زانكۆى سەلاحەدین - هەولێر
Salahaddin University-Erbil

Some Types of Tree

Research Project

Submitted to the department of (Mathematics) in partial fulfillment of the requirements for the degree of BSc.in (Graph Theory)

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April - 2023-

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Acknowledgment

- My God who gave me everything.
- My supervisor, Lecturer Ivan
- To Salahaddin University College of Education Mathematics Department.
- All the staff of the department
- The noble spirits of my Parents.
- My brothers with my love

Amal Qasem Yasen

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Abstract

In this work we study a particular type of connected graph called a tree and we illustrate some types of the trees.

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Introduction

Trees appeared implicitly in the 1847 work of the German physicist Gustav Kirchhoff in his study of currents in electrical networks, while Arthur Cayley used trees in 1857 to count certain types of chemical compounds. Trees are important to the understanding of the structure of graphs and are used to systematically visit the vertices of a graph. Trees are also widely used in computer science as a means to organize and utilize data. The simplest organic chemical molecules are the alkanes. Alkanes are hy-drocarbons and so their molecules consist only of carbon and hydrogen atoms, denoted by the symbols C and H, respectively.

This work consists two chapter

In chapter one we present some concept definition of graphs.

In chapter two we present a definition of tree and some basic properties of a tree. and present definitions of some types of tree.

Chapter One

Basic notations and Definitions

Definition1.1[1 (Maata', 2013)]A **graph** $G = (V(G), E(G))$ or $G = (V, E)$ consists of two finite sets $V(G)$ or V the vertex set of the graph ,which is a non-empty set of elements called vertices and $E(G)$ or E ,the edge set of graph, which is a possibly empty set of elements called edges, such that each edge e in E is assigned as an unordered pair of vertices (u , v) called the end vertices of e.

Definition1.2[(Lesniak, 1986)]A digraph D consists of a finite nonempty set V of objects called vertices and a set E of ordered pairs of distinct vertices. Each element of E is an arc or a directed edge.

Definition1.3[(ZHANG, 1979)**]** A digraph H is called a subdigraph of a digraph D if $V(H) \subseteq V(D)$ and $E(H) \subseteq E(D)$.

Definition1.4[(Ray, First Online :1 Janury 2012)**]** The number of vertices in G is often called the order of G, while

Definition1.5[(Lesniak, 1986)**]** An edge having the same vertex as both of its end vertices is called a self-loop (or simply a loop).

Definition1.6[(Lesniak, 1986)**]** Let H be a graph with vertex set $V(H)$ and edge set $E(H)$, and similarly let G be a graph with vertex set $V(G)$ and edge set $E(G)$.

Definition1.7[(Lesniak, 1986)**]** A graph, that has neither self-loops nor parallel edges, is called a simple graph.

Definition1.8[(ZHANG, 1979)**]**A graph with a finite number of vertices as well as finite number of edges is called a finite graph; otherwise it is an infinite graph.

Definition1.9[(Lesniak, 1986)**]** If a subgraph of a graph G has the same vertex set as G, then it is a spanning subgraph of G.

Definition1.10[(Lesniak, 1986)**]**Let u and v be two vertices of a graph G, then a $u - v$ walk in $G$is a finite alternating sequence of vertices and edges: $(u =)u_o$, e_1 , u_1 , e_2 , u_2 , ... , u_{n-1}, e_n, $u_n(= v)$ so that $e_i = u_{i-1} u_i$ is an edge of G , for $i = 1,2, \dots , n$. The vertices and edges of a $u - v$ walk need not distinct.

Definition1.11[(Lesniak, 1986)**]** We define a $u - v$ trail in a graph G to be a $u - v$ walk in which no edge is traversed more than once.

Definition1.12[]If the vertices$v_1, v_2, \dots, v_k$ of the walk w = ($u = v_0, e_1, v_1, e$ (Lesniak,1986)$_2, \dots, v_k = v$) are distinct then w is called a path.

Definition1.13[(Maata', 2013)**]** A closed path is called cycle.

Definition1.14[(Maata', 2013)**]** Two vertices u and v in a graph G are connected if G contains $u - v$ path.

Definition1.15[(Lesniak, 1986)**]** A graph G is called connected if every two of it is vertices are connected.

Definition1.16[(Maata', 2013)**]** A graph with no cycle is called acyclic graph

Definition1.17[(Lesniak, 1986)**]** A tree is acyclic connected graph.

Definition1.18[(Ray, First Online :1 Janury 2012)**]** A spanning tree of a graph G is a spanning subgraph of G that is a tree.

Definition1.19[(Lesniak, 1986)**]** The degree of a vertex V in a graph G is the number of vertices in G that are adjacent to v.

Definition1.20[(Lesniak, 1986)**]**The level number of v is the length of the unique $r-v$ path in T where T is a rooted tree with root r and v is a vertex of T.

Definition1.22[(Lesniak, 1986)**]** The maximum of the level numbers of vertices of T is called the height of T and denoted by $h(t)$.

Definition1.23[(Lesniak, 1986)**]** Let T is a rooted tree with root r.

For any vertex $\neq r$, the father of $v$is that unique vertex u that is adjacent to v, conversely v is the son of um two vertices having the same father are brothers.

Definition1.24[(Lesniak, 1986)**]**Vertices of a rooted tree having no sons are called leaves.

Definition1.25[(Abdulla, 21 september 2017)**]** Let G_1 and G_2 be two disjoint graphs of orders m and n respectively , let f′=$u_1v_1 \in E(G_1)$ and g′ =$u_2v_2 \in$ E(G_2) an edge identification of graph [8] G_1 and G_2 is denoted by ($G_1 * G_2$) obtained from identifying f′ with g′ where u_1 identifying with u_2 and v_1 with v_2 to get the new edge f^* . it is clear that v($G_{1*}G_2) = v(G_1) + v(G_2) - 2 and\ (G_1 * G_2) = E(G_1) + E(G_2) - 1$.

Chapter Two

Some types of tree

1.1. Tree

Tree is the acycle connected graph. Trees are important to the structural understanding of graphs and to the algorithmic of information processing, and they play a central role in the design and analysis of connected networks. In fact, trees are the backbone of optimally connected networks.

1.2 The Basic Properties of a Tree T on n vertices[(Lesniak, 1986)]

1. T is connected.
2. T contains no cycles.
3. Given any two vertices u and v of T, there is a unique $u - v$ path.
4. Every edge in T is a cut-edge.
5. T contains $n - 1$ edges.
6. T contains at least two vertices of degree 1 if $n \geq 2$.
7. Adding an edge between two vertices of T yields a graph with exactly one cycle.

2. Some type of tree

In this section we illustrate and present some types of tree.

2.1. A directed tree[(Lesniak, 1986)]

A directed tree is a symmetric digraph whose underlying graph is a tree.

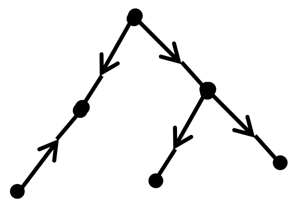

Figure 2.1 T_1 is A directed tree

2.2. Rooted tree[(Lesniak, 1986)]

A rooted tree is a direct tree T with the vertex r (called the root) such that T contains an $r - v$ path for every vertex v of T.

Thus a rooted tree with root r contain no $v - r$ path for each vertex $v \neq r$. Furthermore $id\ r = 0$ and $id\ v = 1$ for all $v \neq r$. Figure 2.2 shows a directed tree T_1 that is not rooted tree and a rooted tree T_2 with root r.

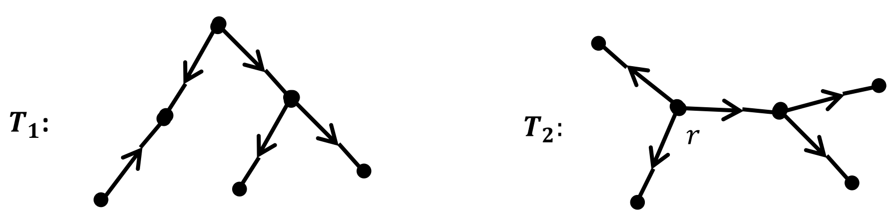

Figure 2.2 T_1 is A directed tree that is not rooted tree and a rooted tree T_2

2.3. n-ary tree and Complete n-ary tree [(abdullah, June 2007)]

In most applications of rooted tree, there is a limit as to how many sons a vertex can have If every vertex of T has n or fewer sons then T is called an n-ary tree . If every vertex of T has ether n or sons then T is called complete n-ary tree. If $n =$ 2 then T is called complete 2 -ary tree**.** Figure 2.3 shows the rooted tree T_1 is a complete 3-ary tree while T_2 is a 3-ary tree is not a complete 3-ary tree and T_3 is complete 2 -ary tree.

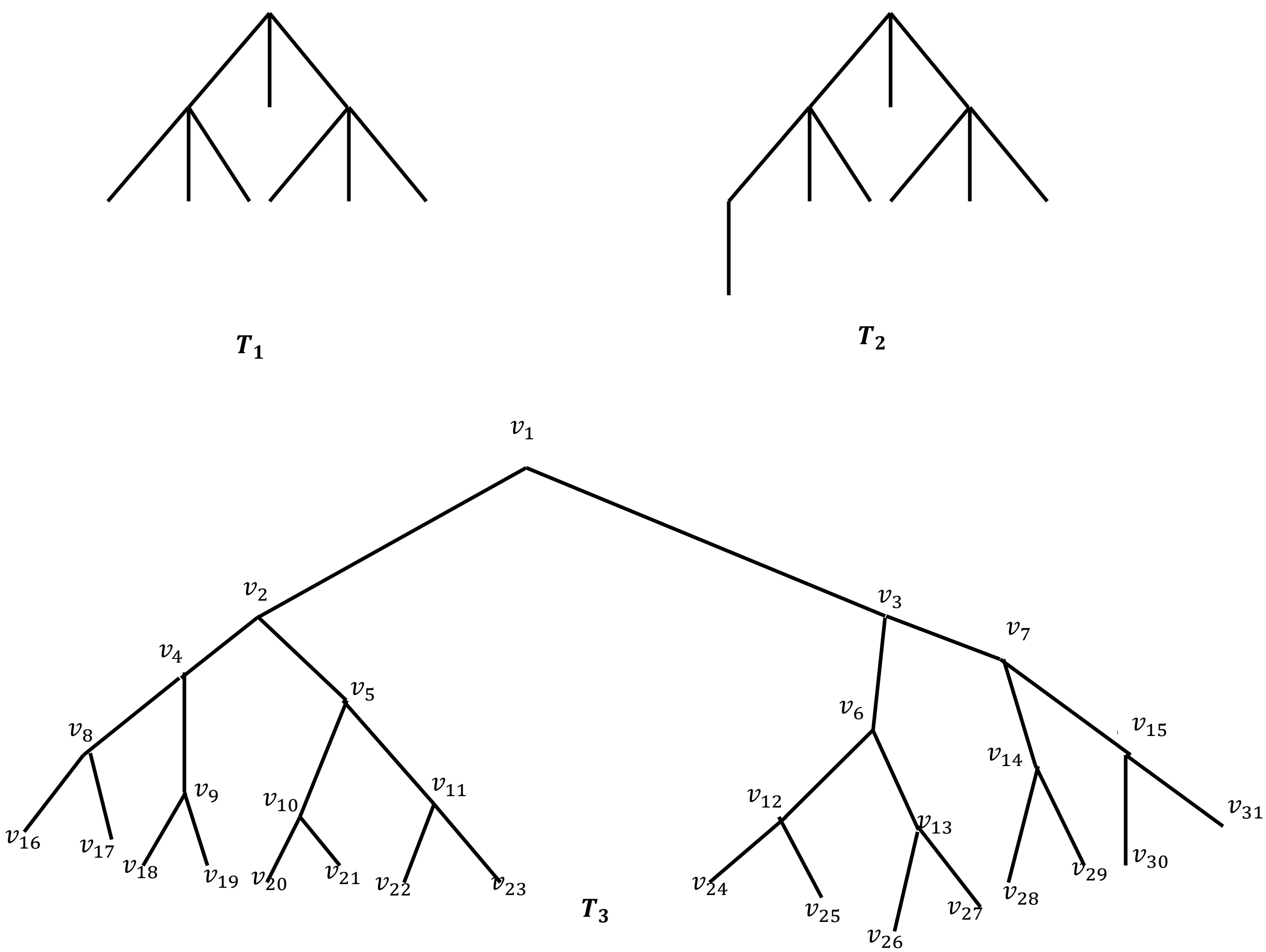

Figure 2.3

2.4. Star tree[(ZHANG G. C., 1979)**]**

Astar tree is a tre which consistof a single internal vertex (and n_1leaves.

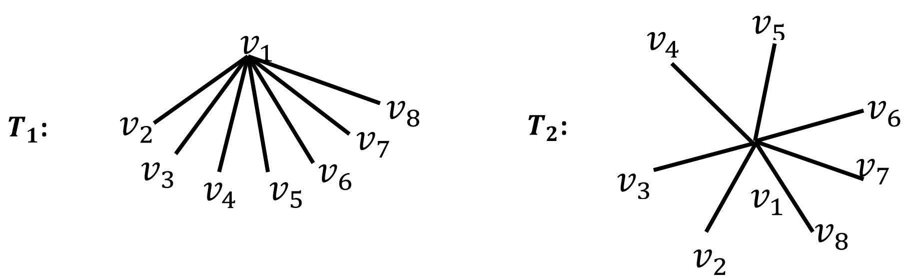

Figure 2.4 T_1 and T_2 are Star Tree

2.5. Double star[(Lesniak, 1986)**]**

A double star $D_{t.s}$ is a graph obtained from an edge $w_o u_0$ by attaching (t-1) terminal vertex to the vertex $w_o$and $(s-1)$ terminal vertices to the vertex $u_o$as shown in Figure 2.5.

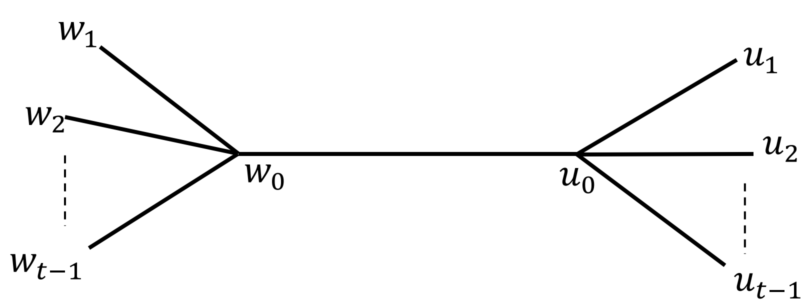

Figure 2.5 Double Star Tree

2.6. A Caterpillar tree[(Lesniak, 1986)]

In a graph theory a caterpillar or caterpillar tree is a tree in which all the vertex are with in distance 1 of a central path .

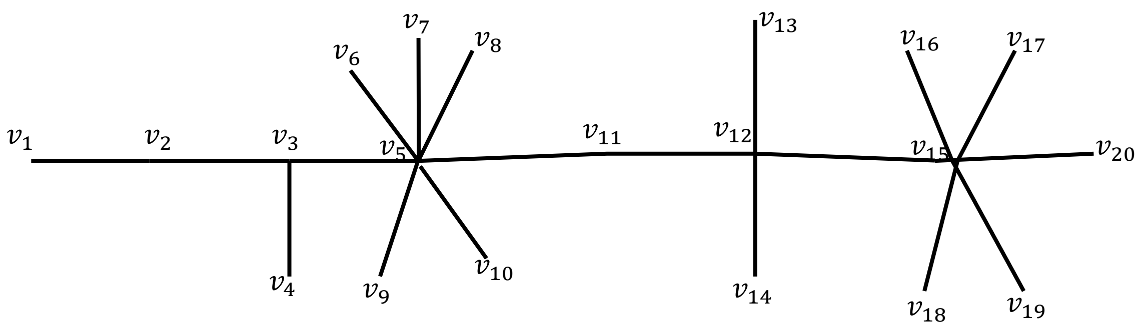

Figure 2.6 Caterpillar tree

2.7. Thorn star[(abdullah, June 2007)]

A thorn star $\boldsymbol{S_{t,k}}$ are graphs obtained from a star S_k by attaching $(t-1)$terminal vertex to each of the end-vertex of $S_k$as shown in Figure 2.7.

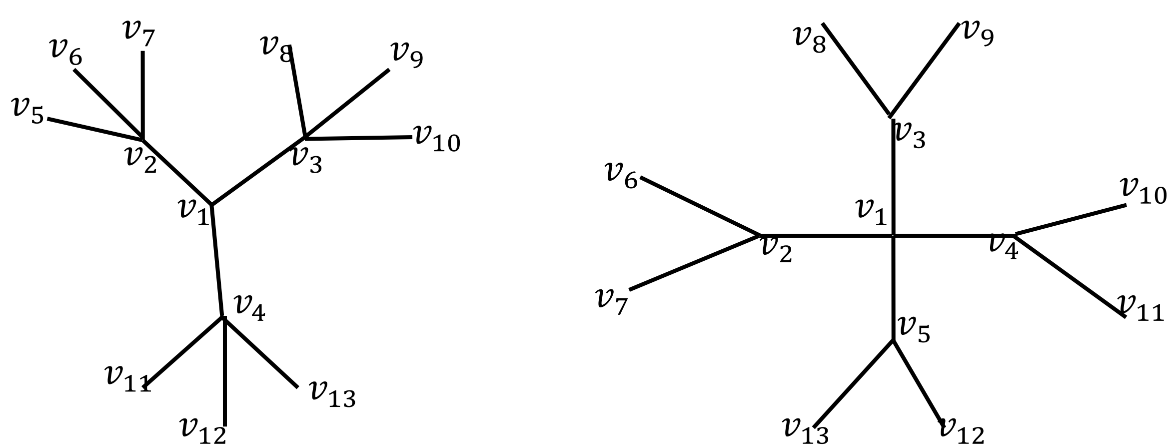

Figure 2.7. thorn star $\boldsymbol{S_{t,k}}$

2.8. Banana trees[(abdullah, June 2007)**]**

A banana tree $B_{t,r}$ consists of a star S_{t+1} each of whose t end-vertices are identified with one of the end- vertices of another star S_{t+1} , as shown in Figure 2.8.

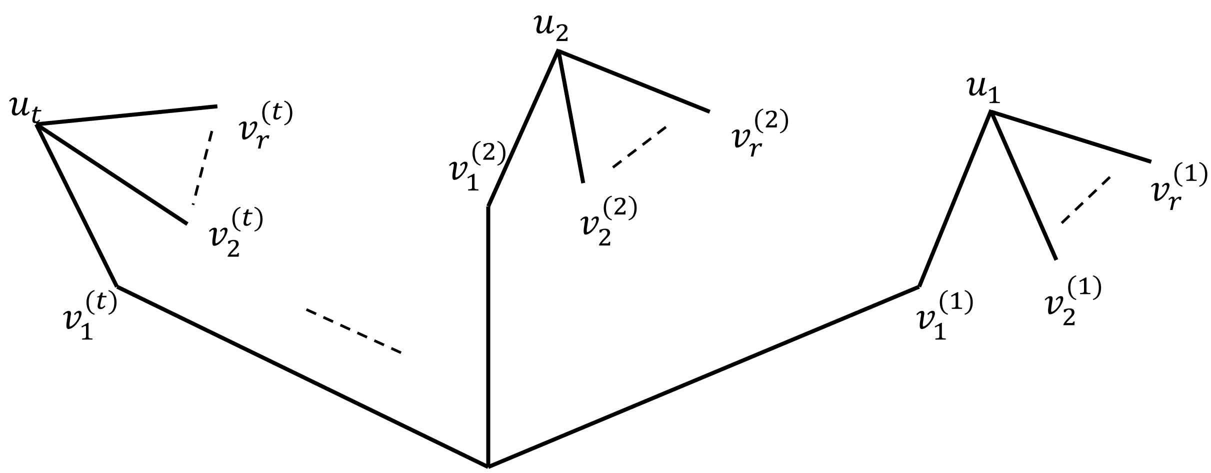

Figure 2.8. **Banana trees**

References

1 (Ray, First Online :1 Janury 2012) Graph Theory With Algoritms and its Application india

2 (ZHANG G. C., 2016) Graph and Digraphs sixth adition .

3 (Abdulla, 21 september 2017) Restricted detour polynomial of edge-identification of two wheel graph.

4 (abdullah, June 2007) Hosoya polynomial of steiner Distance of some Graphs

5 (Lesniak, 1986) GRAPHS &DIGRAPHS second edition

پوختە

لەم کاردا جۆرێکی دیاری کراو لە هێلکاری بەستراو دەخوێنین کە پێی دەوترێت دار وە هەندیك جۆر گرافی دارەکان نیشان دەدەین لەگەڵ پێشکەش کردنی چەند نموونەیەك.

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هەندیك جۆر لە دار

نامەیەكە

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