

Assignment Problem and Hungarian Method

Assignment Problem

Assignment problem:-

is a special kind of transportation problem.

Types of assignment problem

1- Balanced assignment problem

It means no. of line covered zero equal no. of row or column.

2- Unbalanced assignment problem

It means no. of line covered zero not equal no. of row or column.

Hungarian Method

➤ Phase1:- Row and Column reductions:

Step 1: Subtract the minimum value of each row from the entries of that row.

Step 2: Subtract the minimum value of each column from the entries of that column.

➤ Phase2:- Optimization of the problem:

Step 1: Draw a minimum no. of lines to cover all the zeros of the matrix.

❖ Procedurs:-

a) Row Scanning:

i- Starting from the first row, is there exactly one zero in that row mark a square around that zero and draw a vertical line passing through that zero otherwise skip that row.

ii- After scanning the last row, check all the zeros are covered with lines. If yes, go to step 2; otherwise do column scanning.

a) Column Scanning:

i- Starting from the first column, is there exactly one zero in that row mark a square around that zero and draw a horizontal line passing through that zero otherwise skip that column.

ii- After scanning the last column, check all the zeros are covered with lines.

Step 2: Check the no. of square marked is equal to the no. of rows or column of the matrix. If yes, go to step 5; otherwise, go to the step 3.

Step 3: Identify the minimum value of the undeleted cell values.

- a) Add the minimum undeleted cell value at the enter section points of the present matrix.
- b) Subtract the minimum undeleted cell value from all the undeleted cell values.
- c) all other entries remain same.

Step 4: Go to **step 1**.

Step 5: Treat the solution as marked by the squares as the optimal solution.

Example 1 : Solve the following assignment problem using Hungarian method. The matrix entries represent the processing times in hours. Determine the optimal assignment and compute the minimum cost.

| | | Operates | | |
|------|---|----------|----|----|
| | | 1 | 2 | 3 |
| Jobs | A | 11 | 4 | 6 |
| | B | 10 | 8 | 11 |
| | C | 9 | 12 | 7 |

Solution



| Minimum value in Row | |
|----------------------|--|
| (4) | |
| (8) | |
| (7) | |

| | | Operators | | |
|------|---|-----------|---|---|
| | | 1 | 2 | 3 |
| Jobs | A | 7 | 0 | 2 |
| | B | 2 | 0 | 3 |
| | C | 2 | 5 | 0 |



| | | Operators | | |
|------|---|-----------|---|---|
| | | 1 | 2 | 3 |
| Jobs | A | 5 | 0 | 2 |
| | B | 0 | 0 | 3 |
| | C | 0 | 5 | 0 |

| Minimum value in column | |
|-------------------------|-----|
| (2) | (0) |
| (0) | (0) |



Then the no. Of line equal no. of row and column (3=3) it is balance assignment

Optimal assignment

| | <u>operators</u> |
|-------------|------------------|
| <u>Jobs</u> | A = 2 |
| | B = 1 |
| | C = 3 |

$$\text{min cost} = 4 + 10 + 7 = 21$$

Example 2 : A shop has three machinists to be assigned to three machines, the hourly cost of having each machine operated by each machinists is as follows, using Hungarian method to determine the optimal assignment and compute the minimum cost.

Solution

| Machines | Machinists | | |
|----------|------------|----|----|
| | 1 | 2 | 3 |
| A | 10 | 20 | 30 |
| B | 15 | 40 | 40 |
| C | 25 | 45 | 65 |



| Minimum value in Row |
|----------------------|
| (10) |
| (15) |
| (25) |

| Machines | Machinists | | |
|----------|------------|----|----|
| | 1 | 2 | 3 |
| A | 0 | 10 | 20 |
| B | 0 | 25 | 25 |
| C | 0 | 20 | 40 |

| Minimum value in column |
|-------------------------|
| (0) (10) (20) |



| Machines | Machinists | | |
|----------|--------------|----|---------------|
| | 1 | 2 | 3 |
| A | 0 | 0 | 20 |
| B | 0 | 15 | 5 |
| C | 0 | 10 | 20 |



Then the no. Of line not equal no. of row and column ($2 \neq 3$) it is unbalance assignment



Then we choose the minimum least value among undeleted value it is (5), then first step add the value to inter section value , second step subtract the minimum value from all the undeleted cell values, last thing all other value hen covered by line remain same.

| Machines | Machinists | | |
|----------|------------|----|----|
| | 1 | 2 | 3 |
| A | 0 | 0 | 0 |
| B | 0 | 15 | 5 |
| C | 0 | 10 | 20 |



| Machines | Machinists | | |
|----------|------------|----|----|
| | 1 | 2 | 3 |
| A | 5 | 0 | 0 |
| B | 0 | 10 | 0 |
| C | 0 | 5 | 15 |



Then the no. Of line equal no. of row and column (3=3) it is balance assignment

Optimal assignment

| | Machinists |
|----------|-------------------------|
| Machines | A = 2 B = 3 C = 1 |

$$\begin{aligned} \text{min cost} &= 20 + 40 + 25 \\ &= 85 \end{aligned}$$

H.W: Solve the following assignment problem using Hungarian method. The matrix entries represent the processing times in hours. Determine the optimal assignment and compute the minimum cost.

| Jobs | operator | | | | |
|------|----------|----|----|----|----|
| | 1 | 2 | 3 | 4 | 5 |
| A | 9 | 11 | 14 | 11 | 7 |
| B | 6 | 15 | 13 | 13 | 10 |
| C | 12 | 13 | 6 | 8 | 8 |
| D | 11 | 9 | 10 | 12 | 9 |
| E | 7 | 12 | 14 | 10 | 14 |