Chapter one

Linear Programming problems

1-1: Linear Programming problems

Is a class of mathematical model concerned with the efficient allocation of limited resources to known activities with the objective of meeting a desired goal (such as maximizing profit or minimizing cost) . The distinct characteristic of Linear Programming models is that the function representing the objective and the constraints are linear.

*Before formally defining a linear programming problem, we define the concepts of **linear function** and **linear inequality**.*

Definition:

A function $f(x_1, x_2, \ldots, x_n)$ *of* $x_1, x_2, \ldots, x_n$ *is a linear Function if and only if for some set of constants* $C_1, C_2, \ldots, C_n$,

$$f(x_1, x_2, \ldots, x_n) = C_1x_1 + C_2x_2 + \ldots + C_nx_n$$

But

$$f(x_1, x_2) = 2x_1^2\, x_2$$

is not a linear function of x_1 *and* x_2 .

1-2: General form of L.P.:

The L.P. may be of the maximization or minimization type, the constraints may be the type (=, ≥, ≤) and variables may be non-negative or unrestricted in sign. The general L.P. model is usually defined as follows:

Objective function

Maximize or Minimize Z = $\boldsymbol{C_1x_1 + C_2x_2 + \cdots + C_nx_n}$

Subject to restriction:

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n \quad (\le, =, \ge)\ b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n \quad (\le, =, \ge)\ b_2$$

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$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots + a_{mn}x_n \quad (\le, =, \ge)\ b_m$$

Non- negative variables

$$x_1, x_2, \ldots\ldots, x_n \ge 0$$

Example1:

*A company for pieces of car manufacturing has **2 types** of pieces are (**A**, **B**). The production process is going through **3 stages** in order to make pieces of car and each stage has restricted time availability, they are mentioned in the following table:*

Stages of production	Type Piece of car		Available time (hours)
	A	B	
I	8	6	220
II	4	9	280
III	1	2	40
Profit $	120$	80$	

Construct a L. P. model for this problem?

Example 2:

Factory produces two types of tires, the tire made from mixing two types of elastic which is A and B, if each of elastic (rubber) contain four basic components M1, M2, M3, M4, and different quantities which is illustrated by the following table with the required elastic composition of the mixture.

Basic Components	Types of Elastic		Required Components
	Elastic A	Elastic B	
M1	**0**	**0.45**	**1**
M2	**0.5**	**0.3**	**3**
M3	**0.35**	**0**	**1**
M4	**0.15**	**0.2**	**1.5**
Cost of one unit	**32 $**	**24 $**	

H.W:

A company produces two types the company worked for 40 hours work only. The types are processed successively on two machines, the time requirements per unit of the types are given below.

Types / ***Machines***	***A***	***B***
M1	***0.6***	***0.3***
M2	***0.2***	***0.4***

The per unit profit of the first type is 10$ and the second type is 12$. Formulate a L. P. model for this problem?