CHAPTER EIGHT

Propagation of Electromagnetic Wave in different media

Non Homogenous Wave equations for E and H

Wave equations for E:

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times \nabla \times E = -\frac{\partial}{\partial t}\nabla \times B$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = -\mu\frac{\partial}{\partial t}\nabla \times H = -\mu\frac{\partial}{\partial t}\left(J + \frac{\partial D}{\partial t}\right) \quad ,J = \sigma E \;\&\; D = \varepsilon E$$

$$\nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2 E = -\mu\sigma\frac{\partial E}{\partial t} - \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E - \mu\sigma\frac{\partial E}{\partial t} - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} = \nabla\left(\frac{\rho}{\varepsilon}\right) \quad \textit{General wave equation for E}$$

Wave equations for H:

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad \rightarrow \qquad \nabla \times H = \sigma E + \varepsilon\frac{\partial E}{\partial t}$$

$$\nabla \times \nabla \times H = \sigma\nabla \times E + \varepsilon\frac{\partial}{\partial t}\nabla \times E$$

$$\nabla(\nabla \cdot H) - \nabla^2 H = \sigma\left(-\frac{\partial B}{\partial t}\right) + \varepsilon\frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial t}\right)$$

$$\frac{1}{\mu}\nabla(\nabla \cdot B) - \nabla^2 H = -\mu\sigma\frac{\partial H}{\partial t} - \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 H - \mu\sigma\frac{\partial H}{\partial t} - \mu\varepsilon\frac{\partial^2 H}{\partial t^2} = 0 \qquad \textit{General wave equation for H}$$

Plane electromagnetic waves

Theoretical and experimental works investigated that radio waves produced by the antenna, and waves produced by Dipole are *plane electromagnetic waves*.

In plane waves, the two vectors (E & H) are infusing, but perpendicular to each other.

The $(E \times H)$ *gives the direction of propagation*.

If (E) is in the x-direction and (H) in the y-direction, then:

$$E_x = E_0 e^{i(wt-kz)}$$

$$H_y = H_0 e^{i(wt-kz)}$$

$$w = 2\pi f \qquad f = \text{Frequency} \qquad , E_0 \& H_0 \equiv is\ the\ max.intensity$$

1- Propagation of Electromagnetic Wave in Free Space:

$$\nabla^2 E - \mu\sigma\frac{\partial E}{\partial t} - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} = \nabla\left(\frac{\rho}{\varepsilon}\right)$$

$$\nabla^2 H - \mu\sigma\frac{\partial H}{\partial t} - \mu\varepsilon\frac{\partial^2 H}{\partial t^2} = 0$$

$$In\ free\ space \quad \sigma = 0\ , \quad \rho = 0\ , \quad \mu = \mu_0\ , \quad \varepsilon = \varepsilon_0$$

$$\nabla^2 E_x - \mu_0\varepsilon_0\frac{\partial^2 E_x}{\partial t^2} = 0\ , \quad E_x = E_0 e^{i(wt-kz)} \quad \dots\dots\dots\ 1$$

$$or$$

$$\nabla^2 H_y - \mu_0\varepsilon_0\frac{\partial^2 H_y}{\partial t^2} = 0\ , H_y = H_0 e^{i(wt-kz)} \quad \dots\dots\dots\ 2$$

$$\frac{\partial H_y}{\partial z} = \frac{\partial}{\partial z} H_0 e^{i(wt-kz)} = (-ik) H_0 e^{i(wt-kz)} = -ikH_y$$

$$\frac{\partial H_y}{\partial t} = \frac{\partial}{\partial t} H_0 e^{i(wt-kz)} = (iw) H_0 e^{i(wt-kz)} = iwH_y$$

$$\therefore\ \frac{\partial^2 H_y}{\partial z^2} - \mu_0\varepsilon_0\frac{\partial^2 H_y}{\partial t^2} = (-ik)^2 H_y - \mu_0\varepsilon_0 (iw)^2 H_y = 0$$

$$-k^2 + \mu_0\varepsilon_0 w^2 = 0$$

$$\frac{w^2}{k^2} = \frac{1}{\mu_0\varepsilon_0} \quad \rightarrow \quad \frac{w}{k} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$u = \frac{w}{k} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c \quad \text{speed of light} \quad \text{(In free space)}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -\mu_0 \frac{\partial H_y}{\partial t} \ , \qquad H_y = H_0 e^{i(wt-kz)}$$

$$0 + \frac{\partial E_x}{\partial z} - \frac{\partial E_x}{\partial y} = -\mu_0 (iw) H_y$$

$$-ikE_x = -\mu_0 (iw) H_y$$

$$\frac{E_x}{H_y} = \mu_0 \frac{w}{k} = \mu_0 \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\frac{E_x}{H_y} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad \text{Intrinsic impedance}$$

The ratio of E_x to H_y is constant, so that E_x and H_y are in phase.

The square root of $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ is called the intrinsic impedance (η) and has the dimensions of ohms.

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad (\Omega)$$

In free space $\qquad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7} \frac{H}{m}}{8.85 \times 10^{-12} \frac{F}{m}}} = 120\pi \ (\Omega)$

$$\frac{E_x}{\mu_0 H_y} = \frac{1}{\mu_0} \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\frac{E_x}{B_y} = c$$

The Poynting vector and Power consideration($E \times H$):

$\nabla \times H = J + \frac{\partial D}{\partial t}$

$$E \cdot \left(\nabla \times H = J + \frac{\partial D}{\partial t}\right)$$

$$\nabla \cdot (E \times H) = -E \cdot (\nabla \times H) + H \cdot (\nabla \times E) = -E \cdot \left(J + \frac{\partial D}{\partial t}\right) + H \cdot \left(-\frac{\partial B}{\partial t}\right)$$

$$\nabla \cdot (E \times H) = -\varepsilon_0 E \frac{\partial E}{\partial t} - \mu_0 H \frac{\partial H}{\partial t} = -\frac{\varepsilon_0}{2}\frac{\partial}{\partial t}(E^2) - \frac{\mu_0}{2}\frac{\partial}{\partial t}(H^2)$$

$$\nabla \cdot (E \times H) = -\frac{\partial}{\partial t}\left(\frac{\varepsilon_0 E^2}{2}\right) + \frac{\partial}{\partial t}\left(\frac{\mu_0 H^2}{2}\right)$$

$$\int \nabla \cdot (E \times H) dv = -\frac{\partial}{\partial t}\int \left(\frac{\varepsilon_0 E^2}{2} + \frac{\mu_0 H^2}{2}\right) dv$$

$$\underbrace{\oint (E \times H) ds}_{Flux} = \underbrace{-\frac{\partial}{\partial t}\int \left(\frac{\varepsilon_0 E^2}{2} + \frac{\mu_0 H^2}{2}\right) dv}_{Time\ rate\ of\ energy}$$

2- Propagation of Electromagnetic Wave in non conductor(perfect dielectric):

$$\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = \nabla \left(\frac{\rho}{\varepsilon}\right)$$

$$\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0$$

$In\ non\ conductor \quad \sigma = 0 \ , \ \rho = 0 \, , \ \mu = \mu_0\mu_r \ , \quad \varepsilon = \varepsilon_0\varepsilon_r$

If the direction propagation in the z-direction

$$\nabla^2 E - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \qquad E_x = E_0 e^{i(wt-kz)}, \ H_y = H_0 e^{i(wt-kz)}$$

$$(-ik)^2 E_x - \mu\varepsilon(iw)^2 E_x = 0 \quad \rightarrow \quad -k^2 + \mu\varepsilon w^2 = 0$$

$$\frac{w^2}{k^2} = \frac{1}{\mu\varepsilon} \quad \rightarrow \quad \frac{w}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$u = \frac{w}{k} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \cdot \frac{1}{\sqrt{\mu_r\varepsilon_r}} \qquad \text{speed of wave}$$

In perfect dielectric $\mu_r = 1$

$$u = \frac{c}{\sqrt{\varepsilon_r}} \quad or \quad \varepsilon_r = \frac{c^2}{u^2} = n^2$$

$$n = \sqrt{\varepsilon_r} \qquad n = \text{Refraction index} = \frac{c}{u}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \quad \rightarrow \quad -ikE_x = -\mu(iw)H_y$$

$$\frac{E_x}{H_y} = \mu \frac{w}{k} = \mu \frac{1}{\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$\frac{E_x}{H_y} = \eta = \eta_0 \times \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$\frac{E_x}{H_y} = \frac{E_x}{\mu B_y} = \frac{w}{k} \qquad \frac{E_x}{B_y} = u$$ Speed of wave in the media

Poynting vector $E \times H$: (H.W)

$$\mu = \mu_0 \mu_r \quad , \qquad \varepsilon = \varepsilon_0 \varepsilon_r$$

3- Propagation of Electromagnetic Wave in Loss Dielectrics (Conductor)

$$\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = \nabla\left(\frac{\rho}{\varepsilon}\right)$$

$$\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0$$

$$In\ non\ conductor \qquad \rho \neq 0\ , \quad \mu = \mu_0\mu_r\ \ , \qquad \varepsilon = \varepsilon_0\varepsilon_r$$

If the direction propagation in the z-direction

$$E_x = E_0 e^{i(wt-kz)},\ \ H_y = H_0 e^{i(wt-kz)}$$

$$\frac{d^2 H_y}{dz^2} H - \mu\sigma \frac{\partial H_y}{\partial t} - \mu\varepsilon \frac{\partial^2 H_y}{\partial t^2} = 0$$

$$(-ik)^2 H_y - \mu\sigma(iw)H_y - \mu\varepsilon(iw)^2 H_y = 0 \quad \rightarrow \quad -k^2 - i\mu\sigma w + \mu\varepsilon w^2 = 0$$

$$k^2 = \mu\varepsilon w^2 - i\mu\sigma w \qquad \rightarrow \qquad k^2 = \mu\varepsilon w^2\left(1 - i\frac{\sigma}{w\varepsilon}\right)$$

$$\left|\frac{J_d}{J_c}\right| = \left|\frac{\frac{\partial D}{\partial t}}{\sigma E}\right| = \left|\frac{\varepsilon\frac{\partial E}{\partial t}}{\sigma E}\right| = \left|\frac{iw\varepsilon E}{\sigma E}\right| = \frac{w\varepsilon}{\sigma} = \varphi$$

$1-\ \varphi \leq 0.02$ $\quad$ *Good conductor*

$2-\ \varphi \approx \infty$ $\quad$ *Perfect dielectric*

$3-\ \infty > \varphi > 0.02$ $\quad$ *Loosy dielectric (conductor)*

$$k^2 = \mu\varepsilon w^2\left(1 - \frac{i}{\varphi}\right) = \mu\varepsilon w^2 - i\frac{\mu\varepsilon w^2}{\varphi} \quad \ldots\ldots\ldots\ldots\ 1$$

$k = k_r - ik_i$

$k \equiv propagation\ \ constant$

$k_r \equiv attenuation\ factor$

$k_i \equiv phase\ shift\ constant$

$$k^2 = k_r^2 - 2ik_ik_r - k_i^2 \qquad \rightarrow \ \ k^2 = k_r^2 - k_i^2 - 2ik_ik_r \ \ \ldots\ldots\ldots \quad 2$$

By Compare between the (1&2)equations we get:

$$k_r^2 - k_i^2 = \mu\varepsilon w^2$$

$$2ik_ik_r = i\frac{\mu\varepsilon w^2}{\varphi}$$

$$\text{Real part} \quad k_r = w\sqrt{\left(\frac{\mu\varepsilon}{2}\right)}\ \sqrt{\left(\sqrt{1+\frac{1}{\varphi^2}}+1\right)}$$

$$\text{Imaginary part} \quad k_i = w\sqrt{\left(\frac{\mu\varepsilon}{2}\right)}\ \sqrt{\left(\sqrt{1+\frac{1}{\varphi^2}}-1\right)}$$

Q1/ The electric field intensity E of a uniform plane wave in air has an amplitude of (800 volt/m) and wave length ($\lambda = 2foot$) find:

a- The frequency b- The period c- The value of (k) if the field is expressed in the form ($E = Acos(wt - kz)$) d- The amplitude of H

Q2/ A (9.4GHz) plane wave is propagation in polyethylem if the amplitude of the H is $\left(7\frac{mA}{m}\right)$, and the material is assumed to be Lossless. Find

a- u b- λ in polyethlylem c- The phase constant

d- The intensity amplitude of (H) d- The intensity amplitude of (E)

4- Propagation of Plane Wave in Conductor media (Lossy dielectric)

$$k = w(\mu\varepsilon)^{1/2} \cdot (1+\frac{1}{\varphi})^{1/4} \cdot e^{-i\theta} \qquad \theta = \tan^{-1}\left(\frac{k_i}{k_r}\right)$$

The phase of E with respect to H

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\frac{E_x}{H_y} = \frac{w\mu}{k} = \frac{w\mu}{w(\mu\varepsilon)^{1/2} \cdot (1+\frac{1}{\varphi})^{1/4} \cdot e^{-i\theta}}$$

$$\frac{E_x}{H_y} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{1}{(1+\frac{1}{\varphi})^{1/4}} \cdot e^{i\theta}$$

$$\frac{E_x}{H_y} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \quad for\ free\ space$$

$$\frac{E_x}{H_y} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} \quad perfect\ dielectric$$

$$E_x = E_0 e^{i(wt-kz)} = E_0 e^{i(wt-(k_r-ik_i)z)} = E_0 e^{i(wt-k_r z)-k_i z} \qquad \ldots\ldots.. \quad 1$$

$$H_y = H_0 e^{i(wt-kz-\theta)} = H_0 e^{i(wt-(k_r-ik_i)z-\theta)} = H_0 e^{i(wt-k_r z-\theta)-k_i z} \qquad \ldots\ldots \quad 2$$

$$\frac{E_x}{H_y} = \frac{E_0}{H_0 e^{-i\theta}} \qquad \rightarrow \qquad \frac{E_0}{H_0} = \frac{E_x}{H_y} e^{-i\theta}$$

$$\frac{E_0}{H_0} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{1}{(1+\frac{1}{\varphi})^{1/4}} \cdot e^{i\theta} \cdot e^{-i\theta}$$

$$\eta^0 = \frac{E_0}{H_0} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{1}{(1+\frac{1}{\varphi})^{1/4}}$$

$$w = 2\pi f = 2\pi \frac{c}{\lambda_0} = \frac{c}{\lambda_0'} \qquad \lambda_0 = befor\ propagation \ \ , \ \ \lambda_0' = \frac{\lambda_0}{2\pi}$$

$$(\mu\varepsilon)^{1/2} = (\mu_r\varepsilon_r)^{1/2}(\mu_0\varepsilon_0)^{1/2}$$

$$k_r = \frac{c}{\lambda_0'}(\frac{\mu_r\varepsilon_r}{2})^{1/2} \cdot \frac{1}{c}\left[(1+\frac{1}{\varphi^2})^{1/2}+1\right]^{1/2}$$

$$k_r = \frac{1}{\lambda_0'}(\frac{\mu_r\varepsilon_r}{2})^{1/2} \cdot \left[(1+\frac{1}{\varphi^2})^{1/2}+1\right]^{1/2}$$

For perfect dielectric $\mu_r = 1 \ \ , \ \ \varphi = \infty$

$$k_r = \frac{1}{\lambda_0'}(\varepsilon_r)^{1/2}$$

$$k_r\lambda_0' = (\varepsilon_r)^{1/2} \qquad n^2 = \varepsilon_r$$

$$n = n_c - iK \qquad \begin{array}{l} n = Refractive\ index \\ n_c = \ Real\ part \\ K = Imaginery\ part \end{array}$$

$$n = k\lambda_0' = \lambda_0'[k_r - ik_i]$$

$$n = \lambda_0'k_r - i\lambda_0'k_i \qquad n_c = \lambda_0'k_r\ , \ \ K = \lambda_0'k_i$$

Poynting vector $E \times H$:

$\nabla \times H = J + \frac{\partial D}{\partial t}$

$$E \cdot \left(\nabla \times H = J + \frac{\partial D}{\partial t}\right)$$

$$\nabla \cdot (E \times H) = H \cdot (\nabla \times E) - E \cdot (\nabla \times H)$$

$$\nabla \cdot (E \times H) = H \cdot \left(-\mu \frac{\partial H}{\partial t}\right) - E \cdot \left(\varepsilon_0 \frac{\partial E}{\partial t} + J\right)$$

$$\nabla \cdot (E \times H) = -\frac{\partial}{\partial t}\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right) - E \cdot J$$

$$\int \nabla \cdot (E \times H) dv = \int \left(-\frac{\partial}{\partial t}\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right) - E \cdot J\right) dv$$

$$\underbrace{-\oint (E \times H) ds =}_{Flux} \quad \frac{\partial}{\partial t} \int \underbrace{\left(\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right)\right) dv}_{Time\ rate\ of\ energy} + \int (E \cdot J) dv \leftarrow loss\ heat\ energy$$

5- Propagation of Plane Electromagnetic Wave in Good Conductors

In conductors (Lossy dielectrics)

$$k_r = w\sqrt{\left(\frac{\mu\varepsilon}{2}\right)}\ \sqrt{\left(\sqrt{1+\frac{1}{\varphi^2}}+1\right)}$$

$$k_i = w\sqrt{\left(\frac{\mu\varepsilon}{2}\right)}\ \sqrt{\left(\sqrt{1+\frac{1}{\varphi^2}}-1\right)}$$

$$\varphi = \frac{w\varepsilon}{\sigma} \le \frac{1}{50} \qquad \textit{in Good conductors}$$

$$\varphi = \frac{w\varepsilon}{\sigma} << \frac{1}{50}$$

$$\therefore \quad w\sqrt{\left(\frac{\mu\varepsilon}{2}\right)}\ \sqrt{\left(\sqrt{1+\frac{1}{\varphi^2}}\mp 1\right)} \cong w\sqrt{\left(\frac{\mu\varepsilon}{2}\right)}\sqrt{\frac{1}{\varphi}}$$

$$Then \quad k_r = k_i = w\sqrt{\left(\frac{\mu\varepsilon}{2}\right)}\cdot\sqrt{\frac{1}{\varphi}} = w\sqrt{\left(\frac{\mu\varepsilon}{2}\right)}\cdot\sqrt{\frac{\sigma}{w\varepsilon}}$$

$$= \sqrt{\left(\frac{w\mu\sigma}{2}\right)} \quad \textit{in Good condcutors}$$

$$\nabla^2 E - \mu\sigma\frac{\partial E}{\partial t} - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} = \nabla\left(\frac{\rho}{\varepsilon}\right) \qquad But \quad \rho = J_d = 0$$

If the direction propagation in the z-direction

$$\frac{d^2E_x}{dz^2} - \mu\sigma\frac{\partial E_z}{\partial t} = 0$$

$(-ik)^2 E_x - \mu\sigma(iw)E_x = 0 \quad \rightarrow \quad -k^2 + i\mu\sigma w = 0$

$k^2 = -iw\mu\sigma$

$$k = \sqrt{w\mu\sigma}\sqrt{-i} = \sqrt{w\mu\sigma}\,\frac{1-i}{\sqrt{2}}$$

$$\sqrt{-1} = e^{-i\frac{\pi}{4}}$$
$$= \cos\frac{\pi}{4} - i\sin\frac{\pi}{4}$$
$$= \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}} = \frac{1-i}{\sqrt{2}}$$

$$k = \sqrt{\frac{w\mu\sigma}{2}}\,(1-i) = \sqrt{\frac{w\mu\sigma}{2}} - i\sqrt{\frac{w\mu\sigma}{2}}$$

$$k = k_r - ik_i$$

$$\therefore\ k_r = k_i = \sqrt{\frac{w\mu\sigma}{2}}$$

The attenuation constant:

$$\delta = \lambda' = \frac{1}{k_r} = \frac{1}{k_i} = \sqrt{\frac{2}{w\mu\sigma}}$$

δ is the distance the amplitude of the wave attenuation by a factor of $\left(\frac{1}{2}\right)$ in one radial.

- As $\quad \sigma - inceased \ \rightarrow \ \delta\ decreased$

  So Good conductors are there for always opaque of light. Except in the form of extremely thin films.

- The non-conducting media at optical frequency are necessary transport.

$$\nabla \times E_x = -\frac{\partial B_y}{\partial t}$$

$$\frac{E_x}{H_y} = \frac{w\mu}{k} = \frac{w\mu}{\sqrt{w\mu\sigma}}\frac{1}{\sqrt{-1}} = \frac{w\mu}{\sqrt{w\mu\sigma}}\frac{1}{e^{-i\frac{\pi}{4}}}$$

$$\frac{E_x}{H_y} = \sqrt{\frac{w\mu}{\sigma}}\ e^{+i\frac{\pi}{4}} \qquad Impedence$$

There for E leads H by$\left(\frac{\pi}{4}\right)$ radian in Good conductors

$$\frac{E_{0x}}{H_0 y} = \sqrt{\frac{w\mu}{\sigma}} \qquad (H.w)$$

The phase velocity u

$$u = \frac{w}{k_r} = \frac{w}{\left(\frac{w\mu\sigma}{2}\right)^{\frac{1}{2}}} = \left(\frac{2w}{\mu\sigma}\right)^{\frac{1}{2}} = \delta w$$

$$n = \lambda'_0 k = \lambda'_0(k_r - ik_i) = \lambda'_0 k_r - i\lambda'_0 k_i$$

$$n = n_c - K$$

Poynting vector $E \times H$

$$\nabla \cdot (E \times H) = H \cdot (\nabla \times E) - E \cdot (\nabla \times H)$$

$$\nabla \cdot (E \times H) = H \cdot \left(-\frac{\partial B}{\partial t}\right) - E \cdot (J_c + J_d)$$

$$\nabla \cdot (E \times H) = -\mu H \cdot \frac{\partial H}{\partial t} - E \cdot J_c$$

$$\nabla \cdot (\mathrm{E} \times \mathrm{H}) = -\frac{\partial}{\partial t}\left(\frac{\mu H^2}{2}\right) - E \cdot J_c$$

$$\int \nabla \cdot (\mathrm{E} \times \mathrm{H}) dv = -\frac{\partial}{\partial t} \int \left(\frac{\mu H^2}{2}\right) dv - \int (E \cdot J_c) dv$$

$$\underbrace{-\oint (\mathrm{E} \times \mathrm{H}) \mathrm{ds} =}_{Flux} \quad \frac{\partial}{\partial t} \int \underbrace{\left(\frac{\mu H^2}{2}\right) dv}_{rate\ \ increas\ of\ magnetic\ energy} + \int (E \cdot J_c) dv \leftarrow loss\ heat\ energy$$

6- Propagation of Plane Electromagnetic Wave in low pressure ionized gas:

We shall use the general result of a homogenous (Isotropic) medium, which its properties do not vary from point to point.

Isotropic: The properties are same in all direction from any given point.

The wave equation for (E) and (H) in such media follows:

$$\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = \nabla \left(\frac{\rho}{\varepsilon}\right)$$

$$\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0$$

We assume the Gas pressure is low so that collations

And energy loses can be neglected . We all so neglect thermal agitation by setting T=0

Under these conditions the pressure in ionized gases is different from that in metals. Let us consider a plane electromagnetic wave travelling in z-direction through ionized gas, which an ion own properties (*mass=m, charge=q&velocity=v)*,situated at *(x,y,z)* then:

$$F_E = qE_x \quad , \quad F_m = q\left(v \times B_y\right)$$

$$F_T = F_E + F_m = qE_x + qv \times B_y = q(E_x + v \times B_y)$$

$$F = ma = m\left(\frac{\partial^2 x}{\partial t^2} + \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2 z}{\partial t^2}\right), \qquad v = \frac{dx}{dt} a_x + \frac{dy}{dt} a_y + \frac{dz}{dt} a_z$$

$$v \times B = \begin{vmatrix} a_x & a_y & a_z \\ \frac{dx}{dt} & \frac{dy}{dt} & \frac{dz}{dt} \\ \underbrace{B_x}_{0} & B_y & \underbrace{B_z}_{0} \end{vmatrix} = -\frac{dz}{dt} B_y a_x + \frac{dx}{dt} B_y a_z$$

$$\therefore \quad F = m\left(\frac{\partial^2 x}{\partial t^2} + \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2 z}{\partial t^2}\right) = q(E_x + v \times B_y)$$

$$\left(\frac{\partial^2 x}{\partial t^2} + \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2 z}{\partial t^2}\right) = \frac{q}{m}(E_x + v \times B_y)$$

$$\left(\frac{\partial^2 x}{\partial t^2} + \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2 z}{\partial t^2}\right) = \frac{q}{m}(E_x - \frac{dz}{dt} B_y a_x + \frac{dx}{dt} B_y a_z)$$

$$\frac{\partial^2 y}{\partial t^2} = 0$$

$$\frac{\partial^2 x}{\partial t^2} = \frac{q}{m}\left(E_x - \frac{dz}{dt} B_y\right) \ ------- \ 1$$

$$\frac{\partial^2 z}{\partial t^2} = \frac{q}{m}\frac{dx}{dt} B_y \ --------- \ 2$$

From eq(1):

$$\frac{\partial^2 x}{\partial t^2} = \frac{q}{m}\left(E_x - \frac{dz}{dt} B_y\right) = \frac{qE_x}{m}\left(1 - \frac{B_y}{E_x}\frac{dz}{dt}\right)$$

$$In\ free\ space\ approximate \approx \frac{E}{B} = c$$

$$\frac{\partial^2 x}{\partial t^2} = \frac{qE_x}{m}\left(1 - \frac{1}{c}\frac{dz}{dt}\right) \ ------ \ 3$$

From eq(2):

$$\frac{\partial^2 z}{\partial t^2} = \frac{q}{m}\frac{dx}{dt}B_y \quad \times \quad \frac{E_x}{E_x}$$

$$\frac{\partial^2 z}{\partial t^2} = \frac{qE_x}{m}\cdot\frac{B_y}{E_x}\cdot\frac{dx}{dt} = \frac{qE_x}{m}\cdot\frac{1}{c}\cdot\frac{dx}{dt}$$

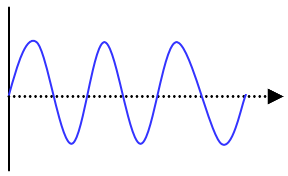
$$\frac{\partial^2 z}{\partial t^2} = \frac{qE_x}{m}\cdot\frac{1}{c}\cdot\frac{dx}{dt} \quad ------- \quad 4$$

Assume that $v << c$

$$\therefore \quad \frac{1}{c}\frac{dz}{dt} << 1$$

Then we can written the eq(3) as:

$$\frac{\partial^2 x}{\partial t^2} = \frac{qE_x}{m} \quad , \qquad E_x = E_0\cos(wt)$$



$$\frac{\partial^2 x}{\partial t^2} = \frac{q}{m}E_0\cos(wt) \qquad (accelaration) ---- \quad 5$$

$$\frac{dx}{dt} = \frac{q}{wm}E_0\sin(wt) \qquad (velocity) ----- \quad 6$$

$$x = -\frac{q}{w^2 m}E_0\cos(wt) \qquad (distance) ----- \quad 7$$

From eq(4&6) we get:

$$\frac{\partial^2 z}{\partial t^2} = \frac{qE_x}{m}\cdot\frac{1}{c}\cdot\frac{q}{wm}E_0\sin(wt) \quad \rightarrow \quad \frac{\partial^2 z}{\partial t^2} = \frac{q^2E_0^2}{m^2wc}\cdot cos(wt)\cdot sin(wt)$$

$$\underbrace{\frac{\partial^2 z}{\partial t^2} = \frac{q^2E_0^2}{m^2wc}\cdot sin(2wt)}_{accelaration} \qquad where \qquad sin(2wt) = cos(wt)\cdot sin(wt)$$

$$\frac{dz}{dt} = \frac{-q^2E_0^2}{4m^2w^2c} \cdot cos(2wt) \qquad (velocity)$$

$$z = \frac{q^2E_0^2}{8m^2w^3c} \cdot sin(2wt) \qquad (distance)$$

$$\frac{dz}{dt} = \frac{-q^2E_0^2}{4m^2w^2c} \cdot cos(2wt) \quad \times \frac{1}{c}$$

$$\frac{1}{c}\frac{dz}{dt} = \frac{-q^2E_0^2}{4m^2w^2c^2} \cdot cos(2wt)$$

$$\frac{1}{c}\left(\frac{dz}{dt}\right)_{Max.} = \frac{-q^2E_0^2}{4m^2w^2c^2} \qquad at\ Max. velocity \quad \cos(\theta) = 1$$

If the charge particle it's an electron:

$$m = 9.1 \times 10^{-31}kg \quad , \quad q = 1.6 \times 10^{-19}c \quad , \quad w = 2\pi f$$

$$\frac{1}{c}\left(\frac{dz}{dt}\right)_{Max.} = 2.17 \times 10^3 \frac{\overbrace{E_0^2}^{Power}}{f^2}$$

$For\ free\ space \qquad S = E \times H = EHsin\theta$

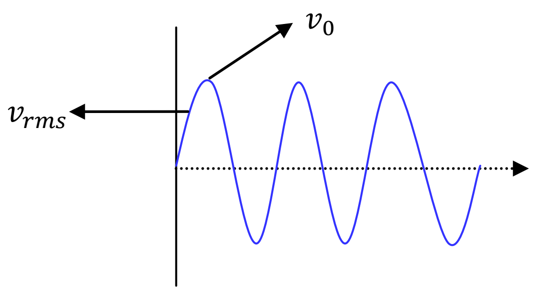
$S = E \times H = EH \qquad \sin 90 = 1$

$$B = \mu H$$

$$S = \frac{E^2}{\mu_0 c} \quad , \qquad \frac{E}{B} = c \ \ in\ free\ space \qquad \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$$

$$S = \varepsilon_0 c E^2$$

$$S_{av} = \varepsilon_0 c E_{rms}^2 = \varepsilon_0 c \left(\frac{E_0}{\sqrt{2}}\right)^2$$



$$S_{av} = \frac{1}{2}\varepsilon_0 c E_0^2 \qquad \rightarrow \qquad E_0^2 = \frac{2S_{av}}{\varepsilon_0 c}$$

$$\frac{1}{c}\left(\frac{dz}{dt}\right)_{Max.} = 2.17\times10^3\,\frac{E_0^2}{f^2} = 2.17\times10^3\,\frac{1}{f^2}\cdot\frac{2S_{av}}{\varepsilon_0 c} = \frac{2.17\times10^3}{2.66\times10^{-3}}\frac{S_{av}}{f^2}$$

$$\frac{1}{c}\left(\frac{dz}{dt}\right)_{Max.} = 1.63\times10^6\ \frac{S_{av}}{f^2}$$

$$\frac{1}{c}\left(\frac{dz}{dt}\right)_{Max.} \propto\ S_{av}$$

$$\frac{1}{c}\left(\frac{dz}{dt}\right)_{Max.} \propto\ \frac{1}{f^2}$$

This seems to indicate that it is possible for $\left(\frac{dz}{dt}\right)_{Max.}$ to exceed the velocity of light source, we have neglected the relativistic effect.

The ratio $\frac{1}{c}\left(\frac{dz}{dt}\right)_{Max.} = \frac{v_z}{c}$

Is usually much smaller than unity.

Example: At 1km from an antenna radiating 50kwatt of power is tropically at frequency of 1MHz.

Sol/ $\frac{1}{c}\left(\frac{dz}{dt}\right)_{Max.} = 1.63\times10^6\ \frac{S_{av}}{f^2} = 1.63\times10^6\ \frac{50\times10^3}{10^{12}} = 10^{-8}$

Example: A laser beam can carry power density of the order of $\left(10^{16}\,\frac{w}{m^2}\ \ at\ f\cong 10^{15}Hz\right)$ (visible light).

Sol/ $\frac{1}{c}\left(\frac{dz}{dt}\right)_{Max.} = 1.63\times10^6\ \frac{S_{av}}{f^2} = 1.63\times10^6\ \frac{10^{16}}{(10^{15})^2} \cong 1.63\times10^{-8}$

The conductivity of an ionized gas

Since v_d (drift velocity)can be a scribed to the E, we can consider the ionized gas as having conductivity (σ) such that:

$$\underbrace{J_c}_{conduction\ current} = \sigma E \qquad \underbrace{J_\rho}_{convection\ current} = \rho v_d \quad for\ Gas$$

$$J = \sum_{i=1}^{n} \overbrace{N_i Q_i}^{\rho} \overbrace{\left(\frac{dx}{dt}\right)}^{v}$$

$$E_x = E_0 \cos(wt) = E_0 e^{iwt}$$

$$\frac{dx}{dt} = \frac{Q}{mw} E_0 \sin(wt)$$

$$J_c = \sigma E = \sum_{i=1}^{n} N_i Q_i \left(\frac{dx}{dt}\right)$$

$$\frac{dx}{dt} = \frac{Q}{mw} E_0 \cos(\mathrm{wt} - \frac{\pi}{2})$$

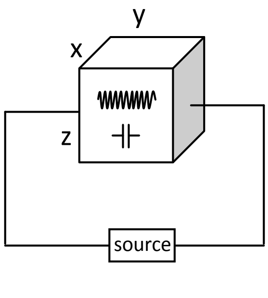
$$\cos(\mathrm{wt} - \frac{\pi}{2}) = e^{i(wt - \frac{\pi}{2})}$$

$$\sigma E_0 e^{iwt} = \sum_{i=1}^{n} \frac{N_i Q_i^2}{w m_i} E_0 e^{i(wt - \frac{\pi}{2})}$$

$$\sigma = \sum_{i=1}^{n} \frac{N_i Q_i^2}{w m_i} e^{-i\frac{\pi}{2}} \qquad e^{-i\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) - i\sin\left(\frac{\pi}{2}\right) = -i$$

$$\sigma = \frac{-i}{w} \sum_{i=1}^{n} \frac{N_i Q_i^2}{m_i} \quad \rightarrow \quad \sigma = \frac{-i N_e Q_e^2}{w m_e} \quad For\ electrons$$

$$R = \frac{V}{I} = \rho \frac{l}{A} = \rho = \frac{1}{\sigma} \qquad let\ \ l = A \qquad such\ as\ \ x = y = z = 1cm$$



$$For\ this\ case\ \ \sigma = \frac{1}{\underbrace{R}_{Resistance}} = G \quad Conductance\ \ in\ (Resistance)$$

$$\sigma = \frac{1}{\underbrace{Z}_{Impedance}} = Y \quad Conductance \quad in\ \ (Coil + Capacitance)$$

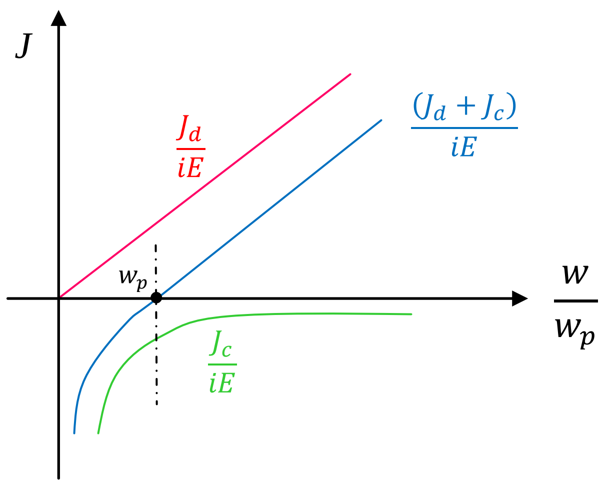
$$Z = \frac{1}{\sigma} = i\frac{wm_e}{N_e Q_e^2} = iwI \qquad Where\ \ I = \frac{m_e}{N_e Q_e^2}$$

$$I = \frac{m_e}{N_e Q_e^2} \quad is\ the\ equivalent\ inductance\ of\ an\ imaginary\ cabic\ of\ plasma$$

$$conduction\ current\ \ J_c = \sigma E = -i\frac{N_e Q_e^2}{wm_e}E = \frac{-i}{wI}E \quad \rightarrow \quad \frac{J_c}{-iE} = \frac{1}{wI}$$

$$displacement\ current \qquad J_d = \frac{dD}{dt} = \varepsilon_0 \frac{dE}{dt} \quad \rightarrow \quad J_d = i\varepsilon_0 wE$$

This means that J_c loges E by ($\frac{\pi}{2}$ Radius) and J_d leads E by ($\frac{\pi}{2}$ Radius), so $J_d\ leads\ J_c$ by ($\pi\ Radius$) , or $J_d\ and\ J_c$ are by ($\pi\ Radius$) out of phase.



$$at\ \ \frac{w}{w_p} > 1 \qquad The\ total\ current\ is\ Capacity$$

$$at\ \ \frac{w}{w_p} < 1 \qquad The\ total\ current\ is\ inductive$$

$$at\ w = w_p \qquad The\ total\ current\ is\ zero$$

Q1/ Copper (Cu) has a conductivity $\sigma \cong 6 \times 10^{7} \frac{1}{\Omega m}$, $\mu_r = 1$ and $\varepsilon_r = 3.5$, at 6MHz. Find: φ, skin depth, phase velocity and $\frac{E_0}{H_0}$. Or $Find\ \varphi, \delta, \frac{\lambda}{\lambda_0}, u, |\eta|$

$$\varphi = \frac{w\varepsilon}{\sigma} = \frac{2*3.14*6*10^{6}*3.5*8.8542*10^{-12}}{6*10^{7}} \cong 19.46 * 10^{-12}$$

$$\delta = \sqrt{\frac{2}{w\mu\sigma}} = \sqrt{\frac{2}{2*3.14*f*4*3.14*10^{-7}*6*10^{7}}} = \frac{0.066}{\sqrt{6*10^{6}}} = 2.694 * 10^{-5}\ m$$

$$u = \frac{w}{k_r} = \frac{w}{\left(\frac{w\mu\sigma}{2}\right)^{\frac{1}{2}}} = \left(\frac{2w}{\mu\sigma}\right)^{\frac{1}{2}} = \delta w = 2.694 * 10^{-5} * 2 * 3.14 * 6 * 10^{6} =$$

$1107.79\ m/s$

$$\frac{E_{0x}}{H_0 y} = \sqrt{\frac{w\mu}{\sigma}} = \sqrt{\frac{2*3.14*6*10^{6}*4*3.14*10^{-7}}{6*10^{7}}} = 8881.26$$

Q2/ If the 20MHz laser beam passes through a glass whose index of Refraction is n=1.6 find:

$1 -\ \varepsilon_r\ , \mu_r$
$2 -\ E_0\ in\ the\ glass$
$3 -\ H_0\ in\ the\ glass$

$n = \text{Refraction index} = \frac{c}{u}$ $\qquad u = \frac{c}{n} = \frac{3 * 10^{8}}{1.6} = 1.875 * 10^{8}\ \frac{m}{s}$

$n = \sqrt{\varepsilon_r}$ $\qquad \varepsilon_r = n^2 = (1.6)^2 = 2.56$

$$\frac{E_x}{B_y} = u = \eta_0 \times \sqrt{\frac{\mu_r}{\varepsilon_r}} = 120 * 3.14 \times \sqrt{\frac{\mu_r}{\varepsilon_r}} = 1.875 * 10^{8}$$

$$\sqrt{\mu_r} = \frac{1.875 * 10^{8}}{120 * 3.14} = 497611.46 * \sqrt{\varepsilon_r} = 629434.24$$

$$\mu_r = 396187464197.33$$