Relation Ship Between Primal and Dual model

Example(1):- let we have the following linear programming below.

$$\textit{Min } Z = x_1 - 3x_2 - 2x_3$$

Subject to:

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$-2x_1 + 4x_2 \leq 12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 18$$

$$x_1, x_2, x_3 \geq 0$$

And the optimal solution in the table below.

B.V	x_1	x_2	x_3	S_1	S_2	S_3	Solution
Z	0	0	0	-0.36	-0.72	-0.16	-14.04
x_1	1	0	0	0.32	0.14	-0.08	2.48
x_2	0	1	0	0.16	0.32	-0.04	4.24
x_3	0	0	1	0.10	-0.05	0.10	1.90

Find the value for dual decision variable.

Solution:

$$W = Z = -14.04$$

$$s_1 = y_1 = -0.36$$

$$s_2 = y_2 = -0.72$$

$$s_3 = y_3 = -0.16$$

Example(2):- let we have the following linear programming below.

$$Max\ Z = 2x_1 + x_2 - 5x_3$$

Subject to:

$$-x_2 + 3x_3 \leq 10$$

$$5x_1 + 2x_2 \leq 11$$

$$x_1 + 8x_2 + x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

And the optimal solution in the table below.

B.V	x_1	x_2	x_3	S_1	S_2	S_3	Solution
Z	0	0	5.03	0	0.39	0.03	4.61
S_1	0	0	3.13	1	-0.03	0.13	11.03
x_1	1	0	-0.05	0	0.21	-0.05	1.79
x_2	0	1	0.13	0	-0.03	0.13	1.03

Find the value for dual decision variable.

Solution:

$$W = Z = 4.61$$

$$s_1 = y_1 = 0$$

$$s_2 = y_2 = 0.39$$

$$s_3 = y_3 = 0.03$$

Example(3):- let we have the following linear programming below.

$$\textit{Min } Z = 3x_1 + 2.5x_2$$

Subject to:

$$2x_1 + 4x_2 \geq 40$$

$$3x_1 + 2x_2 \geq 50$$

$$x_1, x_2 \geq 0$$

And the optimal solution in the table below.

B.V	x_1	x_2	S_1	S_2	R_1	R_2	Solution
Z	0	0	(-3-M)/2	(-7/8)-M	(3/8)-(M/2)	(7/8)-2M	51.25
x_2	0	1	-3/8	3/8	-1/4	1/4	2.5
x_1	1	0	1/4	-1/4	1/2	-1/2	15

Find the value for dual decision variable.

Solution:

$$W = Z = 51.25$$

$$R_1 = y_1 = \frac{3}{8}$$

$$R_2 = y_2 = 7/8$$

Example(4):- let we have the following linear programming below.

$$Max\ Z=5x_1 + 2x_2 + 3x_3$$

Subject to:

$$x_1 + 5x_2 + 2x_3 = 30$$

$$x_1 - 5x_2 - 6x_3 \leq 40$$

$$x_1, x_2, x_3 \geq 0$$

And the optimal solution in the table below.

B.V	x_1	x_2	x_3	R_1	S_1	Solution
Z	0	23	7	5+M	0	150
x_1	1	5	2	1	0	30
S_1	0	-10	-8	1	1	10

Find the value for dual decision variable

Solution:

$$W = Z = 150$$
$$R_1 = y_1 = 5$$
$$S_1 = y_2 = 0$$