $A + (B + C) = (A + B) + C$ (b) $(AB)C = A(BC)$

(c) $(a + b)C = aC + bC$ (d) $a(B - C) = aB - aC$

4- Compute the inverse of the following matrices

, $B = \begin{pmatrix} 2 & -3 \\ 4 & 4 \end{pmatrix} A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$

5- Use the matrix A and B in exercise 4 to verify

(a) $(A + B)^t = A^t + B^t$ (b) $(B^t)^{-1} = (B^{-1})^t$

6- Use the matrix A in exercise 4 to compute A^3 *and* $A^2 - 2A + I$.

7- Show that if a square matrix A satisfies $A^2 - 3A + I = 0,$ *then* $A^{-1} = 3I - A$.