Question Bank

Electromagnetic Theory

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Example: Find $\nabla|\vec{r}|$

$\vec{r} = \mathrm{x}\hat{\imath} + y\hat{\jmath} + z\hat{k}$ $\qquad |\vec{r}| = \sqrt{rr} = \sqrt{x^2 + y^2 + z^2}$

$$\nabla|\vec{r}| = \frac{\partial|\vec{r}|}{\partial x}\hat{\imath} + \frac{\partial|\vec{r}|}{\partial y}\hat{\jmath} + \frac{\partial|\vec{r}|}{\partial z}\hat{k}$$

$$= \frac{\frac{1}{2}(2x)}{\sqrt{x^2 + y^2 + z^2}}\hat{\imath} + \frac{\frac{1}{2}(2y)}{\sqrt{x^2 + y^2 + z^2}}\hat{\jmath} + \frac{\frac{1}{2}(2z)}{\sqrt{x^2 + y^2 + z^2}}\hat{k}$$

$$= \frac{\mathrm{x}\hat{\imath} + y\hat{\jmath} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}} = \frac{\vec{r}}{|\vec{r}|} = r$$

Example:

The region ($r \leq a$) in spherical coordinates has an Electric intensity, $E = \frac{\rho r}{3\epsilon} a_r = E_r$ which $\mathrm{E} = E_r a_r + E_\theta a_\theta + E_\varphi a_\varphi$. Examine of both sides of divergence theorem.

$$\oint_s A \cdot n da = \int_v (\nabla \cdot A) dV$$

$$n = a_r$$

$\oint_s \; E \cdot nda \; = \iint \left(\frac{\rho r}{3\epsilon}\right) a_r \cdot (r^2 \sin\theta \, d\theta \, d\varphi) a_r = \frac{\rho r^3}{3\epsilon} \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi =$
$\frac{\rho r^3}{3\epsilon} |-\cos\theta|_0^{\pi} |\varphi|_0^{2\pi} = \frac{\rho r^3}{3\epsilon} [-(-1-1)\,.\,2\pi] = \frac{4\pi\rho r^3}{3\epsilon}$ L. H. S

But

$$\text{R. H. S} = \int_v (\nabla \cdot A) dV$$

$$= \int (\nabla \cdot E) \cdot r^2 \sin\theta \, dr d\theta d\varphi = \iiint (\nabla \cdot E) \cdot r^2 \sin\theta \, dr d\theta d\varphi$$

$E_r = \frac{\rho r}{3\epsilon} a_r$ and for spherical $\left[\nabla \cdot E = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r)\right] = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\rho r}{3\epsilon}\right) = \frac{\rho}{\epsilon}$

$$\therefore \int_v \int (\nabla \cdot E) \cdot dV = \int \frac{\rho}{\epsilon} \cdot r^2 \sin\theta \, dr d\theta d\varphi$$

$$= \frac{\rho}{\epsilon} \int_0^r r^2 dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi = \frac{\rho}{\epsilon}\frac{4}{3}\pi r^3 \qquad \text{R. H. S}$$

$$\therefore \text{L. H. S} = \text{R. H. S}$$

Given the general vector field $[A = (y \cos ax) a_x + (y + e^x) a_z]$. Find $(\nabla \times A)$.

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y \cos ax & 0 & y + e^x \end{vmatrix}$$

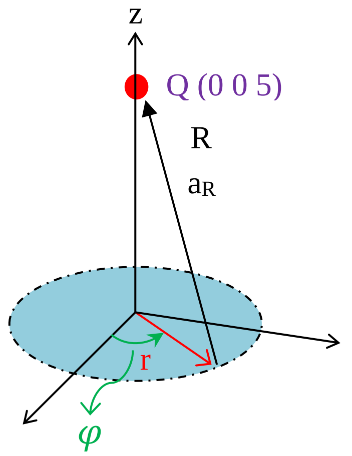
$$= \left(\frac{\partial}{\partial y} y + e^x - \frac{\partial}{\partial z} 0\right) \hat{\imath} - \left(\frac{\partial}{\partial x} y + e^x - \frac{\partial}{\partial z} y \cos ax\right) \hat{\jmath} + \left(\frac{\partial}{\partial x} 0 - \frac{\partial}{\partial y} y \cos ax\right) \hat{k}$$

$$= \left(\frac{\partial}{\partial y}(y + e^x)\right) \hat{\imath} - \left(\frac{\partial}{\partial x}(y + e^x)\right) \hat{\jmath} + \left(-\frac{\partial}{\partial y}(y \cos ax)\right) \hat{k}$$

$$= \hat{\imath} - e^x \hat{\jmath} - \cos ax \, \hat{k}$$

Example:

Find the force on a point charge of 50μc at(0,0,5) due to a charge of 500π μc, which uniformly distributed over the circular disk ($r \le 5 \quad \varphi = 2\pi \quad z = 0$).



Sol/

$$F = Eq$$

$$E = \int \frac{\rho_s ds}{4\pi\varepsilon_0 R^2} a_R$$

$$F = qE = q\int \frac{\rho_s ds}{4\pi\varepsilon_0 R^2} a_R \quad , \qquad \rho_s = \frac{q}{s} = \frac{500\pi\ 10^{-6}}{\pi r^2} = \frac{500\pi\ 10^{-6}}{\pi\ 25} = 20 \times 10^{-6}$$

$$R = -r_{a_r} + z_{a_z} \quad , \qquad dl = dra_r + rd\varphi a_\varphi + dza_z \ , \qquad ds = (rdr\ d\varphi)a_z$$

$$F = q\int \frac{\rho_s rdrd\varphi}{4\pi\varepsilon_0 R^2} a_R = q\int \frac{\rho_s rdrd\varphi\ a_z}{4\pi\varepsilon_0 (r^2+25)} . \frac{(-r_{a_r} + 5_{a_z})}{\sqrt{r^2+25}}$$

$$= \frac{5q\rho_s}{4\pi\varepsilon_0}\int_0^5 \frac{rdr}{(r^2+25)^{\frac{3}{2}}}\int_0^{2\pi} d\varphi$$

$$= \frac{5.(50\times10^{-6})(20\times10^{-6})}{4\pi(\frac{10^{-9}}{36\pi})} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{r^2+25}}\Big|_0^5 \cdot 2\pi = 16.56N$$

Example:

Find the work done in carrying the charge from r_1 to r_2 along a radial path due to infinite line charge.

$$w = -\int Eq \cdot dl = -q\int \frac{\rho_l}{2\pi\varepsilon_0 r} a_r \cdot dra_r = -\frac{q\rho_l}{2\pi\varepsilon_0}\int_{r_1}^{r_2} \frac{dr}{r} = -\frac{q\rho_l}{2\pi\varepsilon_0} \ln\frac{r_2}{r_1}$$

Example: Find the work done in moving a point charge q=5μc from the origin to $(2,\frac{\pi}{4},\frac{\pi}{2})$ in the field $\left(E = 5e^{-\frac{r}{4}} a_r + \frac{10}{\text{rsin}\,\theta} a_\varphi\right)$.

Sol\

$$w = -q\int_{r_1}^{r_2} Edl = -q\int_{r_1}^{r_2} \left(5e^{-\frac{r}{4}} a_r + \frac{10}{\text{rsin}\,\theta} a_\varphi\right) \cdot (dra_r + rd\theta a_\theta + r\sin\theta\, d\varphi a_\varphi)$$

$$w = -5 \times 10^{-6}\left[\int_0^2 5e^{-\frac{r}{4}}\,dr + \int_0^{\frac{\pi}{2}} 10\,d\varphi a_\varphi\right] = -5 \times 10^{-6}\left[20\left(e^{\frac{-2}{4}} - 1\right) + 5\pi\right]$$
$$= 117.9 \times 10^{-6} J$$

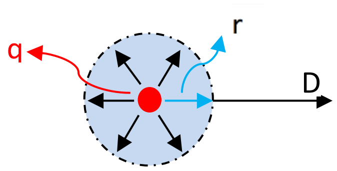
$$V_{12} = \frac{w}{q} = \frac{117.9 \times 10^{-6} J}{5 \times 10^{-6}} = 2.358\, volt.$$

The ratio between flux density and electric field intensity

Relation between D and E :

$$D \cdot ds = \varphi_{enc} \quad \rightarrow \quad \int D \cdot ds\, a_r = \varphi_{enc} \quad \rightarrow \quad Dr^2 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi = \varphi_{enc}$$

$$D 4\pi r^2 = \varphi_{enc} \quad \rightarrow \quad D = \frac{\varphi_{enc}}{4\pi r^2}\, a_r$$



For appoint charge:

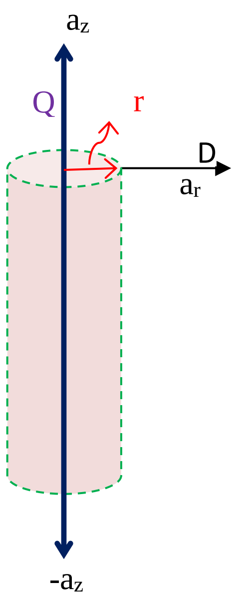
$$E = \frac{\varphi_{enc}}{4\pi r^2 \varepsilon_0}\, a_r \qquad , \frac{D}{E} = \varepsilon_0$$

$$D = \varepsilon_0 E \quad - in\ free\ space \qquad D = \varepsilon\, E \quad - in\ homogenous\ iso\ topic\ medium$$
$$\varepsilon = \varepsilon_r \varepsilon_0$$

<u>Example:</u> Find D of a uniform finite line charge.

Sol/

The appropriate Gauss surface for a line as a clyndrical



$$\int D \cdot ds = \varphi_{enc}$$

$$\int D a_r \cdot ds\, a_z + \int D a_r \cdot ds\, a_r + \int D a_r \cdot ds\,(-a_z) = \varphi_{enc}$$

$$Dr \int_0^{2\pi} d\varphi \int_0^{l} dz = \varphi_{enc}$$

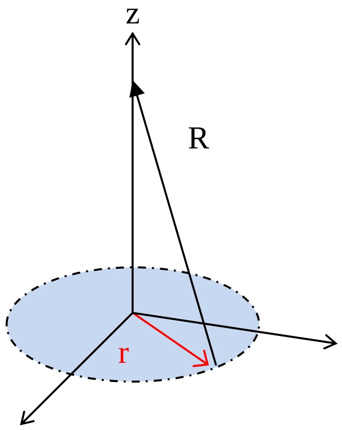
$$\therefore D = \frac{\varphi_{enc}}{2\pi r l}$$

Q/ Find D and E due to a uniform finite plane charge.
Sol./

$$\int D\cdot ds = \varphi = D\cdot rdrd\varphi = D\int_0^r rdr\int_0^{2\pi} d\varphi = D\frac{r^2}{2}2\pi = D\pi r^2$$

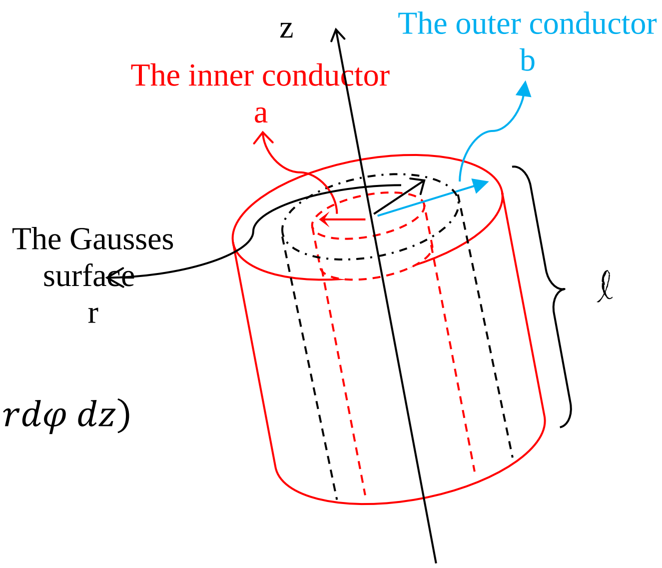
$$D = \frac{\varphi}{\pi r^2}$$

$$D = \varepsilon_0 E \quad \rightarrow \quad E = \frac{D}{\varepsilon_0} \qquad \therefore E = \frac{\varphi}{\pi r^2 \varepsilon_0}$$



Example: Find the energy stored in the electrostatic field of a section of a coaxial cable.

Sol.\



$$\int D\cdot ds = \varphi_{enc} = \int \rho_s ds$$

$$rD\int_0^{2\pi} d\varphi \int_0^{l} dz = \int \rho_s ds$$

$$= \rho_s a\int_0^{2\pi} d\varphi \int_0^{l} dz \qquad wher\ (ds = rd\varphi\ dz)$$

$$D.2\pi rl = \rho_s 2\pi al$$

$$D = \frac{\rho_s a}{r} \qquad \therefore E = \frac{\rho_s a}{\varepsilon r}a_r \qquad where\ \ \varepsilon = \varepsilon_0\varepsilon_r \qquad then:$$

$$W = \frac{1}{2}\varepsilon_0\int E^2 dv = \frac{1}{2}\int \varepsilon_0\left(\frac{\rho_s a}{\varepsilon r}\right)^2 rdr\ d\varphi\ dz = \frac{1}{2}\frac{{\rho_s}^2 a^2}{\varepsilon_0\varepsilon_r^2}\int_a^b \frac{dr}{r}\int_0^{2\pi} d\varphi\int_0^l dz$$

$$W = \frac{{\rho_s}^2 a^2}{2\varepsilon_0\varepsilon_r^2 r}.\ln\frac{b}{a}.2\pi.l \quad \rightarrow \quad W = \frac{\pi{\rho_s}^2 a^2 l}{\varepsilon_0\varepsilon_r^2 r}.\ln\frac{b}{a} \quad energy\ store$$

The energy stored per unit volume is :

$$u = \frac{W}{V} = \frac{\frac{\pi {\rho_s}^2 a^2 l}{\varepsilon_0 \varepsilon_r^2 r} . \ln\frac{b}{a}}{(\pi b^2 l - \pi a^2 l)} \qquad wher \begin{pmatrix} inner\ vol. & outer.vol. \\ \pi a^2 l & \pi b^2 l \end{pmatrix}$$

$$u = \frac{{\rho_s}^2 a^2}{\varepsilon_0 \varepsilon_r^2 r (b^2 - a^2)} . \ln\frac{b}{a}$$

Exa: Find the energy stored in a parallel plate capacitor.

Sol\

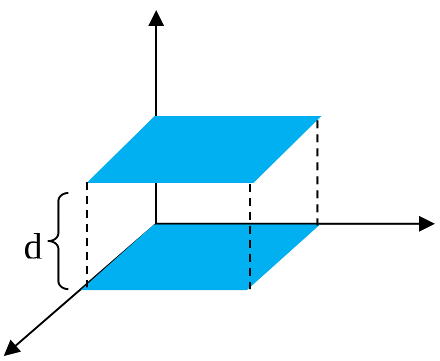
*w*her E=V\d for capacitor then:

$$W = \frac{1}{2}\int \varepsilon_0 E^2 dv$$

$$W = \frac{1}{2}\int \varepsilon_0 \left(\frac{V}{d}\right)^2 dxdydz$$

$$W = \frac{1}{2}\left[\varepsilon_0 \left(\frac{V}{d}\right)^2 Ad\right]$$

$$W = \frac{1}{2}\varepsilon_0 \frac{V^2 A}{d}$$

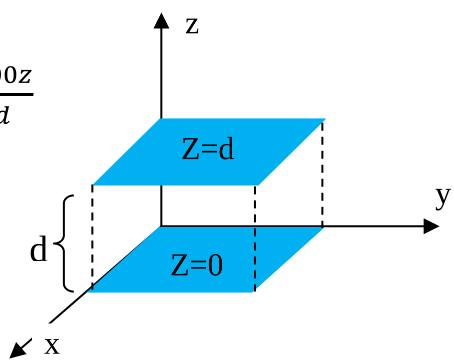


Example: Consider the parallel conductors where *V=0 at Z=0 , V=100 volt at Z=d.*

Sol\ $V = \frac{V_1(z-z_2) - V_2(z-z_1)}{z_1 - z_2} = \frac{V(z-d) - 100(z-0)}{0-d} = \frac{100z}{d}$

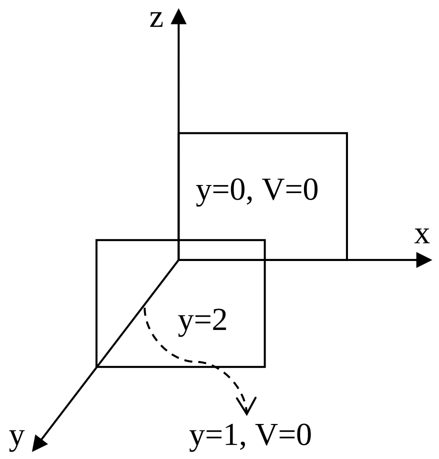
$$E = -\nabla V = -\frac{dV}{dz} a_z = -\frac{d}{dz}\left(\frac{100z}{d}\right) = \frac{100}{d}$$

$$D = \varepsilon_0 E = \frac{100}{d}\varepsilon_0$$



Ex.: Two Parallel conducting planes in free space are at y=0 and y=2 cm, and the zero voltage reference is at y=1 cm, if$D = 2.53j\,\frac{nc}{m^2}$ between the two conductors. Determine the conductor voltage.

Sol.\ $\nabla^2 V = \frac{\partial^2 V}{\partial y^2} = 0$



$$\int \frac{d}{dy}\left(\frac{dV}{dy}\right) = \int 0 \quad \rightarrow \quad V = Ay + B$$

$$E = -\nabla V \quad \rightarrow \quad E = -Aj \ \ldots\ldots\ldots..1$$

$$D = \varepsilon_0 E \quad \rightarrow \quad E = \frac{D}{\varepsilon_0} \quad in\ free\ space$$

$$E = \frac{253 \times 10^{-9}}{8.8542 \times 10^{-12}} = 2.86 \times 10^4 \frac{V}{m}$$

From eq. 1 we get valueof A

$$A = -2.86 \times 10^4 \frac{V}{m} \quad , \quad \therefore V = -2.86 \times 10^4\, y + B$$

Boundary conditionat y=1cm

$$0 = -2.86 \times 10^4 \times 10^{-2} + B \qquad \therefore B = 286$$

$$V = -2.86 \times 10^4\, y + 286$$

at y=0

$$V_1 = -2.86 \times 10^4 \times 0 + 286 \quad \rightarrow \quad V_1 = 286\ volt$$

at y=2 cm

$$V_2 = -2.86 \times 10^4 \times 2 \times 10^{-2} + 286 \quad \rightarrow \quad V_2 = -286\ volt$$

Example: In cylindrical coordinate there are two plane charged ($\varphi = constant$), then planes are insulated along z-axis. Find the expression for D between the planes where: $V = 100v \quad at \quad \varphi = \alpha \quad and \quad V = 0 \quad at \quad \varphi = 0$

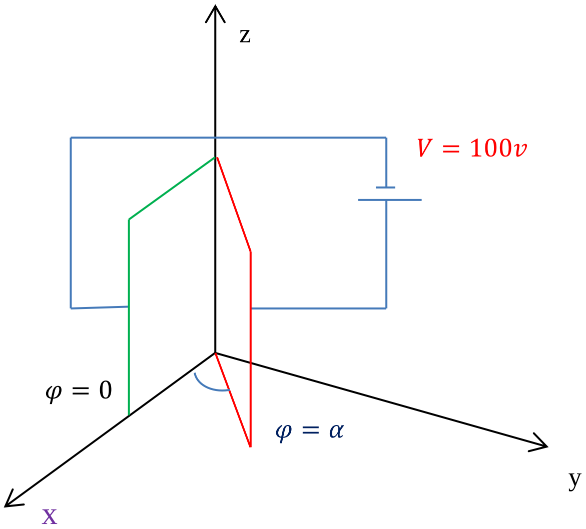
Sol/ $V = A\varphi + B$

$$\left.\begin{aligned} 100 &= A\alpha + B \\ 0 &= 0 + B \end{aligned}\right\} \therefore \quad \begin{aligned} B &= 0 \\ A &= \frac{100}{\alpha} \end{aligned}$$

$$V = \frac{100\varphi}{\alpha}$$

$$E = -\nabla V = -\frac{1}{r}\frac{dV}{d\varphi} a_\varphi = \frac{-100}{r\alpha} a_\varphi$$

$$D = \frac{-100\varepsilon_0}{r\alpha} a_\varphi$$



The equipotential surfaces aresemicircul radial planes, $V =$

Example: In spherical coordinates $V = 0 \ at \ r = 0.1m \ and \ V = 100v \ at \ r = 2m$. Find E and D.

Sol./

$$V = -\frac{A}{r} + B$$

$$\left.\begin{aligned} 0 &= -\frac{A}{0.1} + B \\ 100 &= -\frac{A}{2} + B \end{aligned}\right\} 100 = -\frac{A}{2} + \frac{A}{0.1} = A\left(\frac{20-1}{2}\right) \rightarrow A = 9.5$$

$$B = \frac{A}{0.1} = \frac{9.5}{0.1} = 95$$

$$\therefore \ V = \frac{-9.5}{r} + 95$$

Example: The region between two concentric cylindrical contains a uniform charge density (ρ).

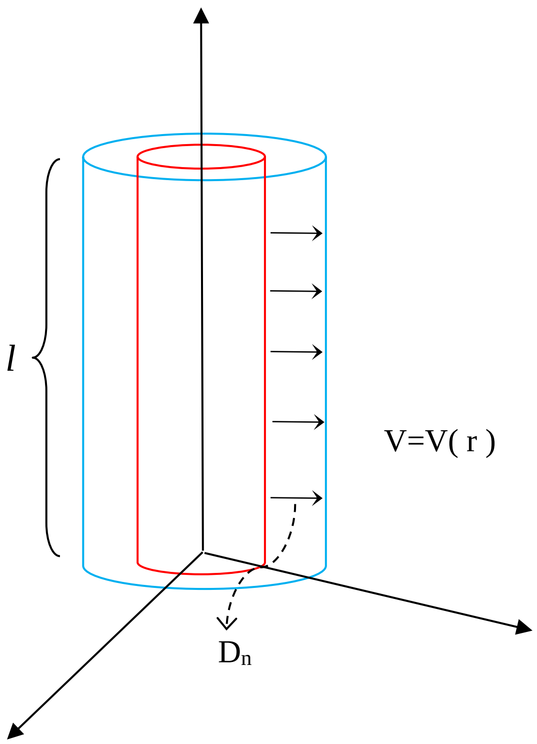
Sol./ For concentric cylindrical equipotential surfaces. $V = V(r)$

$$\nabla^2 V = \frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = \frac{-\rho}{\varepsilon}$$

$$\int d\left(r\frac{dV}{dr}\right) = \int \frac{-\rho}{\varepsilon} r dr$$

$$r\frac{dV}{dr} = \frac{-\rho}{2\varepsilon}r^2 + A$$

$$\therefore \frac{dV}{dr} = \frac{-\rho r}{2\varepsilon} + \frac{A}{r}$$



$$\int dV = \int \left(\frac{-\rho r}{2\varepsilon} + \frac{A}{r}\right) dr$$

$$V = -\frac{\rho r^2}{4\varepsilon} + A \ln r + B$$

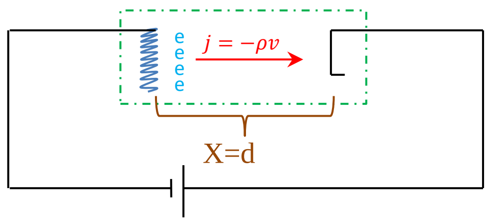
$$E = -\nabla V = -\frac{dV}{dr} a_r$$

$$\therefore E = \frac{\rho r}{2\varepsilon} - \frac{A}{r}$$

$$D = \varepsilon E = \varepsilon\left(\frac{\rho r}{2\varepsilon} - \frac{A}{r}\right) = \frac{\rho r}{2} - \frac{\varepsilon A}{r}$$

Example: A parallel plate Diode, operating condition x-axis, during the prose's space charge build up in the region between the electrons. Find the un expression for the potential function between the plates if the current density $j = -\rho v$ where: $V = 0 \quad at \ x = 0$, $V = V_0 \ at \ x = d$

Sol./ charge density $\rho = -\frac{j}{v}$



$$eV = \frac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2eV}{m}}$$

$$\rho = \frac{-j}{\sqrt{\frac{2eV}{m}}}$$

$$\nabla^2 V = \frac{-\rho}{\varepsilon} \quad \rightarrow \quad \frac{d^2V}{dx^2} = \frac{1}{\varepsilon}\frac{+j}{\sqrt{\frac{2eV}{m}}} = \frac{j}{\varepsilon}\left(\frac{2e}{m}\right)^{\frac{-1}{2}} V^{\frac{-1}{2}}$$

$$\frac{d^2V}{dx^2} = k\,V^{\frac{-1}{2}} \qquad where\ \ k = \frac{j}{\varepsilon}\left(\frac{2e}{m}\right)^{\frac{-1}{2}}$$

$$2\left(\frac{dV}{dx}\right)\cdot\frac{d^2V}{dx^2} = k\,V^{\frac{-1}{2}}\ \cdot\ 2\left(\frac{dV}{dx}\right)$$

$$\int \frac{d}{dx}\left(\frac{dV}{dx}\right)^2 dx = \int 2k\,\left(\frac{dV}{dx}\right)V^{\frac{-1}{2}}\,dx$$

$$\left(\frac{dV}{dx}\right)^2 = 4kV^{\frac{+1}{2}} \qquad \rightarrow \qquad \frac{dV}{dx} = 2\sqrt{kV^{\frac{+1}{2}}}$$

$$\int V^{-\frac{1}{4}}\,dV = 2\sqrt{k}\ dx$$

$$\frac{4}{3}V^{\frac{3}{4}} = 2\sqrt{k}\,x + A$$

First boundary condition: $V = 0 \quad at\ \ x = 0$

$$\frac{4}{3}(0)^{\frac{3}{4}} = 2\sqrt{k}\,0 + A \qquad \rightarrow \qquad A = 0$$

$$\therefore \quad \frac{4}{3}V^{\frac{3}{4}} = 2\sqrt{k}\,x \qquad \dots\dots\dots\ \ 1$$

Second boundary condition: $V = V_0 \quad at\ \ x = d$

$$\frac{4}{3}(V_0)^{\frac{3}{4}} = 2\sqrt{k}\,d \qquad \dots\dots\dots\ \ 2$$

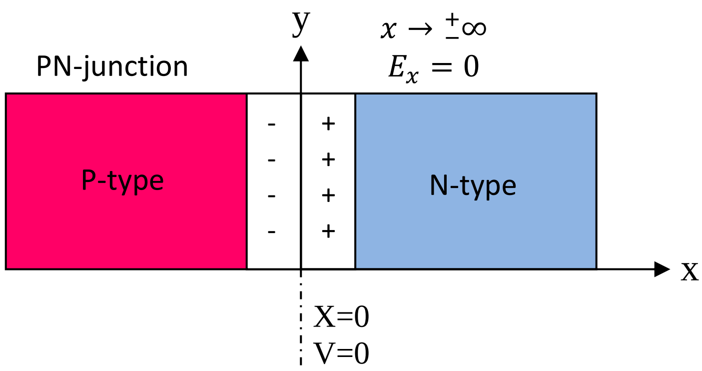
From eq. 1 &2

$$\frac{\frac{4}{3}V^{\frac{3}{4}}}{\frac{4}{3}(V_0)^{\frac{3}{4}}} = \frac{2\sqrt{k}\,x}{2\sqrt{k}\,d} \qquad \rightarrow \qquad \frac{V^{\frac{3}{4}}}{(V_0)^{\frac{3}{4}}} = \frac{x}{d}$$

$$\therefore \quad V = V_0\left(\frac{x}{d}\right)^{\frac{4}{3}}$$

Q1/

A p-n junction between two halves of a semiconductor bar extended in the x-direction assume that the region for $x < 0$ is p-type and that the region for $x > 0$ is n-type. A charge distribution of this form may be approximated by the expression $\left(\rho = 2\rho_0 sech\frac{x}{a}\ tanh\frac{x}{a}\right)$, with $\rho_{max} = \rho_0 \quad where\ \rho_{max} \equiv maximum\ charge\ density.$ Find V.



$$\nabla^2 V = -\frac{\rho}{\varepsilon}$$

$$\nabla^2 V = \frac{2\rho_0 sech\frac{x}{a}\ tanh\frac{x}{a}}{\varepsilon}$$

$$\frac{dV}{dx} = \frac{2a\rho_0\ sech\frac{x}{a}}{\varepsilon} + C_1$$

$$E_x = \frac{2a\rho_0\ sech\frac{x}{a}}{\varepsilon} + C_1$$

$$at\ \ x \to \pm\infty \qquad E_x = 0\,, \qquad \therefore C_1 = 0$$

$$E_x = \frac{dV}{dx} = \frac{2a\rho_0\ sech\frac{x}{a}}{\varepsilon}$$

Integrating again:

$$V = \frac{4a^2\rho_0}{\varepsilon}\tan^{-1} e^{\frac{x}{a}} + C_2$$

Let us arbitrarily select our zero reference of potential at the center of the junction, $= 0$;

$$0 = \frac{4a^2\rho_0}{\varepsilon}\frac{\pi}{4} + C_2 \qquad\qquad \therefore\ C_2 = -\frac{4a^2\rho_0}{\varepsilon}\frac{\pi}{4}$$

$$V = \frac{4a^2\rho_0}{\varepsilon}\left(\tan^{-1} e^{\frac{x}{a}} - \frac{\pi}{4}\right)$$

Example:Find the current in the circular wire if $[J = 15(1 - e^{-1000r})a_z]$ $at\ r = 2mm$.

$$I = \int J.ds = \int J.ds\ \cos\theta \quad , \quad \theta = 0$$

$$I = \int 15\,(1 - e^{-1000r})a_z.rdrd\varphi a_z$$

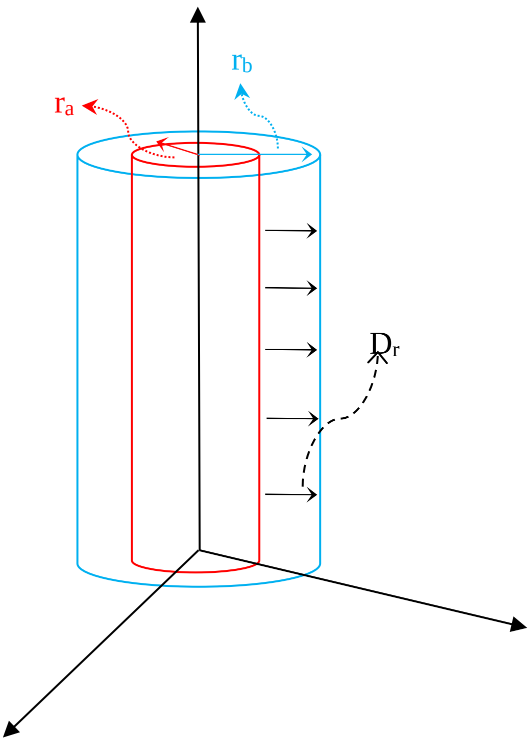
$$I = 15\int_0^{2\pi} d\varphi \int_0^r (1 - e^{-1000r})rdr$$

$$I = 1.33 \times 10^{-4} Amp$$

Example:Tow concentric cylinder $(r_a = 0.01m\ ,\ r_b = 0.08m)$where$\left(\rho_{sa} = 40\frac{pc}{m^2}\right)$, $D\&E$ exst between the cylinders, and equal to zero elsewhere. Find (ρ_{sb}).

Sol/

Between cylinders $\rho = 0$



$$\nabla.D = \rho = 0$$

$$\nabla.D = \frac{1}{r}\frac{d}{dr}(rD_r) = 0$$

$$\int d\,(rD_r) = 0$$

$$(rD_r) = A$$

$$\therefore\ D_a = \frac{A}{r_a} = \rho_{sa} \qquad \rightarrow\ A = 0.4\frac{pc}{m}$$

$$\rho_{sb} = D_b = \frac{A}{r_b} = \frac{0.4\frac{pc}{m}}{0.08m} = 5\frac{pc}{m^2}$$

Example: Find the polarization within a material which has;

$a)\ D = \frac{1.5\mu c}{m^2}, E = 15\frac{kv}{m}$

$b)\ D = \frac{2.8\mu c}{m^2}, X_e = 1.7$

$c) n = 10^{20}\frac{molecular}{m^3}, \quad p = 1.5 \times 10^{-26}\ cm\ and\ \ E = 10^5\frac{v}{m}$

$D)\ E = 50\frac{kv}{m}, \varepsilon_r = 4.4$

Sol/

a- $D = \varepsilon_0 E + P$

$$P = D - \varepsilon_0 E = 1.5 \times 10^{-6} - 8.8442 \times 10^{-12} \times 15000 = 1.367 \times 10^{-6}\frac{c}{m^2}$$

b- $P = X_e \varepsilon_0 E$

$$D = \varepsilon_r \varepsilon_0 E$$

$$P = \frac{X_e \varepsilon_0 D}{\varepsilon_0 (1 + X_e)} \quad D = (1 + X_e)\varepsilon_0 E$$

$$E = \frac{D}{(1 + X_e)\varepsilon_0}$$

$\therefore\ P = \frac{X_e D}{(1 + X_e)} = 1.763 \times 10^{-6}\frac{c}{m^2}$

c- $P = nqd = np = 10^{20} \times 1.5 \times 10^{-26} = 1.5 \times 10^{-6}\frac{c}{m^2}$

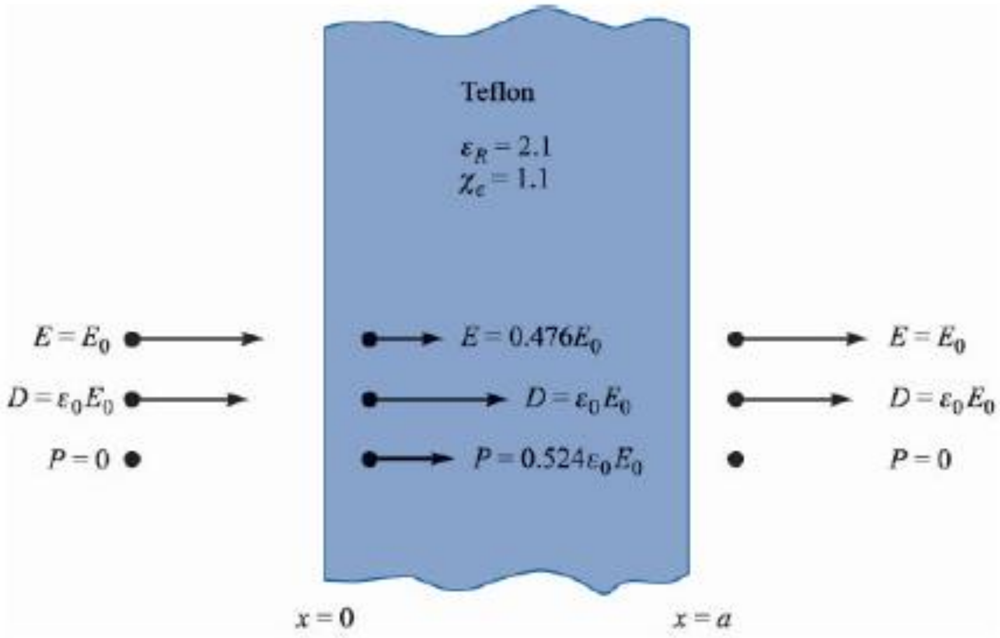
d- $P = X_e \varepsilon_0 E \qquad P = D - \varepsilon_0 E = \varepsilon_r \varepsilon_0 E - \varepsilon_0 E = \varepsilon_0 E(\varepsilon_r - 1)$

$$P = (\varepsilon_r - 1)\varepsilon_0 E = 1.505 \times 10^{-6}\frac{c}{m^2}$$

Example:

A slab of Teflon ($\varepsilon_r = 2.1$, $\chi_e = 1.1$) extending from(x=0) to (x=a) with free space on both sides of it, and external field $E_{ex} = E_0 a_x \& D_{ex} = \varepsilon_0 E_0 a_x \quad and \quad P_{ex} = 0$. Find E ,D , P in the side the material and why $P_{ex} = 0$.

Sol/



$$E_t = D_t = 0$$

$$D_n = \rho_s = \varepsilon_0 E_n = \varepsilon_0 E_0 \, a_x$$

$$D_{n1} = D_{n2} \qquad \rightarrow \qquad D_{ex} = D_{in} = \varepsilon_0 E_0 \, a_x$$

$$E_{in} = \frac{D_{in}}{\varepsilon_0 \varepsilon_R} = \frac{D_{in}}{\varepsilon} = \frac{\varepsilon_0 E_0 \, a_x}{\varepsilon_0 \times 2.1} = 0.476 E_o a_x$$

$$P = D_{in} - \varepsilon_0 E_{in} = \varepsilon_0 E_0 - \varepsilon_0 \times 0.476 E_o a_x$$

$$P = 0.524 \varepsilon_0 E_0 \, a_x$$

Example: At 1km from an antenna radiating 50kwatt of power is tropically at frequency of 1MHz.

Sol/ $\frac{1}{c}\left(\frac{dz}{dt}\right)_{Max.} = 1.63 \times 10^6 \, \frac{S_{av}}{f^2} = 1.63 \times 10^6 \, \frac{50 \times 10^3}{10^{12}} = 10^{-8}$

Example: A laser beam can carry power density of the order of $\left(10^{16} \frac{w}{m^2} \;\; at \; f \cong 10^{15} Hz\right)$ (visible light).

Sol/ $\frac{1}{c}\left(\frac{dz}{dt}\right)_{Max.} = 1.63\times10^{6}\ \frac{S_{av}}{f^2} = 1.63\times10^{6}\ \frac{10^{16}}{(10^{15})^2} \cong 1.63\times10^{-8}$

Copper (Cu) has a conductivity $\sigma \cong 6\times10^{7}\frac{1}{\Omega m}$, $\mu_r = 1$ and $\varepsilon_r = 3.5$, at 6MHz. Find: φ, skin depth, phase velocity and $\frac{E_0}{H_0}$. Or $Find\ \varphi, \delta, \frac{\lambda}{\lambda_0}, u, |\eta|$

$$\varphi = \frac{w\varepsilon}{\sigma} = \frac{2*3.14*6*10^{6}*3.5*8.8542*10^{-12}}{6*10^{7}} \cong 19.46*10^{-12}$$

$$\delta = \sqrt{\frac{2}{w\mu\sigma}} = \sqrt{\frac{2}{2*3.14*f*4*3.14*10^{-7}*6*10^{7}}} = \frac{0.066}{\sqrt{6*10^{6}}} = 2.694*10^{-5}\ m$$

$$u = \frac{w}{k_r} = \frac{w}{\left(\frac{w\mu\sigma}{2}\right)^{\frac{1}{2}}} = \left(\frac{2w}{\mu\sigma}\right)^{\frac{1}{2}} = \delta w = 2.694*10^{-5}*2*3.14*6*10^{6} =$$

$1107.79\ m/s$

$$\frac{E_{0x}}{H_0 y} = \sqrt{\frac{w\mu}{\sigma}} = \sqrt{\frac{2*3.14*6*10^{6}*4*3.14*10^{-7}}{6*10^{7}}} = 8881.26$$

If the 20MHz laser beam passes through a glass whose index of Refraction is n=1.6 find:

$1-\ \varepsilon_r\ ,\mu_r$
$2-\ E_0\ in\ the\ glass$
$3-\ H_0\ in\ the\ glass$

$n = \text{Refraction index} = \frac{c}{u}$ $\qquad u = \frac{c}{n} = \frac{3*10^{8}}{1.6} = 1.875*10^{8}\ \frac{m}{s}$

$n = \sqrt{\varepsilon_r} \qquad \varepsilon_r = n^2 = (1.6)^2 = 2.56$

$$\frac{E_x}{B_y} = u = \eta_0\times\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120*3.14\times\sqrt{\frac{\mu_r}{\varepsilon_r}} = 1.875*10^{8}$$

$$\sqrt{\mu_r} = \frac{1.875*10^{8}}{120*3.14} = 497611.46*\sqrt{\varepsilon_r} = 629434.24$$

$\mu_r = 396187464197.33$