

# Chapter 5

## Game Theory

### 1. Introduction to Game Theory

Game theory is a type of decision theory in which one's choice of action is determined after taking into account all possible alternatives available to an opponent playing the same game, rather than just by the possibilities of several outcome results. Game theory does not insist on how a game should be played but tells the procedure and principles by which action should be selected. Thus it is a decision theory useful in competitive situations.

Game is defined as an activity between two or more persons according to a set of rules at the end of which each person receives some benefit or suffers loss. The set of rules defines the game. Going through the set of rules once by the participants defines a play.

### 2. Properties of a Game

- a) There are finite numbers of competitors called 'players'
- b) Each player has a finite number of possible courses of action called 'strategies'
- c) All the strategies and their effects are known to the players but player does not know which strategy is to be chosen.
- d) A game is played when each player chooses one of his strategies. The strategies are assumed to be made simultaneously with an outcome such that no player knows his opponents strategy until he decides his own strategy.
- e) The game is a combination of the strategies and in certain units which determines the gain or loss.
- f) The figures shown as the outcomes of strategies in a matrix form are called 'pay-off matrix'.
- g) The player playing the game always tries to choose the best course of action which results in optimal pay off called 'optimal strategy'.
- h) The expected pay off when all the players of the game follow their optimal strategies is known as 'value of the game'. The main objective of a problem of a game is to find the value of the game.
- i) The game is said to be 'fair' game if the value of the game is zero otherwise it's known as 'unfair'.

### 3. Characteristics of Game Theory

a) Competitive game: A competitive situation is called a competitive game if it has the following four properties

- There are finite number of competitors such that  $n \geq 2$ . In case  $n = 2$ , it is called a two- person game and in case  $n > 2$ , it is referred as n-person game.
- Each player has a list of finite number of possible activities.
- A play is said to occur when each player chooses one of his activities. The choices are assumed to be made simultaneously i.e. no player knows the choice of the other until he has decided on his own.
- Every combination of activities determines an outcome which results in a gain of payments to each player, provided each player is playing uncompromisingly to get as much as possible. Negative gain implies the loss of same amount.

b) Strategy: The strategy of a player is the predetermined rule by which player decides his course of action from his own list during the game.

The two types of strategy are

- Pure strategy
- Mixed strategy

Pure Strategy: If a player knows exactly what the other player is going to do, a deterministic situation is obtained and objective function is to maximize the gain. Therefore, the pure strategy is a decision rule always to select a particular course of action.

Mixed Strategy: If a player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained and objective function is to maximize the expected gain. Thus the mixed strategy is a selection among pure strategies with fixed probabilities.

c) Number of persons: A game is called 'n' person game if the number of persons playing is 'n'. The person means an individual or a group aiming at a particular objective.

Two-person, zero-sum game: A game with only two players (player A and player B) is called a 'two-person, zero-sum game', if the losses of one player are equivalent to the gains of the other so that the sum of their net gains is zero.

Two-person, zero-sum games are also called rectangular games as these are usually represented by a payoff matrix in a rectangular form.

d) Number of activities: The activities may be finite or infinite.

e) Payoff: The quantitative measure of satisfaction a person gets at the end of each play is called a payoff

f) Payoff matrix: Suppose the player A has 'm' activities and the player B has 'n' activities. Then a payoff matrix can be formed by adopting the following rules

- Row designations for each matrix are the activities available to player A

- Column designations for each matrix are the activities available to player B

- Cell entry  $V_{ij}$  is the payment to player A in A's payoff matrix when A chooses the activity i and B chooses the activity j.

- With a zero-sum, two-person game, the cell entry in the player B's payoff matrix will be negative of the corresponding cell entry  $V_{ij}$  in the player A's payoff matrix so that sum of payoff matrices for player A and player B is ultimately zero.

g) Value of the game: Value of the game is the maximum guaranteed game to player A (maximizing player) if both the players use their best strategies. It is generally denoted by 'V' and it is unique.

## 4. Classification of Games

All games are classified into

- Pure strategy games

- Mixed strategy games

Strategy: It is the pre-determined rule by which each player decides his course of action from his list available to him. How one course of action is selected out of various courses available to him is known as strategy of the game.

Types of Strategy: Generally, two types of strategy are employed

(i) Pure Strategy

(ii) Mixed Strategy

(i) Pure Strategy: It is the predetermined course of action to be employed by the player. The players knew it in advance. It is usually represented by a number with which the course of action is associated.

(ii) Mixed Strategy: In mixed strategy the player decides his course of action in accordance with some fixed probability distribution. Probability are associated with each course of action and the selection is done as per these probabilities.

In mixed strategy the opponent cannot be sure of the course of action to be taken on any particular occasion. Pure strategy games can be solved by saddle point method.

Decision of a Game. In Game theory, best strategy for each player is determined on the basis of some rule. Since both the players are expected to be rational in their approach this is known as the criteria of optimality. Each player lists the possible outcomes from his action and selects the best action to achieve his objectives. This criteria of optimality are expressed as Maximin for the maximizing player and Minimax for the minimizing player.

## 5. The Maximin-Minimax Principle

a) Maximin Criteria: The maximizing player lists his minimum gains from each strategy and selects the strategy which gives the maximum out of these minimum gains.

b) Minimax Criteria: The minimizing player lists his maximum loss from each strategy and selects the strategy which gives him the minimum loss out of these maximum losses.

Example (1): - consider a two person zero sum game involving the set of pure strategy for Maximizing Player A say A1 A2 & A3 and for player B, B1 & B2, with the following payoff Player B

		Player B	
		I	II
Player A	I	9	2
	II	8	6
	III	6	4

Solve the payoff for this matrix. (Find the Saddle point, Game value and the Optimal strategies)

Example (2): - consider a two person zero sum game involving the set of pure strategy for Maximizing Player A say A1 A2 & A3 and for player B, B1, B2 & B3, with the following payoff Player B

		Player B		
		I	II	III
Player A	I	3	1	5
	II	8	4	6
	III	-2	4	2

Solve the payoff for this matrix