APPLICATION ON MOMENTUM EQUATION

LOSSES DUE TO SUDDEN EXPANSION IN PIPE

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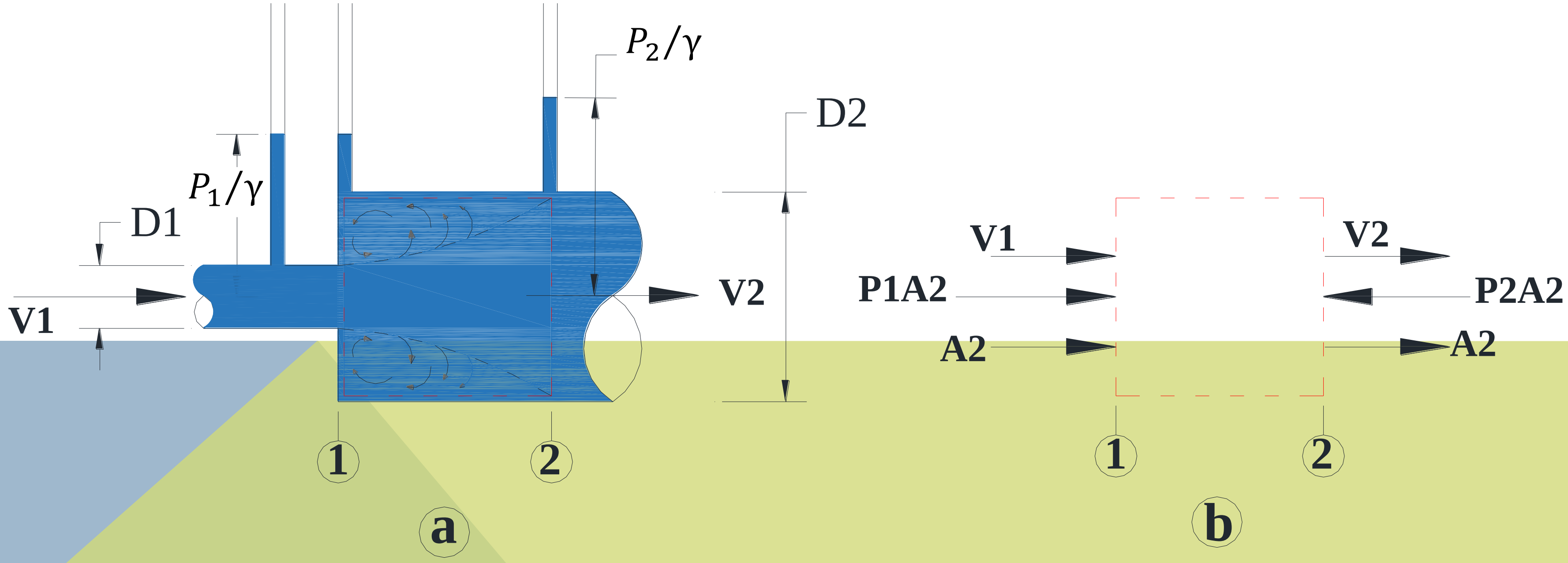
The impulse- principle may be employed to predict the fall of the energy line (that is, the energy loss due to a rise in the internal energy of the fluid caused by viscous dissipation). The losses due to sudden enlargement in a pipeline may be calculated with both the energy and momentum equations. For steady, incompressible, turbulent flow through the control volume between sections 1 and 2 of the sudden expansion of fig. below.



Applying Momentum equation between 1 & 2

$$P_1 A_2 - P_2 A_2 = \rho\, V_2\, Q\ - \rho\, V_1\, Q \qquad ; \ \rho = \frac{\gamma}{g}$$

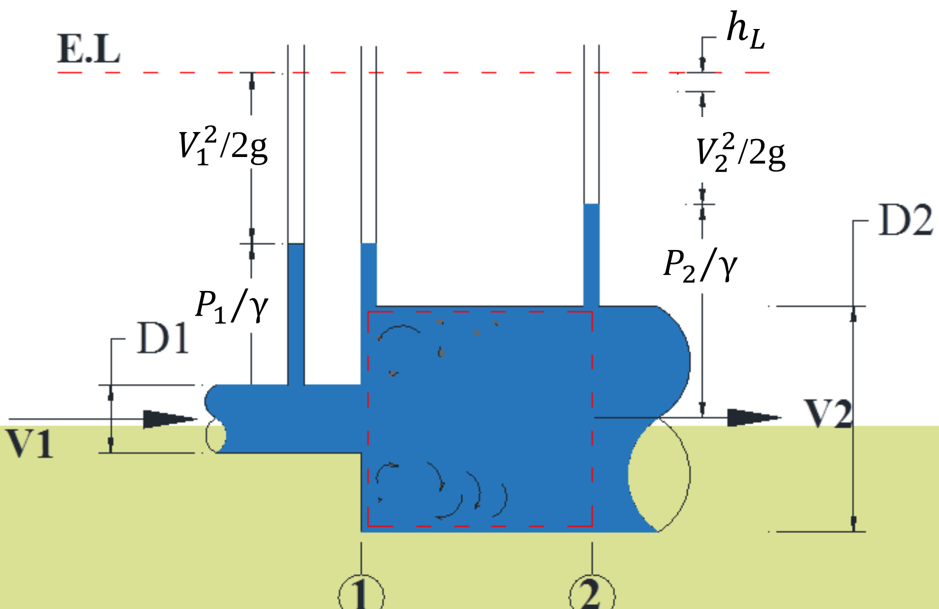
$$Q = V_1 A_1 = V_2 A_2$$

$$(P_1 - P_2) A_2 = \frac{\gamma}{g} Q (V_2 - V_1) \qquad \Rightarrow \qquad (P_1 - P_2) A_2 = \frac{\gamma}{g} V_2 A_2 (V_2 - V_1)$$

$$\frac{P_1 - P_2}{\gamma} = \frac{V_2}{g}\ (V_2 - V_1) \quad \dots\dots\dots.\ 1$$

Applying energy equation between 1 & 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$



$$\frac{P_1 - P_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} + h_L \quad \ldots\ldots\ldots. \; 2$$

Equating equation 1 & 2

$$\frac{V_2^2 - V_2 V_1}{g} = \frac{V_2^2 - V_1^2}{2g} + h_L \quad \Rightarrow \quad h_L = \frac{2(V_2^2 - V_2 V_1)}{2g} - \frac{V_2^2 - V_1^2}{2g}$$

$$h_L = \frac{1}{2g}[2V_2^2 - 2V_2 V_1 - V_2^2 + V_1^2] \quad \Rightarrow \quad h_L = \frac{1}{2g}[V_2^2 - 2V_2 V_1 + V_1^2]$$

$V_2^2 - 2V_2 V_1 + V_1^2 = (V_1 - V_2)^2$; But $V_1 A_1 = V_2 A_2 \Rightarrow V_2 = V_1 \frac{A_1}{A_2}$

$$h_L = \frac{(V_1 - V_2)^2}{2g} \quad \Rightarrow \quad h_L = \frac{(V_1 - V_1 \frac{A_1}{A_2})^2}{2g}$$

$$h_L = \frac{V_1^2 (1 - \frac{A_1}{A_2})^2}{2g}$$

Example: A pipe increases suddenly in diameter from (0.5 m) diameter to (1.0 m), if P1/γ = 1.158 m; P2/γ = 1.6 m. find the flow rate and head loss due to sudden expansion. Neglect the head loss due to friction.

Solution:- $\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$ ……. 1

$Q = V_1 A_1 = V_2 A_2$; $V_1 \frac{\pi\, 0.5^2}{4} = V_2 \frac{\pi\ 1.0^2}{4}$

$V_2 = 0.25\, V_1$ …… 2

$h_L = \frac{V_1^2 (1-\frac{A_1}{A_2})^2}{2g} = \quad h_L = \frac{V_1^2}{2g}\left[1 - \frac{\pi \frac{0.5^2}{4}}{\pi \frac{1.0^2}{4}}\right]^2$

$h_L = 0.5625\, \frac{V_1^2}{2g}$ … … . 3

Substituting Equation 2 & 3 in equ. 1

$$1.158 + \frac{V_1^2}{2g} + 0 = \frac{(0.25\, V_1)^2}{2g} + 1.6 + 0 + 0.5625\, \frac{V_1^2}{2g}$$

$$\frac{V_1^2}{2g} - 0.0625\ \frac{V_1^2}{2g} - 0.5625\ \frac{V_1^2}{2g} = 1.6 - 1.158$$

$$V_1 = 4.8\ m/sec$$

$$V_2 = 0.25\ * 4.8 = 1.2\ m/sec$$

$$Q = V_1 A_1 = 4.8\ \frac{\pi}{4} 0.5^2 = 0.942\ m^3/sec$$

From equ. 3

$$h_L = 0.5625\ \frac{V_1^2}{2g} = 0.5625\ \frac{(4.8)^2}{2g} = 0.66\ m$$