APPLICATION ON MOMENTUM EQUATION

FORCE EXERTED BY A JET

a- Impact on a flat plate

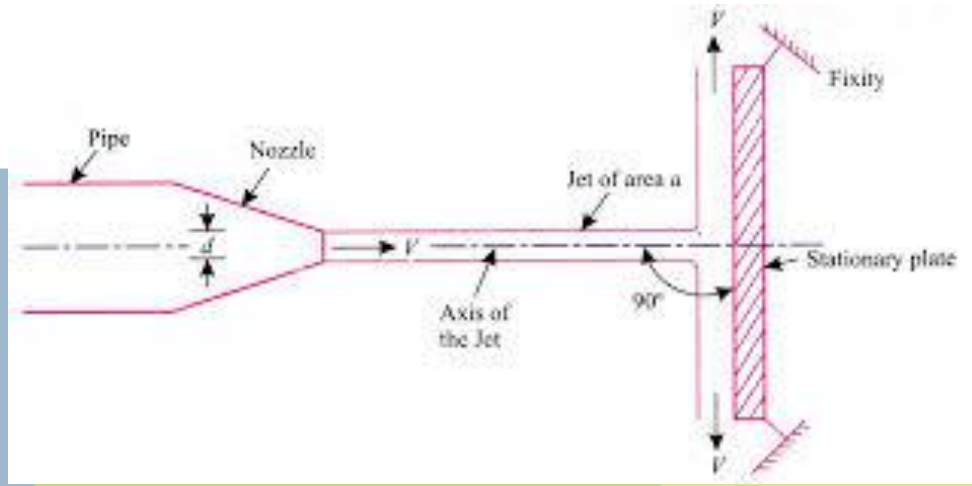
A jet of water issuing from a nozzle has a velocity and hence it possesses a kinetic energy. If this jet strikes a plate then it is said to have an impact on the plate. The jet will exert a force on the plate which it strikes. This force is called a dynamic force exerted by the jet. This force is due to the change in the momentum of the jet as a consequence of the impact. This force is equal to the rate of change of momentum, the force is equal to (mass striking the plate per second) x (change in velocity).

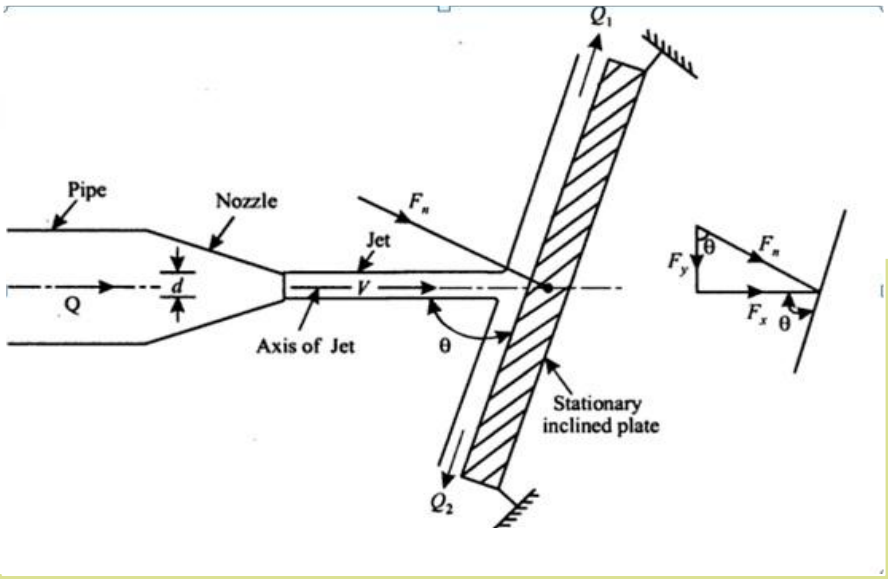
I)- Fixed flat plate:

$\sum F_X = 0$; $\rho\, Q\, \Delta V - F = 0$; $\rho\, Q\, V_1 - \rho\, Q\, V_2 - F = 0$; $V_2 = 0\ for\ fixed\ plate$

$\rho\, Q\, V_1 - F = 0\ F = \rho\, Q\, V_1$; $V_1 = V\ of\ jet\ and$; $Q = VA$; $A = area\ of\ jet$

$$\boldsymbol{F = \rho\, A\, V^2}$$





II) Moving flat plate:

$$F = \rho\, Q\, V$$

For moving plate of velocity u

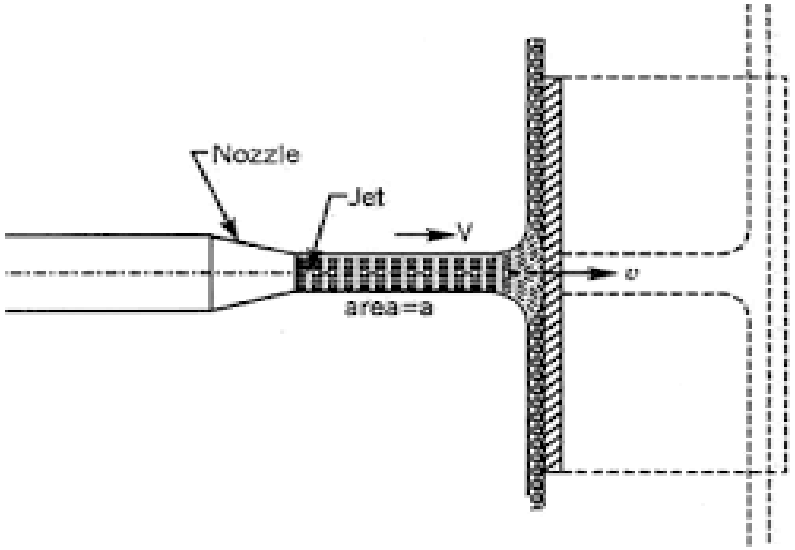
$$V = V_r = V_1 - u$$

$$\boldsymbol{F = \rho\, Q\, V_r}$$

Where: u = velocity of plate

V1 = velocity of the jet

Vr = relative velocity



b- Jet on Curved Vanes:

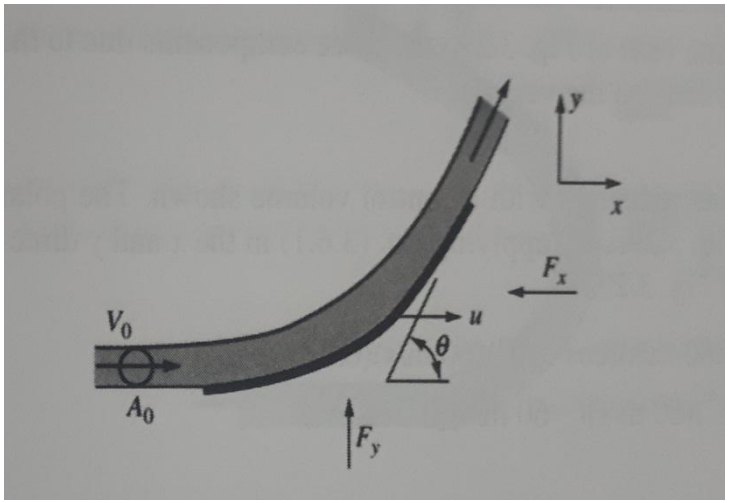
I)- Fixed curved vane:

$\sum F_X = 0$ $\quad -F_X = \rho\, Q\, V_{1X} - \rho\, Q\, V_o \quad$ $F_X = \rho\, Q\, V_o - \rho\, Q\, V_1 \cos\theta$;

$V = V_o = V_1 = velocity\ of\ jet;$

$F_X = \rho\, Q\, (V - V cos\, \theta)$;

$\boldsymbol{F_X = \rho\, Q\, V\, (1 - cos\, \theta)}$; $\quad \boldsymbol{for\ Q = VA}$;



$\boldsymbol{F_X = \rho\, A\, V^2\, (1 - cos\, \theta)}$

$\sum F_Y = 0$

$F_Y = \rho\, Q\, (V_1\, sin\theta - V_{oY})$

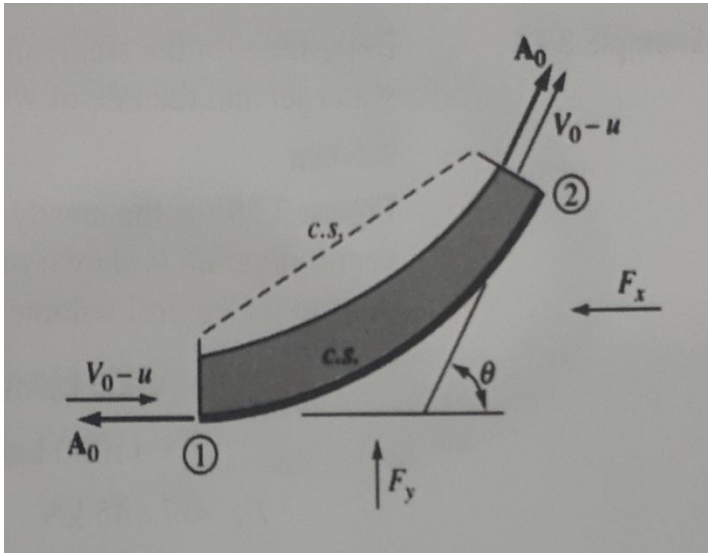
$F_Y = \rho\, Q\, V_1\, sin\theta$; $\quad Q = VA$; $\quad V = V_o = V_1$

$\boldsymbol{F_Y = \rho\, A\, V^2\, sin\theta}$

$$F = \sqrt{F_X^2 + F_Y^2} \quad ; \quad \alpha = tan^{-1}\frac{F_Y}{F_X}$$

II- For Moving Vanes:

a- For Single Moving Vanes:



$$F_X = \rho\, A\, (V - U)^2\, (1 - cos\, \theta\,) \quad ;$$

$$V_r = V - U$$

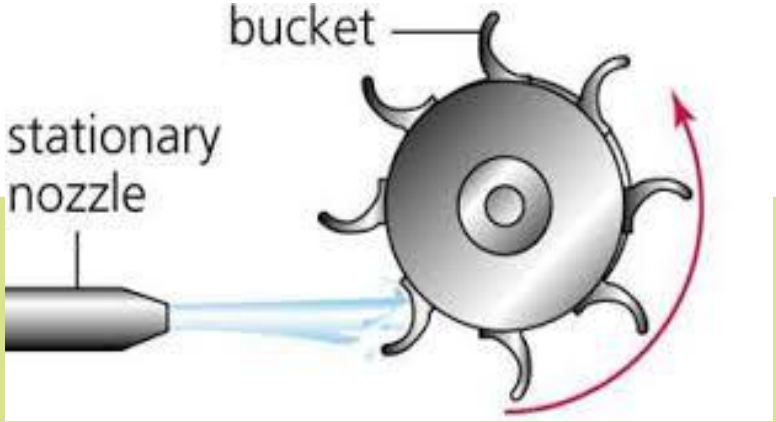
$$F_Y = \rho\, A\, (V - U)^2\, (sin\, \theta\,)$$

Where: U = is Vane velocity

V = is Jet velocity

θ = angle of Vane ; A = Area of Jet

b- For a Series of Vanes:



$$F_X = \rho\, Q\, (V - U)\, (1 - cos\, \theta\,) \quad ;$$

$$V_r = V - U$$

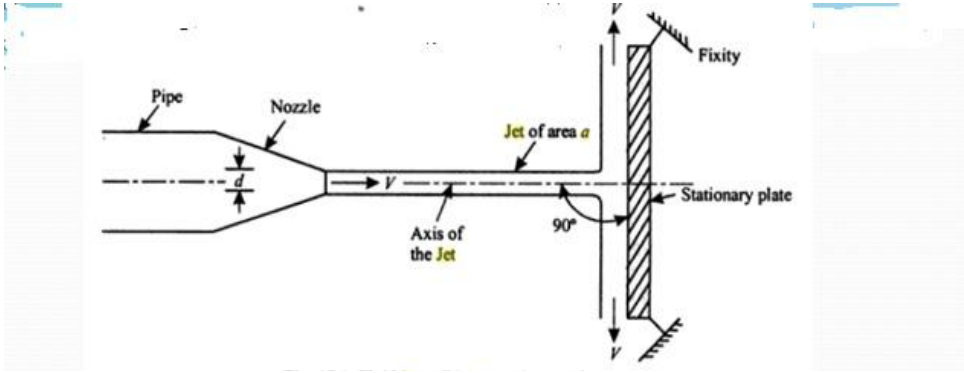
$$F_Y = \rho\, Q\, (V - U)\, (sin\, \theta\,)$$

Example : A jet of water from a nozzle of 50mm diameter impacts normally, with velocity 6.3 m/sec on a stationary fixed flat plate. Calculate the exerted by the water on the plate.

Solution:-

$$\boldsymbol{F = \rho A V^2}$$

$$\boldsymbol{F} = 1000 * \frac{\pi}{4}\left(\frac{50}{1000}\right)^2 * 6.3^2 = 77.89\ N$$



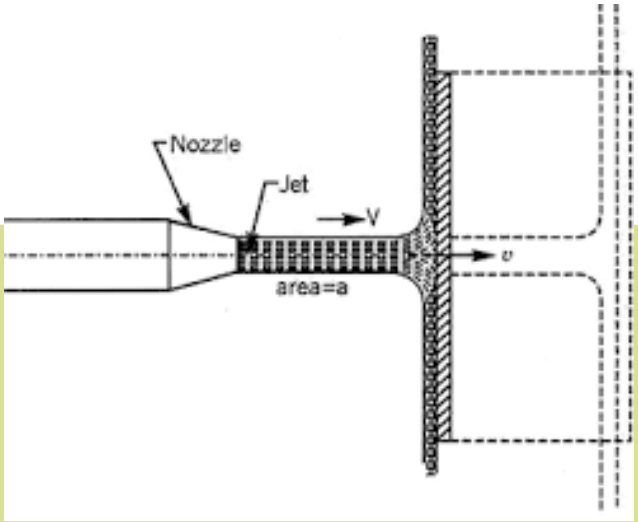
Example : A jet of water 22.5 cm diameter impacts normally on a flat plate moving at 0.6 m/sec in the same direction, if the discharge of water is 0.14 m^3/sec. find the force on plate.

Solution:- $V_r = V - u$

$$V = \frac{0.14}{\frac{\pi}{4}(0.225)^2} = 3.521\ m/sec$$

V_r =3.521 – 0.6 = 2.921 m/sec

$$\boldsymbol{F} = 1000 * 2.921 * 0.14 = 408.94\ N$$



Example:- A single vane moves with a velocity 60 m/sec, subjected to a water jet having velocity 120 m/sec and the vanes makes an angle of 120° with the horizontal, find the horizontal and vertical components of the exerted force. A= 0.001 m^2

Solution:-

$$F_X = \rho\, A\, (V-U)^2\, (1-cos\,\theta\,)$$

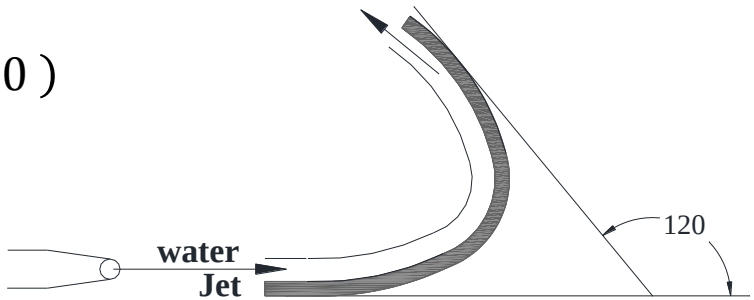
$$F_X = 1000 * 0.001(120-60)^2\, (1-cos\,120\,)$$

$$F_X = 5400\ N$$

$$F_Y = \rho\, A\, (V-U)^2\, (sin\,\theta\,)$$

$$F_Y = 1000 * 0.001(120-60)^2\, (sin\,120\,)$$

$$F_Y = 3117.7\ N$$



**** Solve the same example for fixed Vane U= 0**

$$F_X = \rho\, A\, (V-U)^2\, (1-cos\,\theta\,) \quad \Rightarrow \quad F_X = 1000 * 0.001(120-0)^2\, (1-cos\,120\,)$$

$$F_X = +\,21600\ \text{N}$$

$$F_Y = \rho\, A\, (V-U)^2\, (sin\,\theta\,) \quad \Rightarrow \quad F_Y = 1000 * 0.001(120-0)^2\, (sin\,120\,)$$

$$F_Y = 12470\ N$$