The quantity of fluid set in motion in time t originally occupied a section of the cylinder with length vt, cross-sectional area A, volume vtA, and mass ρvtA. Its longitudinal momentum (that is, momentum along the length of the pipe) is:
Longitudinal momentum $=(\rho vtA)\, v_x$

The pressure in the moving fluid is $P+\Delta P$, and the force exerted on it by the piston is $(P+\Delta P)A$. The net force on the moving fluid is ΔPA and the longitudinal impulse is **Longitudinal impulse** = $\Delta PAt = B\frac{v_x}{v}At$

Because the fluid was at rest at time $t = 0$, the change in momentum up to time t is equal to the momentum at that time. Applying the impulse-momentum theorem, we find:

$$B\frac{v_x}{v}At=(\rho vtA)\, v_x \rightarrow B\frac{1}{v} = =\rho v = P\frac{1}{v}, \text{ because } P(Gas) = B \text{ (Liquid)}$$

where, B is the bulk modulus of the air. If B is the bulk modulus of the air, v is velocity and ρ is the density, then, velocity is given by:

$$v = \sqrt{\frac{P}{\rho}} \quad \text{..................5.2}$$

Speed of a longitudinal wave in a fluid (v)

Bulk modulus or the Pressure of the fluid (P)

Density of the fluid (ρ)

While we derived Eq. (5.2) for waves in a pipe, it also applies to longitudinal waves in a bulk fluid, including sound waves traveling in air or water. The above equation is known as **Newton's formula** for the velocity of sound waves in a gas.

Example 1

Find the speed of sound in air at NTP?

Hint: NTP: **N**ormal **T**emperature and **P**ressure: is defined as air at T=20°C and P=1 atm,76 cm.Hg, 101293 N/m², 101.325 kPa, 14.7 psia,, 760 torr. Density of air ρ=1.204 kg/m³.

Solution:

At NTP, P = 76 cm of mercury=76 cm.Hg

$$P = \frac{F}{A} = \frac{mg}{A} = (0.76 \times 13.6 \times 10^3 \times 9.8)\ \text{N/m}^2 = 101293\ \text{N/m}^2 \quad \text{and } \rho = 1.293\ \text{kg/m}^3.$$

∴ Velocity of sound in air at NTP is,

$$v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{101293\ \text{N/m}^2}{1.293\ \text{kg/m}^3}} = 280\ meter/sec$$

The experimental value for the velocity of sound in air is 332 meter/sec . But the theoretical value of 280meter/sec is 15% less than the experimental value. This discrepancy could not be explained by Newton's formula.

5) Velocity of Sound in Solid

When a longitudinal wave propagates in a solid rod or bar, the situation is somewhat different. The rod expands sideways slightly when it is compressed longitudinally, while a fluid in a pipe with constant cross section cannot move sideways. Using the same kind of reasoning that led us to Eq. (5.2), we can show that the speed of a longitudinal pulse in the rod is given by:

$$v = \sqrt{\frac{Y}{\rho}} \quad \text{.................} 5.3$$

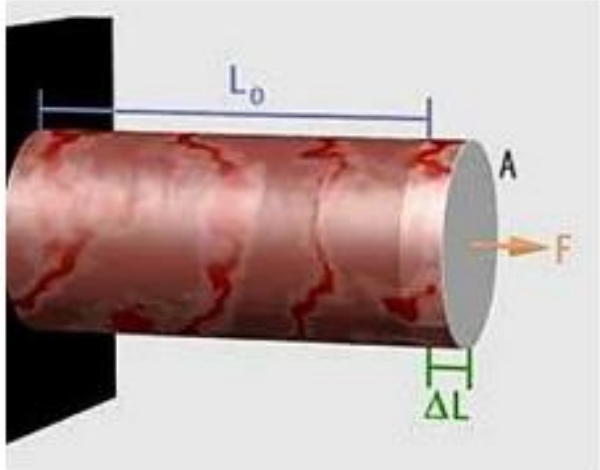
Speed of a longitudinal wave in a solid rod (v)

Young's modulus of rod material (Y)

Density of rod material (ρ)

Young's modulus, denoted by Y:

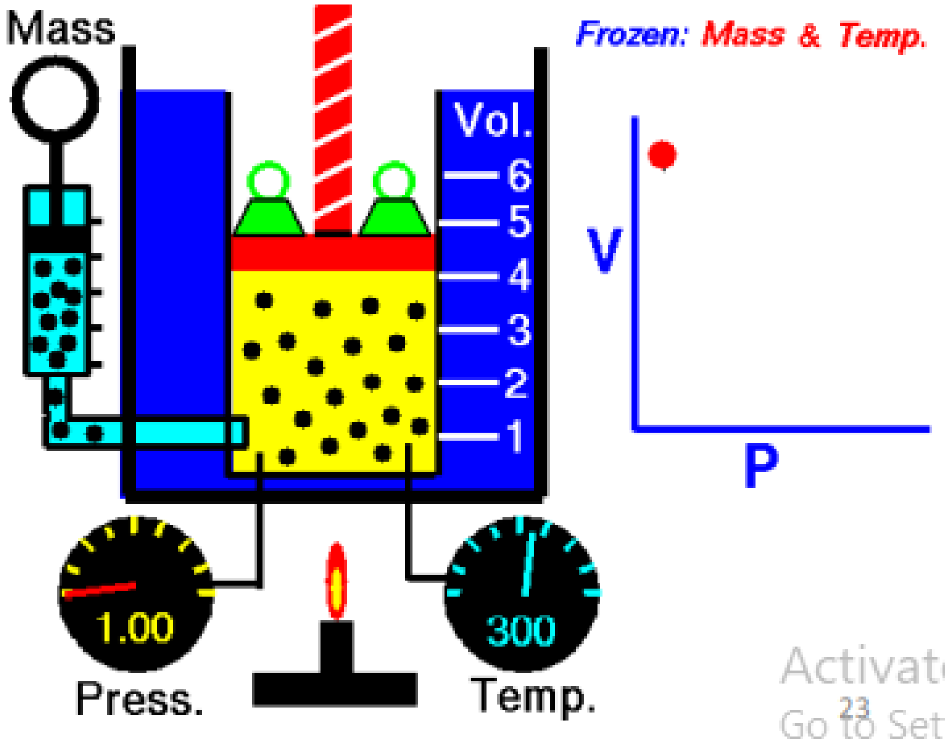
$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_\perp / A}{\Delta l / l_o} = \frac{F_\perp}{A} \frac{l_o}{\Delta l}$$



Laplace's correction

A correction to the calculation of the speed of sound in a gas. **Newton** assumed that the pressure-volume changes that occur when a sound wave travels through the gas are isothermal. **Laplace** was subsequently able to obtain agreement between theory and experiment by assuming that pressure-volume changes are adiabatic. An **adiabatic** process occurs without transfer of heat or mass of substances between a thermodynamic system and its surroundings. In an **adiabatic** process, energy is transferred to the surroundings only as work.

Isothermal Process (Newton)	Adiabatic Process (Laplace)
An isothermal process is a thermodynamic process, in which the temperature of the system remains constant (T = const).	An adiabatic process is a thermodynamic process, in which there is no heat transfer into or out of the system (Q = 0).



velocity of sound in **air:** $v = \sqrt{\frac{\gamma P}{\rho}}$

velocity of sound in **solid:** $v = \sqrt{\frac{Y}{\rho}}$

γ: is an adiabatic index=**1.41**

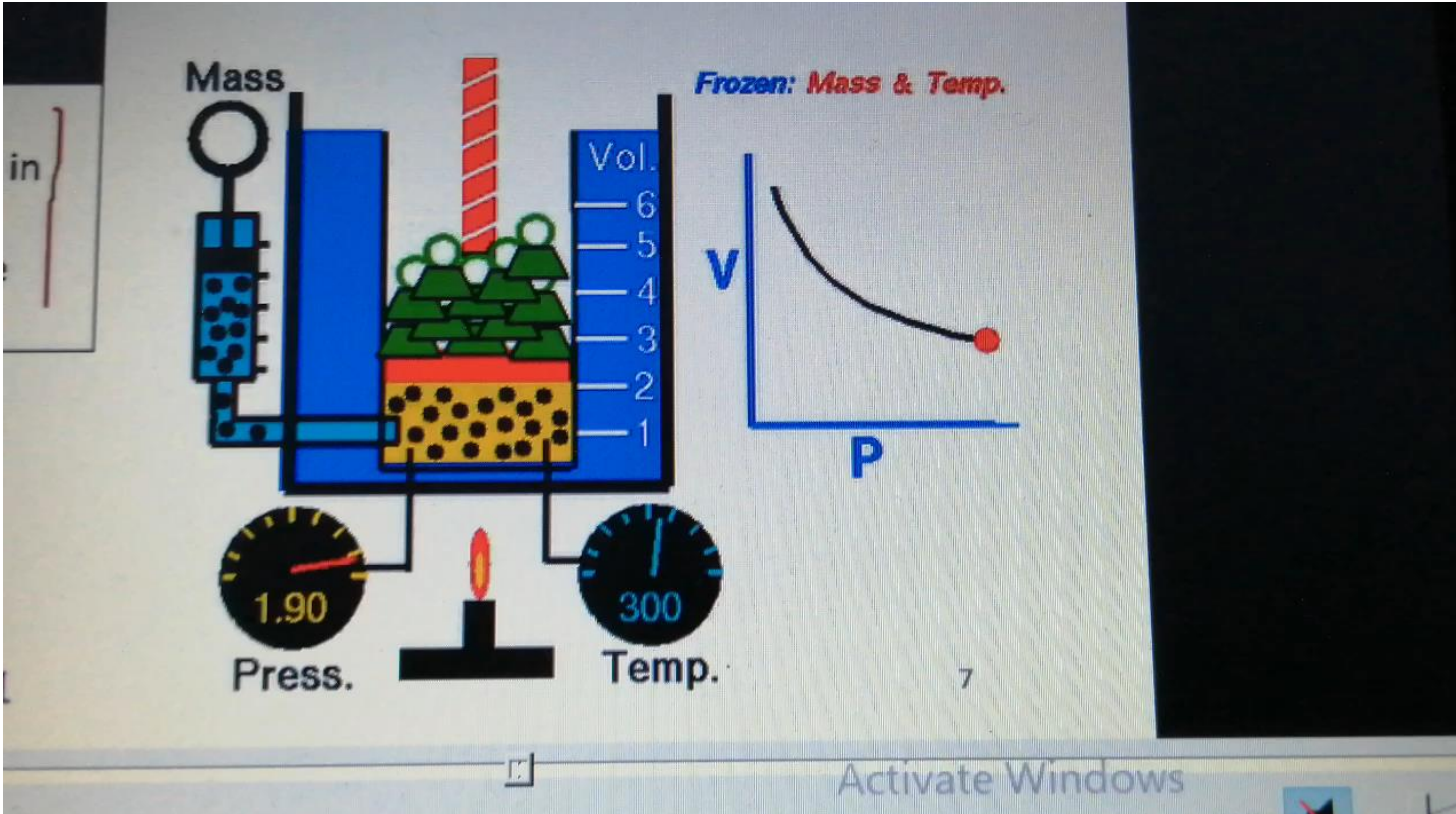

Mass
Frozen: Mass & Temp.
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Table of Speed of Sound in Various Medium

Speed of sound in various substances (CRC Handbook)

Gasses (*0°C*)	Substance	Speed of Sound (*m/s*)
	Carbon Dioxide	259
	Hydrogen	1284
	Helium	965
	Nitrogen	334
	Oxygen	316
	Air (21% Oxygen, 78% Nitrogen)	331
	Air (20°C)	344
Liquids (*25°C*)	Glycerol	1904
	Sea Water (3.5% salinity)	1535
	Water	1493
	Mercury	1450
	Kerosene	1324
	Methyl Alcohol	1103
	Carbon Tetrachloride	926
Solids	Diamond	12000
	Pyrex Glass	5640
	Iron	5960
	Granite	6000
	Aluminum	5100
	Brass	4700
	Copper (annealed)	4760
	Gold	3240
	Lead (annealed)	2160
	Rubber (gum)	1550

$$\text{in Gasses } v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma \times \text{Pressure of Gas}}{\text{density of gas}}}$$

$$\text{in Liquides } v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\text{bulk modulus}}{\text{density of liquid}}}$$

$$\text{in Solids } v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{\text{Young's moduIds}}{\text{density of solid}}}$$

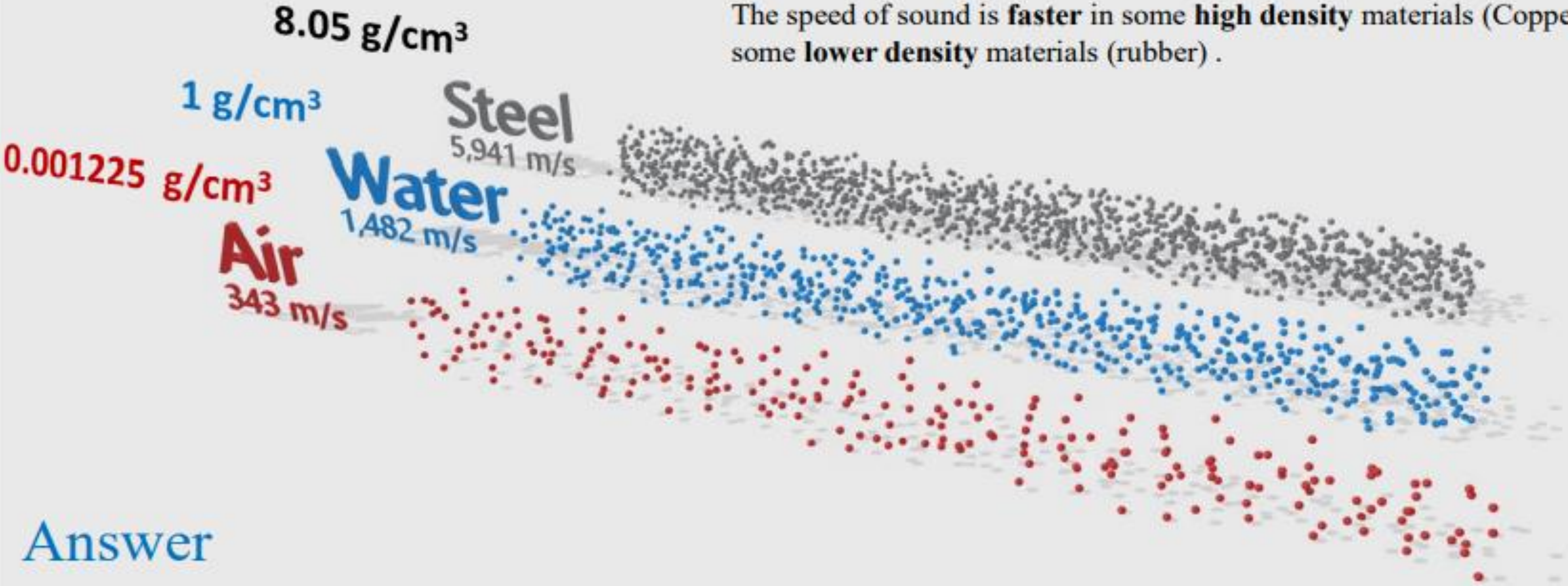
https://byjus.com/physics/speed-of-sound-propagation/

https://www.sciencetopia.net/physics/velocity-sound-gas-newton-formula

Why

The speed of sound is **faster** in materials that have some stiffness like steel and **slower** in flexibility materials like rubber.

The speed of sound is **faster** in some **high density** materials (Copper) is faster than some **lower density** materials (rubber) .



Answer

At the particle level, a rigid material is characterized by atoms and/or molecules with strong forces of attraction for each other. These forces can be thought of as springs that control how quickly the particles return to their original positions. Particles that return to their resting position quickly are ready to move again more quickly, and thus they can vibrate at higher speeds. Therefore, sound can travel faster through mediums with higher elastic properties like copper than it can through solids like rubber, which have lower elastic properties.

Example 2

Find the speed of sound in air at T=20°C, and find the range of wavelengths in air to which the human ear (which can hear frequencies in the range of 20–20,000 Hz) is sensitive. The mean molar mass for air (a mixture of mostly nitrogen and oxygen) is $M=28.8\times10^{-3}$Kg/mol and the ratio of heat capacities is $\gamma=1.4$
(R=8.314 J/mol=Gas constant-Boltzmann constant).

Solution:

We use Eq. (5.5) to find the sound speed from γ, T, and M, and we use to find the wavelengths corresponding to the frequency limits. Note that in Eq. (5.5) temperature T must be expressed in kelvins, not Celsius degrees.

At T=20°C=273+20=293 K, we find:

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.4)(8.314\,J/mol)(293K)}{28.8\times10^{-3}Kg/mol}} = 344\ meter/sec$$

Using this value of v in $\lambda = \frac{v}{f}$, we find that at 20°C the frequency f=20Hz corresponds to $\lambda = 17$ meter, and f=20.000Hz to λ =1.7 meter

.

EVALUATE: Our calculated value of agrees with the measured sound speed at T = 20°C to within 0.3%.

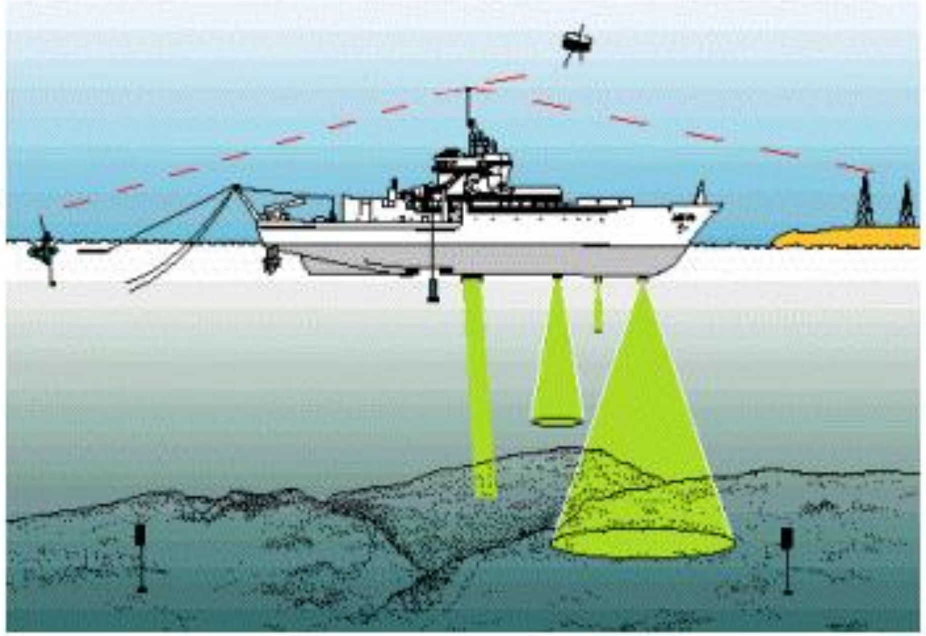
Book:University Physics with Modern Physics Technology by Hugh D. Young Roger A. Freedman 13th Edition, P:573

Example 3

A ship uses a sonar system to locate underwater objects. Find the speed of sound waves in water using, and find the wavelength of a 262Hz wave. Where ρ=1000Kg/m^3 and the compressibility of water is B=2.18x10^9 Pa.

Solution:

Our target variables are the speed and wavelength of a sound wave in water. In Eq. (5.2), we use the density of water, , and the bulk modulus of water, which we find from the compressibility. Given the speed and the frequency f=265 Hz, we find the wavelength from $v = \lambda f$



SONAR Technologies used to study the Hydrosphere

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.18\text{x}109}{1000}} = \mathbf{1476}\ \boldsymbol{meter/sec}$$

$$\lambda = \frac{v}{f} = \frac{\mathbf{1476}\ \boldsymbol{meter/sec}}{\mathbf{265}\ \boldsymbol{Hz}} = 5.57 \text{ meter}$$