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Lecture 3

Vectors and Matrices

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3.1 Vectors

- ❖ A **vector** is an ordered list of numbers.
- ❖ You can enter a vector of any length in MATLAB by typing a list of numbers, separated by **commas** and/or **spaces**, inside square brackets.
- ❖ For example: `>> z = [1,4,7,18]` `>> y = [4 -3 5 -2 8 1]`
- ❖ Vector of values running from 1 to 9:
`>> x = 1:9` `x = 1 2 3 4 5 6 7 8 9`
- ❖ The increment can be specified as the **middle** of three argument:
`>> x = 0:2:10` `x = 0 2 4 6 8 10`
- ❖ Increments can be **fractional or negative**, for example,
`>> 0:0.1:1` or `100:-1:0`.
- ❖ `linspace(0,10,6)` `ans =` 0 2 4 6 8 10

3.1 Vectors

❖ The elements of the vector **x** can be **extracted** as **x(1)**, **x(2)**, etc. For example: `>> x = 0:7;`

`>> x(3)` `>> x(4:7)` `>> x([4,7])`

❖ To **change** the vector **x** from a row vector to a column vector, put a **prime** (**'**) after **x**: `>> x'`

`>> x1=[5,3,1,23,11], min(x1), max(x1), mean(x1), sort(x1), sum(x1).`

❖ You can perform **mathematical operations** on vectors.

for example, to **square** the elements of the vector **x**,

`>> x = 0:2:10`

`>> x.^2` `ans = 0 4 16 36 64 100;`

❖ The **period**(**.**) in this expression says that the numbers in **x** should be squared individually, or *element-by-element*.

3.1 Vectors

- ❖ Typing \mathbf{x}^2 would tell MATLAB to use matrix multiplication to multiply \mathbf{x} by itself and would produce an **error** message in this case.
- ❖ Similarly, you must type $\mathbf{x}.*\mathbf{y}$ or $\mathbf{x}./\mathbf{y}$ if you want to multiply or divide vectors element-by-element.

```
>> x.*y      ans = 0 -6 20 -12 64 10
```

- ❖ Most MATLAB operations are, by **default**, performed element-by-element. **For example**, you do not type a period(.) before: **addition**, **subtraction** and **exp(x)** (the **matrix exponential** function is **expm**).

Example

The Redlich-Kwong equation of state is given by:

$$P = \frac{RT}{v - b} - \frac{a}{v(v + b)\sqrt{T}}$$

Where $R = 8.31 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$, $T = 171, 181, 191 \text{ K}$, the value of a from 0.1 to 1.9, $b = 0.086$, and $v = 5 \times 10^{-2}$. Find the pressure P .

3.2 Matrices

❖ A **matrix** is a rectangular array of numbers. Row and column vectors are examples of matrices.

❖ **Example 1:** Write the following Matrix: $a = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 7 & 15 & 3 \\ 12 & 9 & 6 & 3 \end{pmatrix}$

Ans: $a = [1:4; 5,7,15,3; 12:-3:3];$

Note The “[]” defines the matrix in MATLAB. The matrix *elements* in any row are separated by **commas**, and the rows are separated by **semicolons**.

❖ The elements in a row can also be separated by **spaces**.

Example 2: Extract: $a(7)$, $a(3,2)$, $a(2,:)$, $a(1:3,2:3)$, $a([2\ 3],[1\ 3])$

❖ If two matrices **A** and **B** are the same size:

sum: $A+B$ **add a scalar** (a single number): $A + c$

difference: $A-B$ **subtracts:** $A - c$

3.2 Matrices

❖ Here are some commands:

`zeros(1,3)`, `ones(2,4)`, `rand(3,5)`, `randn(2,5)`, `eye(n,m)`, `det(A)`,
`size (A)`, `Length (A)`.

Example: If `v=[1, 2, 4, 5]`, `w=[1;2;4;5]`, and `A = [1,2,3;4,-5,6;5,-6,7]`,
try to do the following steps:

```
>> v+2    >>B=A'    >> A*B    >>A+B    >> B=A.'
```

```
>> A(2,3)    >> A([2 3],[1 2])    >> B(:,2) = [ ]
```

```
>> B=A([3 2],[2 1])
```

```
>> B=[A(3,2);A(3,1);A(2,2);A(2,1)]
```

3.2 Matrices

Transpose	$B = A'$
Identity Matrix	<code>eye(n)</code> → returns an $n \times n$ identity matrix <code>eye(m,n)</code> → returns an $m \times n$ matrix with ones on the main diagonal and zeros elsewhere.
Addition and subtraction	$C = A + B$ $C = A - B$
Scalar Multiplication	$B = \alpha A$, where α is a scalar.
Matrix Multiplication	$C = A * B$
Matrix Inverse	$B = \text{inv}(A)$, A must be a square matrix in this case. <code>rank (A)</code> → returns the rank of the matrix A .
Matrix Powers	$B = A.^2$ → squares each element in the matrix $C = A * A$ → computes $A * A$, and A must be a square matrix.
Determinant	<code>det (A)</code> , and A must be a square matrix.

3.3 MATLAB Functions

❖ Functions @ sign:

$g = @(x, y) x^2 + y^2;$ **$g = g(1, 2);$** **$g = 5$**

Example: Use built in functions at the points (1:5,2) and (1:5,2:6) for the function of $g = x^2 + y^2$.

Ans: **$g = @(x, y) x.^2 + y^2;$** **$g(1:5, 2);$**

3.4 Solving Equations

- ❖ You can solve algebraic equations, differential equations and solve equations involving variables with **solve** or **fzero**.
- ❖ The command **solve** can solve:
 1. Higher-degree polynomial equations.
 2. Equations involving more than one variable
- ❖ The input to solve can be **symbolic** expression
- ❖ Solve **algebraic equations** to get either exact analytic solutions or high-precision numeric solutions.

Examples about analytic solutions

1. Solve the following equations:

```
>> syms x    solve(x^2 - 2*x - 4 == 0);    ans = 5^(1/2)+1    1-  
5^(1/2).
```

- ❖ The answer is the exact (symbolic) solutions.
- ❖ To get numerical solutions, type **double(ans)**, or **vpa(ans)** to display more digits.

2. Solve $x^2 - 3x = -7$; `syms x` `solve(x^2 - 3*x + 7)`

```
ans= 3/2+1/2*i*19^(1/2)  
      3/2-1/2*i*19^(1/2)
```

- ❖ To get numerical solutions, type **double(ans)**, or **vpa(ans)**.

3. Solve $(2x - \ln y = 1$ for y in terms of x),

```
type:    syms x y
```

```
solve(2*x - log(y) - 1, y)
```

Examples

4. Solve these two equations: $x^2 - y = 2$, $y - 2x = 5$

Ans: `syms x y [x,y]=solve(x^2-y-2,y-2*x-5);`

$$\begin{array}{lcl} x = & & y = \\ 1+2*2^{(1/2)} & & 7+4*2^{(1/2)} \\ 1-2*2^{(1/2)} & & 7-4*2^{(1/2)} \end{array}$$

❖ You can **extract** the first x and y values by typing:

`>> x(1) ans = 1+2*2^(1/2) >> y(1) ans = 7+4*2^(1/2)`

❖ Some equations **cannot** be solved symbolically, and in these cases **solve** tries to find a numerical answer.

5. `syms x solve(sin(x) == 2 - x)`

