APPLICATION ON MOMENTUM EQUATION

LOSSES DUE TO HYDRAULIC JUMP

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In open channel flow, when liquid at a high velocity discharges into a zone of lower velocity, a rather abrupt rise (a standing wave) occurs in the liquid surface and is accompanied by violent turbulence, eddying, air entrainment, and surface undulations, is steady non uniform flow. In effect, the rapidly flowing liquid jet expends and converts kinetic energy in to potential energy and losses develops on the inclined surface. The surface of the jump is very rough and turbulent, the losses being greater as the jump height is greater. For small heights, the form of the jump changes to a standing wave.

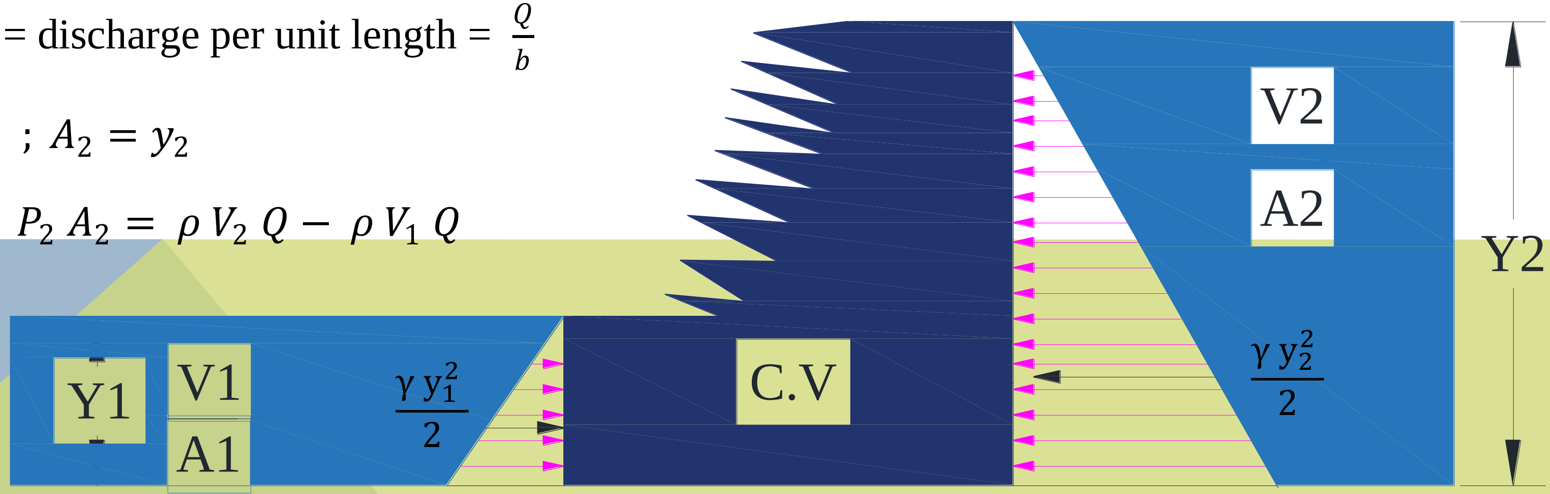
Applying Momentum equation between 1 and 2

$$\sum F_X = 0$$

q = discharge per unit length = $\frac{Q}{b}$

$$A_1 = y_1 \;;\; A_2 = y_2$$

$$P_1 A_1 - P_2 A_2 = \rho V_2 Q - \rho V_1 Q$$



$P_1 = \gamma y_1$ $P_2 = \gamma y_2$

$$\frac{\gamma\, y_1^2}{2} - \frac{\gamma\, y_2^2}{2} = \rho\, V_2\, (y_2\, V_2) - \rho\, V_1\, (y_1\, V_1)$$

$$\boldsymbol{\frac{\gamma\, y_1^2}{2} - \frac{\gamma\, y_2^2}{2} = \rho\, V_2^2\, y_2 - \rho\, V_1^2\, y_1} \quad \ldots\ldots\ldots\ldots \quad \mathbf{1}$$

$$\boldsymbol{F_1 + M_1 = F_2 + M_2}$$

$$\frac{\gamma\, \mathrm{y}_1^2}{2} + \rho\, \mathrm{V}_1^2\, \mathrm{y}_1 = \frac{\gamma\, \mathrm{y}_2^2}{2} + \rho\, \mathrm{V}_2^2\, \mathrm{y}_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2$$

F = hydrostatic force

M = Momentum per second passing the section, by writing for discharge q / unit width

Applying energy equation between 1 & 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_j$$

$$\boldsymbol{\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + h_j} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{3}$$

From Continuity equation

$$q = y_1 V_1 = y_2 V_2 \Longrightarrow \boldsymbol{V_2 = V_1 \frac{y_1}{y_2}}$$

$$\boldsymbol{F_{r1} = \frac{V_1}{\sqrt{g\, y_1}} > 1 \;;\; F_{r2} = \frac{V_2}{\sqrt{g\, y_2}} < 1 \quad \dots\dots\dots \quad 4}$$

$$V_1^2\, y_1^2 = V_2^2\, y_2^2 \quad \Longrightarrow \quad g\, y_1^3\, F_{r1}^2 = g\, y_2^3\, F_{r2}^2$$

$$\boldsymbol{y_1^3\, F_{r1}^2 = y_2^3\, F_{r2}^2 \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad 5}$$

From equation 2

$$y_1^2 \left(1 + 2\,\frac{V_1^2}{g y_1}\right) = y_2^2 \left(1 + 2\,\frac{V_2^2}{g y_2}\right)$$

Substituting equation 4 in 5

$$(\,1 + 2\,F_{r1}^2)\, F_{r1}^{-4/3} = (\,1 + 2\,F_{r2}^2)\, F_{r2}^{-4/3}$$

$$\boldsymbol{F_{r2} = \frac{2\sqrt{2\,F_{r1}}}{[\sqrt{1 + 8\,F_{r1}^2} - 1]^{3/2}}}$$

$$y_2 = \frac{-y_1}{2} + \sqrt{(\frac{y_1}{2})^2 + 2\,\frac{V_1^2\,y_1}{g}}$$

OR

$$2\,\frac{y_2}{y_1} = -1 + \sqrt{1 + 8\,\frac{V_1^2}{g\,y_1}}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8F_{r1}^2}\right]$$

Solving the energy equation for h_j

$$h_j = \frac{(\,y_2 - \,y_1\,)^3}{4\,y_1\,y_2}$$

$h_j = losses\ due\ to\ jump$

Example: water flows in a horizontal open channel at a depth of 0.6 m; the flowrate is 3.7 m^3/sec per meter width. If a hydraulic jump is possible? Calculate the depth just downstream from the jump and the power dissipated in it.

Solution:

$$q = y_1 V_1 \qquad ; \qquad V_1 = \frac{q}{y_1} = \frac{3.7}{0.6} = 6.17\ m/sec$$

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{6.17}{\sqrt{9.806 * 0.6}} = 2.544 > 1$$

$$\therefore \textit{the jump is possible.}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-\mathbf{1} + \sqrt{\mathbf{1} + \mathbf{8}\boldsymbol{F_{r1}^2}}\right] \quad \Rightarrow \quad y_2 = \frac{0.6}{2}\left[-1 + \sqrt{1 + 8 * 2.544^2}\right] = 1.88\ m$$

$$V_2 = \frac{q}{y_2} = \frac{3.7}{1.88} = 1.97\ m/sec$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_j \quad \Rightarrow \quad 0.6 + \frac{6.17^2}{2g} + 0 = 1.88 + \frac{1.97^2}{2g} + 0 + h_j$$

$$h_j = 0.4648\ m$$

$$power = \gamma\, q\, h_j = 16689.8\ watt/m$$

OR

$$\boldsymbol{h_j} = \frac{(\boldsymbol{y_2} - \boldsymbol{y_1})^3}{\mathbf{4}\, \boldsymbol{y_1}\, \boldsymbol{y_2}}$$