Numerical Method

Lecture 4

Dalya Abdullah Anwar
Department of Computer science and information technology
Salahaddin University
dalya.anwer@su.edu.krd
2021-2022

Newton - Raphson Method

An approximation to the root is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Better and successive approximations $x_2, x_3, \ldots, x_n$ to the root are obtained from

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

N-R Formula

Example

Find a real root of the equation $x^3 - x - 1 = 0$

using Newton - Raphson method, start with $x_0=2$.

Solution

Let $f(x) = x^3 - x - 1$,

Then, $f(2) = 5$

$f'(x)=3x^2-1$

$f'(2)=3*4-1$

$=11$

Solution

- The second approximation is computed using Newton-Raphson method as
- $$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{5}{11} = 1.54545$$
-
-
- and

$$f(x_1) = 1.14573$$

The successive approximations are

$$x_2 = 1.54545 - \frac{1.14573}{6.16525} = 1.35961, \qquad f(x_2) = 0.15369$$

$$x_3 = 1.35961 - \frac{0.15369}{4.54562} = 1.32579, \qquad f(x_3) = 4.60959 \times 10^{-3}$$

$$x_4 = 1.32579 - \frac{4.60959 \times 10^{-3}}{4.27316} = 1.32471, \qquad f(x_4) = -3.39345 \times 10^{-5}$$

$$x_5 = 1.32471 + \frac{3.39345 \times 10^{-5}}{4.26457} = 1.324718, \qquad f(x_5) = 1.823 \times 10^{-7}$$

Hence, the required root is 1.3247.

Note

- Methods such as bisection method and the false position method of finding roots of a nonlinear equation *f(x)* = *0* require bracketing of the root by two guesses. Such methods are called bracketing methods.

These methods are always convergent since they are based on reducing the interval between the two guesses to zero in on the root.

In the Newton-Raphson method, the root is not bracketed. Only one initial guess of the root is needed to get the iterative process started to find the root of an equation. Hence, the method falls in the category of open methods.

- Some Familiar Computations Using the Newton-Raphson Method

1. **Computing the Square Root of a Positive Number A:** Compute $\sqrt{A}$, where $A > 0$.

   Computing $\sqrt{A}$ is equivalent to solving $x^2 - A = 0$. The number $\sqrt{A}$, thus, may be computed by applying the Newton-Raphson Method to $f(x) = x^2 - A$.

   Since $f'(x) = 2x$, we have the following Newton iterations to generate $\{x_k\}$:

Newton-Raphson Iterations for Computing $\sqrt{A}$

Input: A - A positive number

Output: An approximation to $\sqrt{A}$.

Step 1. Guess an initial approximation x_0 to $\sqrt{A}$.

Step 2. Compute the successive approximations $\{x_k\}$ as follows:

For $k = 0, 1, 2, \ldots,$ do until convergence

$$x_{k+1} = x_k - \frac{x_k^2 - A}{2x_k} = \frac{x_k^2 + A}{2x_k}$$

End

Example $\quad Let\ A = 2,\ x_0 = 1.5$

Iteration 1. $\quad x_1 = \dfrac{x_0^2 + A}{2x_0} = \dfrac{(1.5)^2 + 2}{3} = 1.4167$

Iteration 2. $\quad x_2 = \dfrac{x_1^2 + A}{2x_1} = 1.4142$

Iteration 3. $\quad x_3 = \dfrac{x_2^2 + A}{2x_2} = 1.4142$

2. Computing the nth Root

It is easy to see that the above Newton-Raphson Method to compute $\sqrt{A}$ can be easily generalized to compute $\sqrt[n]{A}$. In this case $f(x) = x^n - A, \quad f'(x) = nx^{n-1}$.

Thus Newton's Iterations in this case are:

$$x_{k+1} = x_k - \frac{x_k^n - A}{nx_k^{n-1}} = \frac{(n-1)x_k^n + A}{nx_k^{n-1}}.$$

H.W

- **1- Evaluate $\sqrt{12}$, by Newton's formula for 3 iteration.**
- **2-** Derive The Newton -Raphson Iteration for computing $\frac{1}{A}$