Example3: *Find the optimal solution of the following problem by Simplex method.*

Max Z= $4x_1 + 3x_2$

Subject to:

$$x_1 + x_2 \leq 40$$

$$2x_1 + x_2 \leq 60$$

$$x_1, x_2 \geq 0$$

Solution: *transfer the general L.P. model to Standard form.*

general L.P. model → ***Standard form***

Max Z= $4x_1 + 3x_2$ → *Max Z* $-4x_1 - 3x_2 = 0$

Subject to:

$x_1 + x_2 \leq 40$ → $x_1 + x_2 + s_1 = 40$

$2x_1 + x_2 \leq 60$ → $2x_1 + x_2 + s_2 = 60$

$x_1, x_2 \geq 0$ → $x_1, x_2, s_1, s_2 \geq 0$

Initial Table

entering Variable

leaving Variable

B.V	X_1	X_2	S_1	S_2	Solution	Ratio
Z	-4	-3	0	0	0	----
S_1	1	1	1	0	40	(40/1)=40
S_2	2	1	0	1	60	(60/2)=30

$$S_2 \; [\; 2 \quad 1 \quad 0 \quad 1 \quad 60 \;] \; \div 2$$ (Old equation)

$$S_2 \; [\; 2/2 \quad 1/2 \quad 0/2 \quad 1/2 \quad 60/2 \;]$$

$$X_1 \; [\; 1 \quad 1/2 \quad 0 \quad 1/2 \quad 30 \;]$$ (New equation)

Z	-4	-3	0	0	0	(Old equation)
X1	1	1/2	0	1/2	30	* (-4)

Z	-4	-3	0	0	0	
X1	±4	±2	0	± 2	±120	* (-1)

Z	0	-1	0	2	120	(New equation)

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S1	1	1	1	0	40	(Old equation)
X1	1	1/2	0	1/2	30	* (1)

S1	1	1	1	0	40	
X1	∓ 1	∓1/ 2	0	∓ 1/2	∓30	* (-1)

S1	0	1/2	1	-1/2	10	(New equation)

First Table

entering Variable (X_2)

leaving Variable (S_1)

B.V	X_1	X_2	S_1	S_2	Solution	Ratio
Z	0	-1	0	2	120	----
S_1	0	1/2	1	-1/2	10	[10/(1/2)]=20
X_1	1	1/2	0	1/2	30	[30/(1/2)]=60

S_1 [0   1/2   1   -1/2   10] ÷ 1/2 (Old equation)

S_2 [0/(1/2)   (1/2)/(1/2)   1/(1/2)   (-1/2)/(1/2)   10(1/2)]

X_2 [0   1   2   -1   20] (New equation)

Z	0	-1	0	2	120	(Old equation)
X_2	0	1	2	-1	20	* (-1)
Z	0	-1	0	2	120	
X_2	0	± 1	± 2	∓ 1	± 20	* (-1)
Z	0	0	2	1	140	(New equation)

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X_1	1	1/2	0	1/2	30	(Old equation)
X_2	0	1	2	-1	20	* (1/2)
X_1	1	1/2	0	1/2	30	
X_2	0	$\mp 1/2$	∓ 1	$\pm 1/2$	∓ 10	* (-1)
X_1	1	0	-1	1	20	(New equation)

Second Table

B.V	X_1	X_2	S_1	S_2	Solution
Z	0	0	2	1	140
X2	0	1	2	-1	20
X_1	1	0	-1	1	20

Max Z= $4x_1 + 3x_2$ *(140) =* $4(20) + 3(20)$

$\therefore$ The optimal solution is (140) in the point (20 , 20)