University of Salahaddin - Erbil   
College of Administration and Economics   
Department of Statistics and informatics

Probability Models

Fourth stage

**Second Course**  
Academic year 2022 - 2023

**Three units in week**

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**Course overview:**

Probability Models, provides information pertinent to the standard concepts and methods of stochastic modeling. This course begins with an overview of diverse types of stochastic models, which predicts a set of possible outcomes weighed by their likelihoods or probabilities.

For a model to be stochastic, it must have a random variable where a level of uncertainty exists. Due to the uncertainty present in a stochastic model, the results provide an estimate of the probability of various outcomes.

To estimate the probability of each outcome, one or more of the inputs must allow for random variation over time. It results in an estimation of the probability distributions, which are mathematical functions that show the likelihood of different outcomes.

For example, if you are analyzing investment returns, a stochastic model would provide an estimate of the probability of various returns based on the uncertain input (e.g., [market volatility](https://corporatefinanceinstitute.com/resources/knowledge/trading-investing/vix-volatility-index/)). The random variable typically uses time-series data, which shows differences observed in historical data over time. The final probability distributions result from many stochastic projections that reflect the randomness in the inputs.

stochastic models must meet several criteria that distinguish them from other probability models. First, stochastic models must contain one or more inputs reflecting the uncertainty in the projected situation. Generally, the model must reflect all aspects of the situation to project a probability distribution correctly.

Probabilities are correlated to events within the model, which reflect the randomness of the inputs. The probabilities are then used to make predictions or to provide relevant information about the situation.

**Course objective:**

Course objective:

The students who succeeded in this course; • At the end of the course the students are expected to:

1) Know the properties and usage of special probability distributions .

2) Understand the notion of stochastic process and analyze different types of stochastic processes.

4) Model different real-life situations with the help of stochastic processesa topological space instead of limited to real values representing time

**Student's obligation**

* . Closed-book policy: No use of calculators, or books will be allowed during any in-class tests/quizzes.
* Policy related to make-up exams or other work: There will be no opportunities to make up for work not submitted. However, if a student provides a legitimate excuse well in advance, scores will be prorated. Work with due date should be turned in at the beginning of class on the stated due date. Late work will not be accepted and will be deemed work not submitted.
* Policy on class attendance: Requirements for class attendance and make-up exams, assignments, and other work in this course are consistent with university policies that can be found in the online catalog at: <https://catalog.ufl.edu/ugrad/current/regulations/info/attendance.aspx>
* **Forms of teaching**
* Different forms of teaching will be used to reach the objective of the course : power point presentation for the head titles and summary of conclusion with applications by some equations, besides worksheet will be designed to let the chance for practicing on several aspects of the course in the classroom, furthermore students will contents homework . There will be classroom discussions and the lecture will give enough background to translate, solve, analyze, and evaluate problems sets.

**Assessment scheme**

**The student must be examined twice in each course. The last grade is (30).**

**Putting grades for daily activities, homework, for (10) marks.**

**The annual work of the material (40) marks.**

**The final exam out of (60) marks.**

**Student learning outcome:**

* On successful completion of the course, students should be able to:
* Explain fundamentals of probability theory, random variables and random processes.
* Understand the mathematical concepts related to probability theory and random processes.
* Understand the characterization of random processes and their properties.
* Formulate and solve the engineering problems involving random processes.
* Analyze the given probabilistic model of the problem.
* Make precise statements about random processes.
* Use computational techniques to generate simulation results.

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| **Course Reading List and References‌:**  **Stochastic Processes, Estimation, and Control**  **Jason L. Speyer**  **Walter H. Chung**  University of California, Los Angeles  Copyright © 2008 by the Society for Industrial and Applied Mathematics.  **Theory and Problems of Probability, Random Variables, and Random Processes**  **Hwei P. Hsu, Ph.D.** Professor of Electrical Engineering  Fairleigh Dickinson University  Copyright © 1997 by The McGraw-Hill Companies.  **LEVEL SETS AND EXTREMA OF RANDOM PROCESSES AND FIELDS**  **JEAN-MARC AZA¨IS**  Universite´ de Toulouse, Institut de Mathe´matiques, Laboratoire de Statistiques et Probabilite´ s, Toulouse, France  **MARIO WSCHEBOR**  Universidad de la Repu´ blica, Centro de Matema´ tica, Facultad de Ciencias  Montevideo, Urugua  A JOHN WILEY & SONS, INC., PUBLICATION  Copyright © 2009 by John Wiley & Sons, Inc. All rights reserved.  **Probability and Statistics by Example: II Markov Chains: a Primer in Random Processes and their Applications**  Yuri Suhov /*University of Cambridge*  Mark Kelbert /*University of Wales–Swansea*  © Y. Suhov and M. Kelbert 2008 | | |
| **The Topics:** | **Lecturer's name** | |
| |  |  | | --- | --- | | **Week** | **Subject** | | **Week-1** | **concept and definition probability models** | | **Week-2** | **Capter-1 : Bernoulli Processes**  **1.1 Definition of Bernoulli process**  **1.2 Basic properties of Bernoulli process** | | **Week-3** | **1.3 Interarrival Time**  **1.4 The time of the kth success** | | **Week-4** | **1.5 Splitting and Merging of Bernoulli Processes**  **1.6 Exercises** | | **Week-5** | **Capter-2: The Poisson Process**  **2.1 Definition Poisson Process**  **2.2 Counting process**  **2.3 Independent and stationary increments characterization of the Poisson process** | | **Week-6** | **2.4 Time between Arrivals**   1. **Relation of Poisson and exponential distribution:** 2. **Relation of Poisson and gamma distribution** | | **Week-7** | **2.5 Splitting and Merging of Poisson Processes** | | **Week-8** | **2.6 Conditional distribution of the arrival times**  **2.7 Compound Poisson process**  2. 8 Poisson process**has a "**no memory**"**property | | **Week-9** | **Chapter 3: The Birth and Death Process**  **3.1 Definition The Birth and Death Process**  **3.2 [THE CLASSIFICATION OF BIRTH AND DEATH PROCESSES](https://www.ams.org/tran/1957-086-02/S0002-9947-1957-0094854-8/S0002-9947-1957-0094854-8.pdf)**  **3.3 General birth and death process** | | **Week-10** | **3.4 Continuous Time Markov Chains**  **3.5 Pure Birth Process Death Process**  **3.6 Continuous Time Markov Chains** | | **Week-10** | **Capter-4 : Queueing Theory**  **4.1 Definition The Queueing Theory**  **4.2 Applications of Queueing Theory** | | **Week-11** | **4.3 Characteristics of queuing systems** | | **Week-12** | **4.4 The M/M/1 Queuing System**  **4.5 Analysis The M/M/1 Queuing System** | | **Week-13** | **4.6 The M/M/1/1 Queuing System**  **4.7 Analysis The M/M/1/k Queuing System** | | **Week-14** | **4.8 The M/M/k Queuing System**  **4.9 Analysis The M/M/k Queuing System** | | **Week-15** | **Mid-semester exam.** | | | |
| **Practical Topics (If there is any)** | |  |
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| **Examinations:**  **1/** Arrival of virus **attacks** (هيرش كردنة سةر) to a Personal Computer (PC) can be modeled by a Poisson process with rate λ =6 attacks per hour.   1. **What is the probability that exactly one attack will arrive between 1 p.m. and 2 p.m.** 2. **Given that six attacks arrive between 1 p.m. and 2 p.m., what is the probability that the fifth attack will arrive between 1:30 p.m. and 2 p.m.?** 3. **What is the expected arrival time of that fifth attack?**   **2/**It is estimated that 80% of people in Erbil **take(وةردةكرن)** Paracetamol for **headache treatment(ضارةسةري سةر ئيَشة**) ( 40% from the testing is females)   1. **Probability that among 5 patient (نةخؤش) at least 2 people take Paracetamol.** 2. **If you know that a patient is Female، find probability that a 6th patient take Paracetamol from 20th peoples.**   **3//** People arrive at a telephone booth(كؤشكي تةلةفؤن) according to a Poisson process at an average rate of 12 per hour, and the average time for each call is an exponential r.v. with mean 2 minutes. Find   1. **Average number of people in the system** 2. The telephone company decision (كؤمثانياي تةلةفؤن برياريدا) to add a second booth (داناني كؤشكي دووةم) **if** customers wait in the queue an average of 3 or more minutes for the phone (Wq >= 3 minutes). **Find the average arrival rate** needed to justify a second booth?   **12-**  **4/ A** conversation in a wireless Reber network is severely disturbed by interference signals according to a Poisson process of rate λ =0.1 per minute.  1- What is the probability that no interference signals occur within the first two minutes of the conversation?  2- Given that the first two minutes are free of disturbing effects, what is the probability that in the next minute precisely one interfering signal disturbs the conversation?  5/ The counter of a bank branch performs the transactions with a mean time of 2 minutes. The customers arrive at a mean rate of 20 customers/hour. If we assume that arrivals follow a Poisson process and that the service time is exponential, determine:  a) Percentage of the time the bank teller is idle  b) Mean waiting time of the customers  c) Percentage of customers that wait in a queue  6/ A grocery shop is attended by one person. Apparently, the arrival pattern of customers during Saturdays follows a Poisson process with an arrival rate of 10 persons/hour. Customers are attended following a FIFO order and, due to the prestige of the shop, once they arrive, they are willing to wait for the service. The service time is distributed exponentially, with a mean time of 4 minutes. Determine  a) Probability of waiting in line  b) Average length of the waiting line  c) Average waiting time  7/ Customers arrive at a hair salon with a mean rate of 5 customers/hour and interarrival times distributed exponentially. There is always one hairstylist present in the salon at any moment and there are 4 chairs for the customers that arrive when the hairstylist is busy. The law regarding fire prevention limits the total number of customers in the salon, at any moment, to a maximum of 5. Customers that arrive when the salon is full can’t enter in it. Service time is distributed exponentially with a mean that depends on the number of customers.  Determine:  a) Average number of waiting customers  b) Average waiting time  c) Percentage of time that the hairstylist is idle   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | Number of customers in the salon | 1 | 2 | 3 | 4 | 5 | | Mean service time per customer (min.) | 9 | 10 | 10 | 13 | 20 |   8/ In the border between countries A and B, the road from B to A is split into five migration and customs inspection booths. Let’s assume that car arrivals have a Poisson distribution with 15 arrivals/hour, while the number of services follows an exponential distribution of 8 services/hour. By Government decree, there is no treatment priority (is that credible?), so the inspection booths provide the service as soon as they are free, attending the cars by order of arrival to the queue. Check if l m < , and in positive case, interpret this inequality. Moreover, determine:  a) Compute p0 , L , Lq , Wq , W , defining its meaning  b) Study if it would be reasonable to suppress three booths  9/ Customers arrive at a shop as a Poisson process with rate 20 per hour. And 60% of the Customers are female. Find   * + - 1. What is the probability that no Customers arrive in the 20 minutes?       2. Haw many Customers are expected to arrive in 2 hour?       3. What is the probability that the time between the arrival of the 3th Customers and the arrival of the 5th Customers is more than 20 minutes?   10/ At the end of a semester, 15% of the students taking a statistics course have fail (رسوب).  60 % of all students have attended the lectures regularly (ئامةدةبوون لة محاضرة بةشيويةكي ريكوبيك).   1. What is the probability that out of the 50 students at least 2 student's fail? 2. On average how many students must be tested to find the third students has fail. 3. On average how many students must be tested to find the three students fail. 4. What is the probability that more than 2 students attended the lectures regularly? | | |