Chapter One

Q1) Simplify the followings

1- 4(3+2i)- 5(2-6i) + (5+8i)

2- (2+3i) (2-5i) (3+2i).

3- (4, -1) (-3, 3) + (-2, 3)(2, 1)

4- $(7, 11)^{2}$

Q2) Let $Z_1 = 2 + 3i \ \ and \ Z_2 = 3 - i$ **Find**

$\frac{Z_1}{Z_2}$, $\frac{Z_2}{Z_1}$, $\frac{1}{Z_1}$, $\frac{1}{Z_2}$

Q3) Compute $\left(\frac{1}{2+3i}\right)\left(\frac{2}{1-2i}\right)$

Q4) Find the value of x and y of the equation

$(3, 4)^2 - 2((x, -y) = (x, y)$.

Q5) Find the value of x and y of the equation

$\left(\frac{2+i}{2-i}\right)^2 + \frac{1}{2x+iy} = 2 + i$.

Q6) Find the value of x and y of the equation

$(1 + 4i)^2 - 2(x - yi) = x + yi.$

Q7) Express $\frac{2-3i}{4+3i}$ in the form $a + bi$

Q8) If $\frac{x-iy}{x+iy} = a + ib$, then prove that $a^2 + b^2 = 1$.

Q9) Prove that $(\frac{1}{2} + \frac{\sqrt{3}}{2} i\ )^3 = -1.$

Q10) Let $Z_1 = 1 + 3i\ ,\ Z_2 = 4 + 2i \ and \ Z_3 = 2 - i$ **Find**

1- $|Z_1|$ and $\overline{Z_1}$, $|Z_2|$ and $\overline{Z_2}$, $|Z_3|$ and $\overline{Z_3}$.

2- $\frac{\overline{Z_1 Z_2}}{|Z_1|}$, $\frac{\overline{Z_2 Z_3}}{|Z_2|}$, $\frac{\overline{Z_3 Z_2}}{|Z_3|}$.

Q11) Find the values of

i^{n+1}, $\frac{5}{-3+4\mathrm{i}}$, $(1+2i)^4$, $(1+i)^n + (1-i)^n$.

Q12) Let n be a positive integer then prove that

$(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos\frac{n\pi}{6}$

Q13) Find the polar forms of the following complex numbers:

1) $(\frac{-1}{2} + i\ \frac{\sqrt{3}}{2})$

2) $(2 - i)$

3) $(2+2i)$

4- $(-\sqrt{3}-i)$

Q14) Find The value of $Z = (-\sqrt{3}+i)^4$.

Q15) If we have $z_1 = 8(cos55, sin55)$ and $z_2 = 6(cos35, sin35)$, then find

1- $|z_1 z_2|$ 2- $arg(z_1 z_2)$ 3 – $\left|\frac{z_1}{z_2}\right|$ 4- $arg\left(\frac{z_1}{z_2}\right)$.

Chapter Two

Q16) Construct a matrix of degree 3×3 such that its elements are of the form $a_{ij} = i^2 - j^2$. Then determine which type of matrix it is

Q17) Find the matrix A=$(a_{ij})_{3\times2}$ such that a_{ij}=2j+i^2.

Q18) Find A+B, A−C, B+C and A−B+C where

A = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \\ -1 & 2 & 9 & 0 \end{pmatrix}$, B = $\begin{pmatrix} -1 & 6 & 3 & 7 \\ 2 & 1 & 4 & -2 \\ -1 & 3 & 8 & 1 \end{pmatrix}$ and C = $\begin{pmatrix} 1 & 0 & 9 & 4 \\ 4 & 2 & 2 & 11 \\ 5 & 2 & 9 & 0 \end{pmatrix}$

Q19) Find $\sqrt{3}$A where $A = \begin{pmatrix} 6 & 12 \\ 3 & 1 \\ -15 & 0 \end{pmatrix}$.

Q20) Find $A \cdot B$ where $A = \begin{pmatrix} 1 & 2 \\ 6 & -3 \\ 0 & 1 \end{pmatrix}_{3\times2}$ and $B = \begin{pmatrix} -1 \\ -1 \end{pmatrix}_{2\times1}$ and $B \cdot A$ if it is possible

Q21) Determine the matrices A and B where 3A+B= $\begin{pmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{pmatrix}$ and

2A−B= $\begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$.

Q22) Let $A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{pmatrix}$,

$a = 4,\ b = -7$

Show that

(a) $A + (B + C) = (A + B) + C$ $(c)(AB)C = A(BC)$

(b) $(a + b)C = aC + bC$ $(d)\ a(B - C) = aB - aC$

Q23) 5- Use the matrix $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -3 \\ 4 & 4 \end{pmatrix}$

to verify $(A + B)^t = A^t + B^t$

Q24) Define idempotent matrix. Show that the matrix A= $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

is idempotent

Q25) Let A=$\begin{pmatrix} -1 & 0 \\ 4 & 2 \\ 1 & 3 \end{pmatrix}$ $and\ B = \begin{pmatrix} 1 & 3 \\ -1 & 4 \end{pmatrix}$. Find A B and B A if it is possible

Q26) Show that the matrix A is Nilpotent, where A=$\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$

Q27) Let $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$. Assuming det(A) = -7, find det(3A).

Q28) Find the value of x, y, z, w for which the matrices

$\begin{pmatrix} x+y & y-z \\ 5-w & 7+x \end{pmatrix}$ and $\begin{pmatrix} w-x & z-w \\ z-y & x+z+w \end{pmatrix}$ may be equal

Q29) Find p(A), where p(x)= $x^2 + 3x - 2$ and A= $\begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$

Q30) If $A = \begin{pmatrix} 1 & 1 \\ x & y \\ x^2 & y^2 \end{pmatrix}$, find the value of $A\,A^t$. Then determine which type of

Matrix

Chapter Three

Q31) Find the det(A), where A=$\begin{pmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 5 & -3 & 6 \end{pmatrix}$

Q32) Evaluate $|B|$, where $B = \begin{pmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{pmatrix}$

Q33) Find the inverse of the matrix $A = \begin{pmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{pmatrix}$ by using adjoint

Q34) Let A be a 4 by 4 matrix and $det(A) = -2$ then find $det(3A)$

Q35) Use the matrix $A = \begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix}$ to compute A^3 and $A^2 - 2A + I$.

Q36) Use the given information to find A.

(a) $A^{-1} = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}$ (b) $(5A^t)^{-1} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$

Q37) Show that if a square matrix A satisfies $A^2 - 3A + I = 0, then$ $A^{-1} = 3I - A$.

Q38) Let $A = \begin{pmatrix} 2 & 5 & 4 \\ 0 & 3 & 1 \\ 0 & 0 & -2 \end{pmatrix}$. Then find the eigenvalue of 1) A^4 2) A^{-1}

Q39) Solve the equation $\begin{vmatrix} x & 5 & 7 \\ 0 & x+1 & 6 \\ 0 & 0 & 2x-1 \end{vmatrix} = 0$.

Q40) Let A be a 3× 3 matrix. Assuming det(A) = -7, find det(3A) and det($2A^{-1}$).

Chapter Four

Q41) Solve the system of equations by (Cramer's rule, Gaussian –Jordan method)

$$4x + 5y = 2$$
$$11x + y + 2z = 3$$
$$x + 5y + 2z = 1$$

Q42) Using Cramer rule to solve the system of linear equations

x +2y+ $2z$ = 6

$-3x + 4y + 6z = 3$

$-x - 2y + 3z = 8$

Q42) Solve the following linear systems using Gausse – Gordan method and Cramer's method (if possible).

1) $x + 2y - 3z = 6$

$2x - y + 4z = 2$

$4x + 3y - 2z = 14.$

2) $x - y + 3z = 1$

$2x - 3y + z = -1$

$3x - y - z = 2.$

Q43) Find the characteristic equations of the following matrices:

(a) $\begin{pmatrix} 10 & -9 \\ 4 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{pmatrix}$ (c) $\begin{pmatrix} 10 & -9 & 0 & 0 \\ 4 & -2 & 0 & 0 \\ 0 & 0 & -2 & -7 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

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Q44) Let $A = \begin{pmatrix} -2 & 2 & 3 \\ -2 & 3 & 2 \\ -4 & 2 & 5 \end{pmatrix}$. Then find the eigenvalues of

(a) A^{-1} (b) $A - 3I$ (c) $A + 2I$ (d) 3A

Chapter Five

Q45) Prove $2 + 8 + 14 + \cdots + (6n - 4) = n(3n - 1)$ for all positive integer n.

Q46) Use mathematical induction to prove that

$$\sum_{i=1}^{n} i\,(i+1)\,(i+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

Q47) Use mathematical induction to prove that

$$\sum_{i=1}^{n} (i+2)\,(3i+1) = n(n+2)(n+3)$$

Q48) Use mathematical induction to prove that

$$\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n-1)}{r-1}$$

Q49) Use mathematical induction to prove that

$$\sum_{i=1}^{n} \frac{1}{(3i-2)(3i+1)} = \frac{n}{3n+1}$$

Q50) Use mathematical induction to prove that

$$\sum_{i=1}^{n} (4i-3) = n(2n-1)$$