40 Questions in Calculus

Q1/ Find the solution set of $\sqrt{(x^2-5x+6)^2} = x^2-5x+6$.

Q2/ Prove or disprove that: If $x, y \in R$ then $|xy| = |x||y|$.

Q3/ Find $\lim_{x\to a} \frac{x^n-a^n}{x^m-a^m}$.

Q4/ If $\sqrt{5-2x^2} \leq f(x) \leq \sqrt{5-x^2}$ for $-1 \leq x \leq 1$, find $\lim_{x\to 0} f(x)$.

Q5/ State and prove Mean value theorem.

Q6/ If $\varphi(x) = x^{cosx}$, then find $\frac{d\varphi}{dx}$.

Q7/ If b, c and d are constants, for what value of b will the curve $y = x^3 + bx^2 + cx + d$ have the point of inflection at $x = 1$? Give reasons for your answer.

Q8/ Define the natural logarithm function. What is its domain and rang. Find its derivative and sketch its graph.

Q9/ Suppose that $f(x) = x^2$ and $g(x) = |x|$. Then the composites $(f \circ g)_{(x)} = |x|^2 = x^2$ and $(g \circ f)_{(x)} = |x^2| = x^2$ are both differentiable at $x = 0$ even though g itself is not differentiable at $x = 0$. Dose this contradict the Chain Rule? Explain.

Q10/ Prove or disprove that: If f has a derivative at $x = c$, then f is continuous at $x = c$.

Q11/ Prove or disprove that: If f is continuous at $x = c$, then f has a derivative at $x = c$.

Q12/ Suppose that f is an even function and g is an odd function defines on the entire real line. Which of the following are even? Odd? (a) $(f \circ g)_{(x)}$ (b) $(g \cdot f)_{(x)}$

Q13/ Prove that $\lim_{x\to 0^+} \sqrt{x} = 0$.

Q14/ Prove or disprove that: If $x \in R$ then $|x|^2 = x^2$.

Q15/ Find the solution set to $|x - 3| = x + 2$.

Q16/ Find the solution set to $\sqrt{(3x-2)^2} = 2 - 3x$

Q17/ Prove or disprove that: Let $h = fg$ be product of two differentiable function of x. If the graphs of f and g have inflection point at $x = a$, then the graph of h have an inflection point at a.

Q18/ Define one to one function. Let $\tau(x) = secx$ for all x in the domain of τ, is τ one to one function? Explain your answer, then sketch the function of both $\tau(x)$ and $\tau^{-1}(x)$.

Q19/ If $f(x) = 5x - 3$, $x_0 = 1$ and $L = 2$ prove that $\lim_{x\to x_0} f(x) = L$.

Q20/ State and prove Sandwich theorem

Q21/Prove or disprove: If the function $f(x)$has a critical point then $f(x)$ has local minimum.

Q22/ Test the continuity of the function $f(x)$on R, when $f(x) = \begin{cases} x+3 & if\ x < 0 \\ x^2 - 1 & if\ x \geq 0 \end{cases}$

Q23/ Define derivative of a function $f(x)$ on an open interval (a, b). Prove that: if $f(x)$ is differentiable at x_0, then it's continuous at it. Dose the converse of this theorem true in general? Explain your answer by an example.

Q24/ Find limit of the following functions if exist.

1-$\lim_{x\to\infty} \sqrt{x^2 + x} - x$ 2- $\lim_{x\to 0} \frac{1 - secx}{tanx}$

3- $\lim_{x\to-\infty} e^{\frac{3x^2-1}{x^2+1}}$ 4- $\lim_{x\to 0}(3 - x^2 sin\frac{1}{x})$

Q25/ Define continuous function on interval .

Let $f(x)$ be differentiable on (a,b) and continuous on $[a,b]$, then there exists at least on number $c \in (a,b)$ such that $\acute{f}(c) = \frac{f(b)-f(a)}{b-a}$. Explain the theorem by an example. What is the special case of this theorem, state it.

Q26/ Find the solution set of the inequality $\lfloor x\rfloor^2 - 3\lfloor x\rfloor = -2$

Q27/ For what values of x is (i)$\lfloor x\rfloor = 0$? (ii)$\lceil x\rceil = 0$?

Q28/ What real numbers x satisfy the equation $\lfloor x\rfloor = \lceil x\rceil$?

Q29/ Prove that: $coth^2 x = csch^2 x + 1$.

Q30/ If an even function $f(x)$ has a local maximum value at $x = c$, can anything be said about the value of f at $x = -c$? Give reason for your answers.

Q31/ Define a continuous function. State Roll's theorem then explain it by an example.

Q32/ Find the solution set of $-|x - 5| < -7$.

Q33/ Solve $\frac{e^x - e^{-x}}{2} = 1$ for x.

Q34/ Find an equation of the tangent to the curve $y = \frac{1}{x^3}$ at a point $(-2, -\frac{1}{8})$.

Q35/ Prove that: Let h , f , g are functions when $f(x) \leq g(x) \leq h(x)$

for all x in some open interval (I) containing c, except possibly at $x = c$ it self.

Suppose also $\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L$ then $\lim_{x\to c} g(x) = L$.

Q36/ Prove that: $\lim_{x\to x_0} f(x) = L$, where $f(x) = \sqrt{x-1}$, $x_0 = 5$, $L = 2$ and $\varepsilon = 1$.

Q37 // If $f(x) = \frac{x^4}{4} - 2x^2 + 4$ then answer the following:

1. What are the critical points of f?
2. On what intervals is f increasing or decreasing?
3. At what points, if any, dose f assume local maximum and minimum values?
4. What are the inflection points of f?
5. Find where the graph of f is concave up and where it is concave down.

Q38/ Let $f(x)$and $g(x)$ be two differentiable functions. Then Prove that

$D_x[f(x) \cdot g(x)] = f(x)D_x g(x) + g(x)D_x f(x)$.

Q39/ Solve for x without using a calculator. $ln\left(\frac{1}{x}\right) + ln(2x^3) = ln\ 3$.

Q40/ Evaluate the following integrals

1.$\int \frac{e^x}{\sqrt{1+4e^{2x}}} dx$

2. $\int tan^{-1}x\ dx$

3.$\int 6^{cosx}\ sinx\ dx$

4.$\int sin^3x\ cos^{-4}x\ dx$

5.$\int p^4 e^{-p}\ dp$

6. $\int \theta^2 \sqrt{3\theta - 1}\ \ d\theta$

7. $\int \frac{x^2 dx}{(9-x^2)^{\frac{1}{2}}}$

8.$\int \frac{2dx}{x\sqrt{1-4lin^2x}}$