CHAPTER SIX

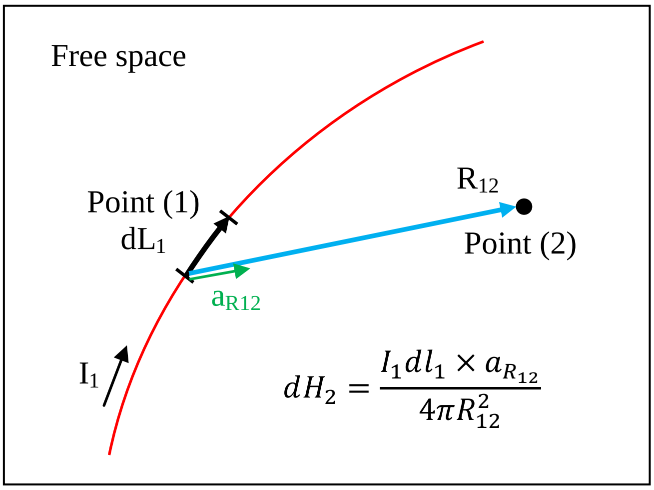
The Steady Magnetic Field

Static electric field:

Source of static electric field are S.E.F stationary charges.

- Whenever +ve or –ve stationary charges exist the static electric field are present.
- The shape and intensity of (S.E.F) is essentially depended on the charge distribution.
- The static electric field intensity arises from a charge distribution.
- The source of stead magnetic field is moving charge.
- This may be a permanent magnets or electric field charged linearly with time D.C current.
- Also the shape and intensity of a steady magnetic field depends on the current distribution.
- The steady of magnetic field arises from a current distribution.
- We assume a differentially current electric as vanishing small section of current carrying filamentary conductor. To derive the laws of magnetic field.

The Biot-savart law:



- Assume a current (I) flowing through a filamentary conductor.
- Let (dl) is the differentially element of length.

The Biot-savart law states that:

$dH \propto Idl \cdot \sin\theta \qquad dH \propto \frac{1}{R^2}$

$$\therefore dH = Idl \cdot \sin\theta \cdot \frac{1}{4\pi R^2}$$

$$dH = \frac{Idl \times a_R}{4\pi R^2}$$

$$dH_2 = \frac{I_1 dl_1 \times a_{R_{12}}}{4\pi R_{12}^2} \quad \text{filamentarycurrent element}$$

- dH the magnitude of the magnetic field produced by the differentially element of a filamentary current.
- This is the similar to coulombs law for differentially element of charge.

$$dE = \frac{d\varphi}{4\pi\varepsilon_0 R^2} a_R$$

The Biot-savart law in t he integral form can be verified experimentally;

$$H = \oint \frac{Idl \times a_R}{4\pi R^2}$$

And also may be expressed in terms of current distribution.

1- Surface current density:

$$H = \oint \frac{k \times a_R}{4\pi R^2} ds \qquad \textit{sheet current element}$$

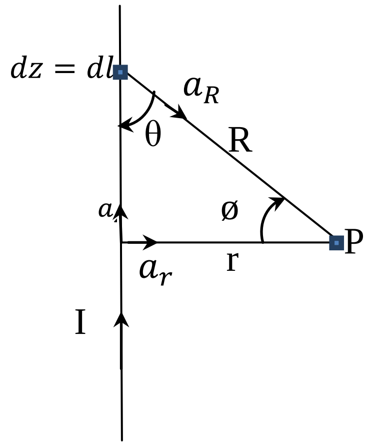
2- Volume current density.

$$H = \oint \frac{J \times a_R}{4\pi R^2} dV \qquad \textit{volume current element}$$

Example: Find Magnetic field intensity (H) due to an infinity long wire carrying the current (I).

$dl = dza_z \quad , \; R = ra_r - za_z$

$$a_R = \frac{R}{|R|} = \frac{ra_r - za_z}{\sqrt{r^2 + z^2}}$$



$$dH = \frac{Idl \times a_R}{4\pi R^2}$$

$$dH = \frac{Idza_z \times \frac{ra_r - za_z}{\sqrt{r^2+z^2}}}{4\pi(r^2 + z^2)}$$

$$dH = \frac{I}{4\pi}\frac{rdza_\varphi}{(r^2 + z^2)^{\frac{3}{2}}}$$

$$H = \frac{I}{4\pi}\int_{-\infty}^{+\infty}\frac{rdza_\varphi}{(r^2 + z^2)^{\frac{3}{2}}} \qquad , \qquad \tan\theta = \frac{r}{z} \;\rightarrow\; z = \frac{r}{\tan\theta} = r\cot\theta$$

$$dz = -r\csc^2\theta\, d\theta$$

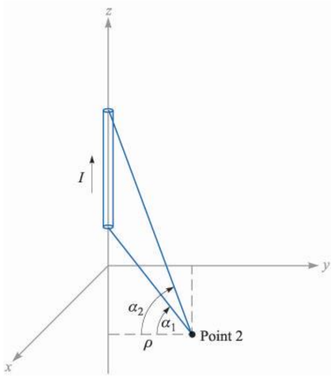
$$H = \frac{I}{4\pi}\int_{-\infty}^{+\infty}\frac{-r\, csc^2\,\theta\; d\theta\; a_\varphi}{(r^2 + (r\,cot\,\theta)^2)^{\frac{3}{2}}} = \frac{I}{4\pi r}\int_{-\infty}^{+\infty}\frac{-\,csc^2\,\theta\; d\theta}{(1 + cot^2\,\theta)^{\frac{3}{2}}}a_\varphi$$

$$H = \frac{I}{4\pi r}\int_{-\infty}^{+\infty}\frac{-\,csc^2\,\theta d\theta}{csc^3\,\theta}a_\varphi = \frac{-I}{4\pi r}\int_{-\pi}^{+\pi}\sin\theta\, a_\varphi = \frac{-I}{4\pi r}[-\cos\theta]_{-\pi}^{+\pi}\; a_\varphi$$

$$\therefore\; H = \frac{I}{2\pi r}\; a_\varphi$$

For finite length current element:

$$H = \frac{I}{2\pi r}[\sin\alpha_2 - \sin\alpha_1]\; a_\varphi$$

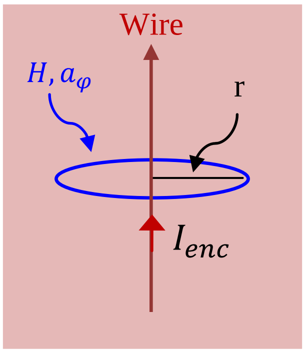


AMPER'S CIRCUITAL LAW

Ampere's circuital law states that the line integral of magnetic intensity about any closed path is exactly equal to the direct current enclosed by that path.

$$\oint H \cdot dl = I_{enc}$$

For an infinite long straight wire



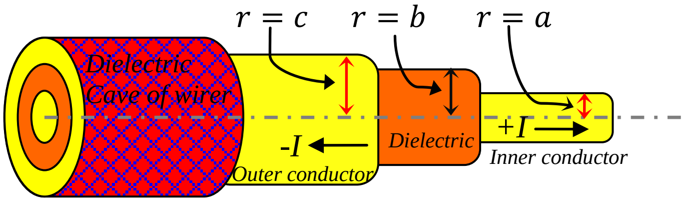
$$\oint H \cdot dl = \oint H \cdot r d\varphi a_{\varphi} = I_{enc}$$

$$H \cdot r \cdot 2\pi a_{\varphi} = I$$

$$H = \frac{I}{2\pi r} a_{\varphi}$$

Example: Consider an infinite along coaxial cable carrying a uniformly total current in the center conductor of (+I) and (-I) in the outer conductor. Find H at:

$1 - a < r < b$
$2 - \ r < a$
$3 - r > c$
$4 - b < r < c$



Infinite coaxial cable

$1 - At\ a < r < b$

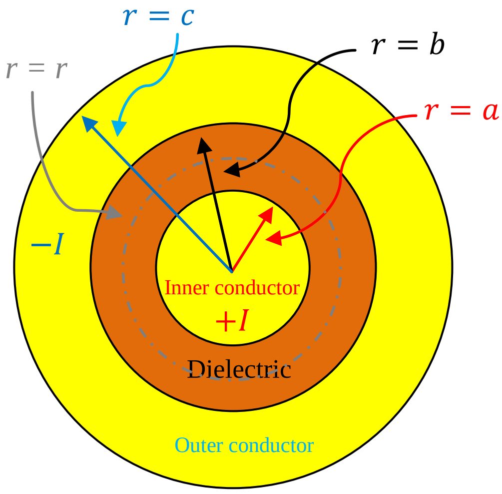
The current enclosed I_{enc}

$I_{enc} = +I$

$$\oint H \cdot dl = I_{enc}$$

$$\oint H \cdot dl = \oint_{0}^{2\pi} H \cdot r d\varphi a_{\varphi} = +I$$

$$H = \frac{I}{2\pi r} a_{\varphi}$$



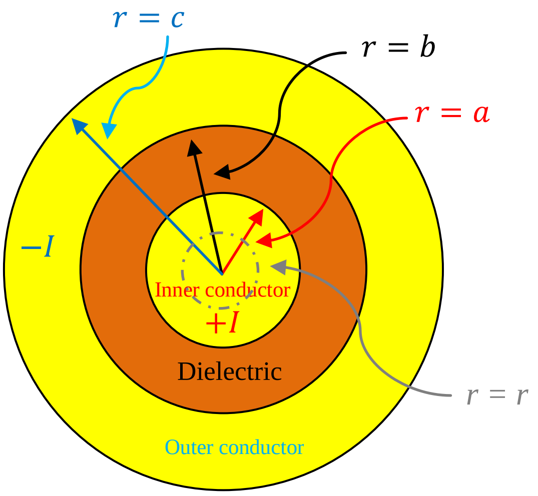
$2 - At\ r < a$

$$\oint H \cdot dl = \frac{Ir^2}{a^2}$$

$$Where\quad \frac{+I}{\pi a^2} = \frac{I_{enc}}{\pi r^2} \rightarrow I_{enc} = \frac{Ir^2}{a^2}$$

$$\oint_0^{2\pi} H \cdot r d\varphi a_\varphi = H \cdot r \cdot 2\pi a_\varphi = \frac{Ir^2}{a^2}$$

$$H = \frac{Ir}{2\pi a^2} a_\varphi$$

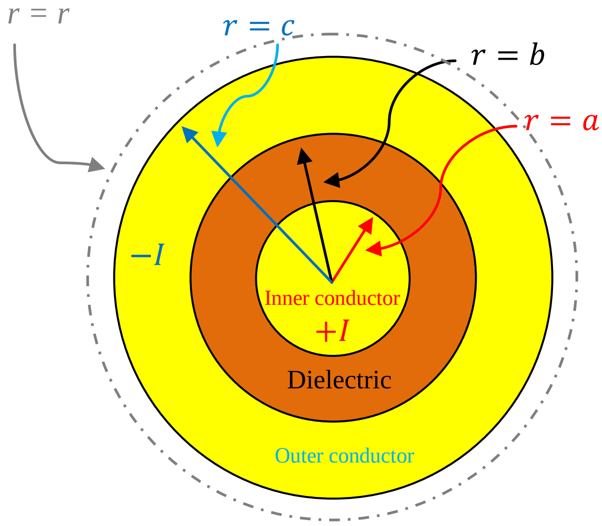


$3 - At\ r > c$

$$I_{enc} = (+I) + (-I) = o$$

$$\oint H \cdot dl = 0$$

$$\therefore H = 0$$



$4 - At\ b < r < c$

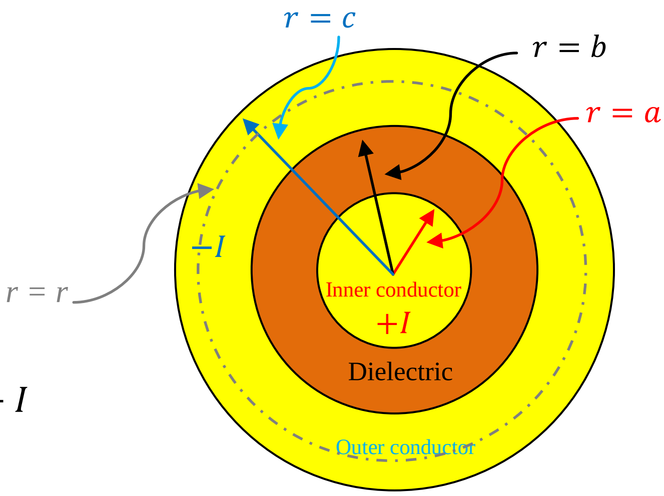
$$\underbrace{\frac{-I}{\pi(c^2 - b^2)}}_{all\ path} = \underbrace{\frac{I'}{\pi(r^2 - b^2)}}_{part\ of\ total\ path}$$

$$I' = \frac{-I(r^2 - b^2)}{(c^2 - b^2)}$$

$$I_{enc} = I' + (+I) = \frac{-I(r^2 - b^2)}{(c^2 - b^2)} + I$$

$$I_{enc} = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$$

$$\therefore\quad I_{enc} = I\left(\frac{c^2 - r^2}{c^2 - b^2}\right)$$



$$\oint H \cdot dl = I_{enc}$$

$$\oint_{0}^{2\pi} H \cdot r d\varphi a_{\varphi} = I\left(\frac{c^2 - r^2}{c^2 - b^2}\right) \qquad \rightarrow \qquad H = \frac{I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right) a_{\varphi}$$

Curl H and Maxwell second eq:

$$\oint H \cdot dl = I \qquad \textit{Ampers circuital law}$$

$$\oint H \cdot dl = \int J.ds$$

$$\oint H \cdot dl = \int_{s} (\nabla \times H)_{n}.ds$$

$$\int (\nabla \times H)_{n}.ds = \int J.ds$$

$$\nabla \times H = J \qquad \begin{array}{c} point\ form\ of\ Ampers\ law \\ or\ (Maxwell's\ \ 2^{nd}\ eq.) \end{array}$$

Magnetic flux and magnetic flux density

Magnetic field density (B):

$$B = \mu H$$

The magnetic flux (φ) is the surfaces integral of (B):

$$\varphi = \int B.ds$$

$$\int B.ds = 0 \qquad for\ magnetic\ pole$$

$$\oint B.ds = \int (\nabla \cdot B) dV \qquad \rightarrow \qquad \int (\nabla \cdot B) dV = 0$$

$$\therefore \quad \nabla \cdot B = 0 \qquad \begin{array}{c} Fourth\ Maxwell'seq. \\ non\ existance\ of\ magnetic\ monopole \end{array}$$

Maxwell eq. in steady and static fields
$\oint D \cdot ds = \varphi = \int \rho dV \quad \rightarrow \quad \nabla \cdot D = \rho$
$\oint H \cdot dl = I = \int J ds \quad \rightarrow \quad \nabla \times H = J$
$\oint E \cdot dl = 0 \quad \rightarrow \quad \nabla \times E = 0$
$\oint B \cdot ds = 0 \quad net\ flux\ magnetic\ pole \quad \nabla \cdot B = 0$

Q1/ Find the incremental contribution to the magnetic field intensity at the origin caused by a current element in free space equal to:

$1 - 3\pi a_z\ \mu A \quad at \quad (3, -4, 0)$
$2 - 3\pi a_z\ \mu A \quad at \quad (3, 2, -4)$
$3 -\ \pi(a_x - 2a_y + 2a_z)\ \mu A \quad at\ (5,0,0)$

Q2/ A current of 0.4 A in the a_z-direction in free space is in a filament parallel to z-axis and passing through the point (2,-4,0). Find H at (0,1,0) if the filament lies then interval $[a) - \infty < z < +\infty \quad b) - 3 < z < +3]$

Q3/ If

1- $H = y^2 z a_x + 2(x+1) yz a_y - (x+1)^2 z^2 a_z$
2- $H = 2r cos\varphi a_r - 4 sin\varphi a_\varphi + 3a_z$
3- $H = 2r cos\theta a_r - 3r sin\theta a_\theta$

Find the current density J

$$\left.\begin{aligned}\varphi &= \int B.ds \\ \oint B.ds &= 0\end{aligned}\right\}$$

For magnetic flux produced by an infinity along wire carrying a current (I)

$$z \to (0 - d)$$

$$r \to (a - b)$$

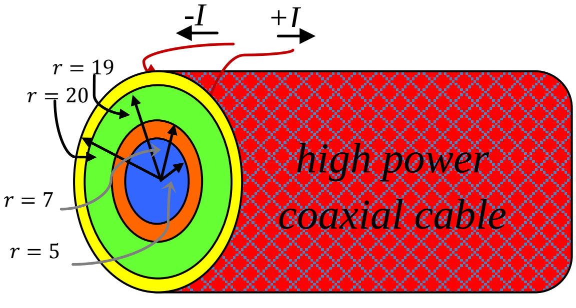
$$\varphi = \int B.ds = \int \mu H.ds = \int \frac{\mu I}{2\pi r} a_\varphi . drdz\, a_\varphi$$

$$\varphi = \int \frac{\mu_0 I}{2\pi r} a_\varphi . drdz\, a_\varphi$$

$$\varphi = \frac{\mu_0 I d}{2\pi} \ln \frac{b}{a}$$

Q/A high power coaxial cable is operated with cooling water passing through a hollow inner conductor with radii of (5 and 7 mm) and the outer conductor has radii of (19 and 20 mm). Each conductor carriers a uniformly distributed total current of (20 amperes), determine the total magnetic flux in a (1m) long of:

a) The inner conductor
b) The space between two conductor
c) The outer conductor



$\varphi = \int B.ds \qquad \varphi = \int \mu_0 H.ds$

a) $\oint Hdl = I_{enc}$

$\oint H.rd\varphi a_\varphi = I_{enc} \qquad I_{enc} = 2\pi r H a_\varphi \quad for\ inner$

b) $\oint H dl = I_{enc}$

$$\frac{I_1}{\pi(r^2-d^2)} = \frac{I_2}{\pi(E^2-d^2)}$$

$$I_1 = I_2\left(\frac{r^2-d^2}{E^2-d^2}\right)$$

$$I_{enc} = I_1 + I_2$$

$$I_{enc} = I_2\left[1 + \frac{r^2-d^2}{E^2-d^2}\right]$$

$$I_{enc} = I_2\left[\frac{r^2-2d^2+E^2}{E^2-d^2}\right]$$

c) $\varphi = \frac{\mu_0 I_{enc} d}{2\pi}\ln\frac{b}{a} = \frac{\mu_0 d}{2\pi} I_2\left[\frac{r^2-2d^2+E^2}{E^2-d^2}\right]\ln\frac{7}{19}$

$$E = \frac{19+20}{2} = 19.5 \qquad d = \frac{7+5}{2} = 6$$

THE SCALAR AND VECTOR MAGNETIC POTENTIALS

1- The Scalar magnetic potential:

We should question whether or not such assistance is available in magnetic fields. Can we define a potential function which may be found from the current distribution and from which the magnetic fields may be easily determined? Can a scalar magnetic potential be defined, similar to the scalar electrostatic potential?

Let assume (V_m)is a scalar magnetic potential whose negative gradient gives the magnetic field intensity.

$$H = -\nabla V_m$$

$\nabla \times H = J \qquad Maxwell\ eq.$

$\nabla \times (-\nabla V_m) = J \quad , \qquad -(\nabla \times \nabla V_m) = J$

From the vector identity curl of each a scalar function$\nabla \times \nabla(scalar\ function) = 0$

$$[-(\nabla \times \nabla V_m) = 0] \qquad \therefore \quad J = 0$$

$$\therefore \ [H = -\nabla V_m] \ \ requires \quad (J = 0)$$

The scalar magnetic potential is also applicable in the case of permanent magnets. This scalar potential also satisfies Laplace's equation. In free space,

$\nabla \cdot B = 0 \qquad B = \mu_0 H$

$\nabla \cdot B = \mu_0 \nabla \cdot H = 0 \qquad , \ \mu_0 \nabla \cdot (-\nabla V_m) = 0$

$\nabla^2 V_m = 0 \qquad Laplace\ equation$ (scalar potential)

2-The vector magnetic potential:

Let us temporarily leave the scalar magnetic potential now and investigate a vector magnetic potential. This vector field is one which is extremely useful in studying radiation from antennas, from apertures, and radiation leakage from transmission lines, waveguides, and microwave ovens. The vector magnetic Potential may be used in regions where the current density is zero or nonzero, and we shall also be able to extend it to the time-varying case later.

Our choice of a vector magnetic potential is indicated by noting that:

$$\nabla \cdot B = 0 \qquad (4th\ max.eq.)$$

$$\nabla \cdot \nabla \times (any\ vector) = 0$$

Let this vector be (A).

$$\nabla \cdot \nabla \times A = 0$$

$$B = \nabla \times A \qquad A - is\ said\ to\ be\ the\ vector\ (magnetic\ potential)$$

$$\nabla \times H = J \qquad (2nd\ Max.eq.)$$

$$B = \nabla \times A \qquad H = \frac{B}{\mu_0} = \frac{1}{\mu_0}\nabla \times A$$

$$\nabla \times \left(\frac{1}{\mu_0}\nabla \times A\right) = J$$

$$\nabla \times \nabla \times A = \mu_0 J$$

$$\nabla(\nabla \cdot A) - \nabla^2 A = \mu_0 J$$

$$-\nabla^2 A = \mu_0 J$$

$$\nabla^2 A = -\mu_0 J \qquad Poissions\ equation\ \text{(vector potential)}$$

Electric field	Magnetic field
$Q = \oint D.ds = d\varphi$ $\nabla \cdot D = \rho$ $E = -\nabla V$ $\nabla^2 V = 0$ $\nabla^2 V = \frac{-\rho}{\varepsilon}$	$\varphi = \oint B.ds$ $\nabla \times H = J$ $\nabla \cdot B = 0$ $H = -\nabla V_m$ $\nabla^2 V_m = 0$ $\nabla^2 A = -\mu_0 J$
$D = \varepsilon E$ $D = \varepsilon_0 \varepsilon_r E$	$I_{enc} = \oint H.dl$ $B = \mu H = \mu_0 \mu_r H$

Maxwell's Four Equations

As they apply to *static electric* fields and *steady magnetic* fields

$$\nabla \cdot D = \rho$$

$$\nabla \times H = J$$

$$\nabla \times E = 0$$

$$\nabla \cdot B = 0$$