1.11 Optimum Number of Rain Gauge

* To get a representative/real distribution of a rainfall event over space and time in a catchment the number of rain gauges should be as large as possible. On the other hand economic considerations, topography, accessibility, maintenance etc restrict the number of gauges to be installed. An optimum density of gauges is desired from which reasonably accurate information about the storm can be obtained.

* If there are already some rain gauge stations in a catchment, the optimal number of stations that should exist to have an assigned percentage of error in the estimation of mean rainfall is obtained by following steps:

1: find mean of rainfall data

$\bar{P} = (P1+P2+P3+\cdots+Pn)/n = \sum Pi/n$

Where:

P : Rainfall in each station (mm or cm).

n: Number of station.

Step 2: find standard deviation of rainfall data.

$$\sigma = \sqrt{\frac{\sum(Pi-\bar{P})^2}{m-1}}$$

Step 3: find coefficient of vibration.

$$C_v = \frac{100\,\sigma}{\bar{P}}$$

Step 4: for a given allowable (n) admissible % error,E.

Optimum number of rain gauges (N) = $\left(\frac{Cv}{E}\right)^2$

Where:

E = % error. It is usual to take ε = 10%. If ε is small N will be large

Example 3.2

A catchment has six rain gauge stations. In year, the annual rainfall recorded by the gauges are as follows:

Station	A	B	C	D	E	F
Rainfall(cm)	82.6	102.9	180.3	110.3	98.8	136.7

For a 10% error in the estimation of the mean rainfall, calculation the optimum number of stations in the catchment.

Solution:

m = 6

$$\bar{P} = \frac{P1+P2+P3+\cdots+Pm}{m} = \frac{\sum Pi}{m} = \frac{82.6+102.9+180.3+110.3+98.8+136.7}{6} = 118.6 \text{ cm}$$

$$\sigma = \sqrt{\frac{\sum(Pi-\bar{P})^2}{m-1}}$$

where:

Pi = precipitation magnitude in the i^{th} station

$$= \sqrt{\frac{(82.6-118.6)^2+(102.9-118.6)^2+(180.3-118.6)^2+(110.3-118.6)^2+(98.8-118.6)^2+(136.7-118.6)^2}{5}} = 35.04$$

$$C_V = \frac{100\,\sigma}{\bar{P}} = \frac{100 \times 35.04}{118.6} = 29.54$$

$$N = \left(\frac{Cv}{E}\right)^2 = \left(\frac{29.54}{10}\right)^2 = 8.7 \text{ , say 9 stations}$$

The optimum number of stations for the catchment is 9. Hence three more additional station are needed.

Example 3.3

A catchment has eight rain gauge stations. In year, the annual rainfall recorded by the gauges (in cm) are 93.8, 106.5, 170.6, 138.7, 87.8, 156.2, 180.9 and 110.3. For a10% error in the estimation of the mean rainfall, find the optimum number of stations in the catchment.

Solution:

m=8

$$P^{-} = \frac{P1+P2+P3+\cdots+Pm}{m} = \frac{\sum Pi}{m} = P^{-} = \frac{93.8+106.5+170.6+138.7+87.8+156.2+180.9+110.3}{8} = 130.6 \text{ cm}$$

$$\sigma = \sqrt{\frac{\sum(Pi-P^{-})^2}{m-1}} =$$

$$\sqrt{\frac{(93.8-130.6)^2+(106.5-130.6)^2+(170.6-130.6)^2+(87.8-130.6)^2+(156.2-130.6)^2+(180.9-130.6)^2+(110.3-130.6)^2}{8-1}}$$

$= 35.91$

$$C_v = \frac{100\,\sigma}{P^{-}} = \frac{100 \ x \ 35.91}{130.6} = 27.5$$

Optimum number of rain gauges= $N = (\frac{Cv}{E})^2 = (\frac{27.5}{10})^2 = 7.56 = 8$.

Example 3.4

Station	1	2	3	4	5
Rainfall(cm)	100	120	190	95	125

Find optimum number of rain gauges? Take the error 10%.

Solution:

m=5

$$P^{-} = \frac{P1+P2+P3+\cdots+Pm}{m} = \frac{\sum Pi}{m} = P^{-} = \frac{100+120+190+138.7+95+125}{5} = 126 \text{ cm}$$

$$\sigma = \sqrt{\frac{\sum(Pi-P^{-})^2}{m-1}} = \sqrt{\frac{(100-126)^2+(120-126)^2+(190-126)^2+(95-126)^2+(125-126)^2}{5-1}} = 37.9$$

$$C_v = \frac{100\,\sigma}{P^{-}} = \frac{100 \ x \ 37.9}{126} = 30.150$$

Optimum number of rain gauges= $N = (\frac{Cv}{E})^2 = (\frac{30.150}{10})^2 = 9.09 \approx 10$

The additional number of rain gauges require N-m= 10-5= 5

Example 3.5

The average annual rainfall in cm at 4 existing rain gauge stations in a basin are 105, 79, 70 and 66. If the average depth of rainfall over basin is to be estimated within 10 % error determine the additional number of rain gauges.

Solution:

m=4

$\bar{P} = \frac{P1+P2+P3+\cdots+Pm}{m} = \frac{\sum Pi}{m} = \bar{P} = \frac{105+79+70+66}{4} = 80\text{cm}$

$\sigma = \sqrt{\frac{\sum(Pi-\bar{P})^2}{m-1}} = \sqrt{\frac{(105-80)^2+(79-80)^2+(70-80)^2+(66-80)^2}{4-1}} = 17.5$

$C_v = \frac{100\,\sigma}{\bar{P}} = \frac{100 \times 17.5}{80} = 21.9$

Optimum number of rain gauges= $N = (\frac{Cv}{E})^2 = (\frac{21.9}{10})^2 = 4.8 \approx 5$

The additional number of rain gauges require N-m= 5-4 = 1

Example 3.6

If the coefficient of variation of rainfall value at 4 rain gauge station is 30% and permissible error in the estimation of mean rainfall is 10%, then calculate the additional number of rain gauge station required in the catchment.

M=4
Cv=30%
E=10%
Optimum number of rain gauges= $N = (\frac{Cv}{E})^2 = (\frac{30}{10})^2 = 9$
Additional number of rain gauge = N-m = 9-4 = 5

Example 1.7

The isohyets due to storm in a catchment were drawn in fig below and the area of the catchment bounded by isohyets were tabulated as below.

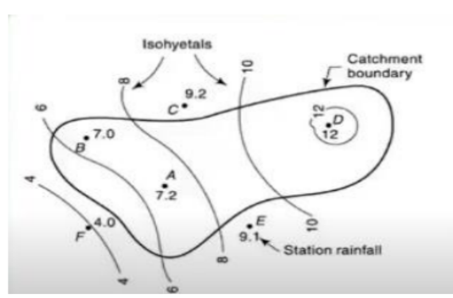
Isohyets(cm)	Area(Km^2)
Station-12	30
12-10	140
10-8	80
8-6	180
6-4	20

Estimate the mean precipitation due to the storm.

Solution:

The ioshyetal of storm is shown in fig.(4.28).

A(total)= 30+140+80+180+20= 450 Km^2

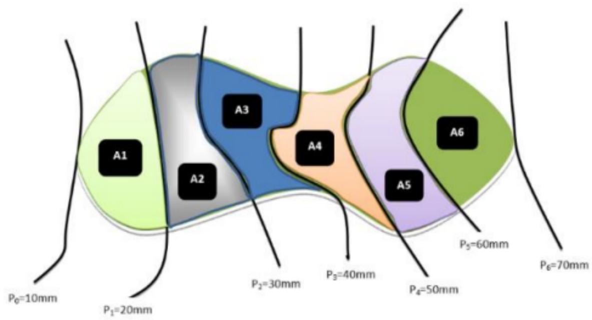


$\bar{P} = \frac{A1\left(\frac{P1+P2}{2}\right)+A2\left(\frac{P2+P3}{2}\right)+\cdots+A_{m-1}\left(\frac{P(m-1)+Pm}{2}\right)}{Total\ Area}$

$$P^{-}= \frac{20\left(\frac{12}{2}\right)+140\left(\frac{12+10}{2}\right)+80\left(\frac{10+8}{2}\right)+180\left(\frac{8+6}{2}\right)+20\left(\frac{6+4}{2}\right)}{450} = 8.84 \text{ cm}$$

Example 1.8

Use the isohyetal method to determine the average precipitation depth within the basin for the storm.



Solution:

The Isohyetal figure is represented Isohetal interval , average precipitation and area which can be used to determine (area x Precipitation) as shown in figure below:

Isohyetal interval	Average Precipitation (mm)	Area (km^2)	rea x Precipitation
< 10	10	0	0
10–20	15	84	1 260
20–30	25	75	1 875
30–40	35	68	2 380
40–50	45	60	2 700
50–60	55	55	3 025
60–70	65	86	5 590
TOTAL		428	16 830

$$P^{-}= \frac{A1\left(\frac{P1+P2}{2}\right)+A2\left(\frac{P2+P3}{2}\right)+\cdots+A_{m-1}\left(\frac{P(m-1)+Pm}{2}\right)}{Total\ Area}$$

Average Precipitation = $\frac{16830}{428}$

Average Precipitation = 39.3mm

1.12 Definitions on Rainfall: the total annual amount of rainfall at a point usual basic precipitation figure available.

1- Intensity (*i*): This is a measure of the quantity of rain falling in a given time, cm per hour, mm per hour or inch per hour.

2- Duration (t): is the period of time during which rain falls.

3- Frequency (N): This refers to the expectation that a given depth of rain fall will fall in a given time. Such an amount may be equaled or exceeded in a given number of days or years.

4- Areal extent: This concerns the area over which a points rain fall can be held to apply.

1.13 Intensity- duration relationship: the greater the intensity of rain fall, in general, the shorter length of time it continues. A formula expressing the connection would be of the type:

$$i = \frac{a}{t+b}$$

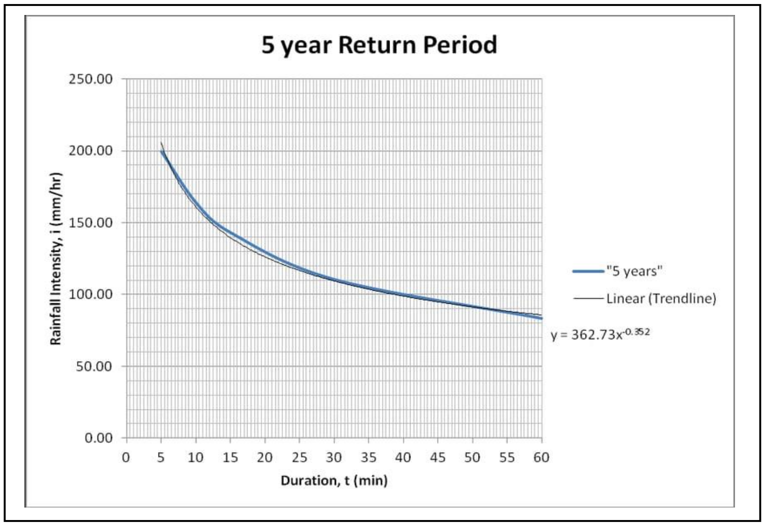
Where: i = intensity mm/hr

t = time (hr)

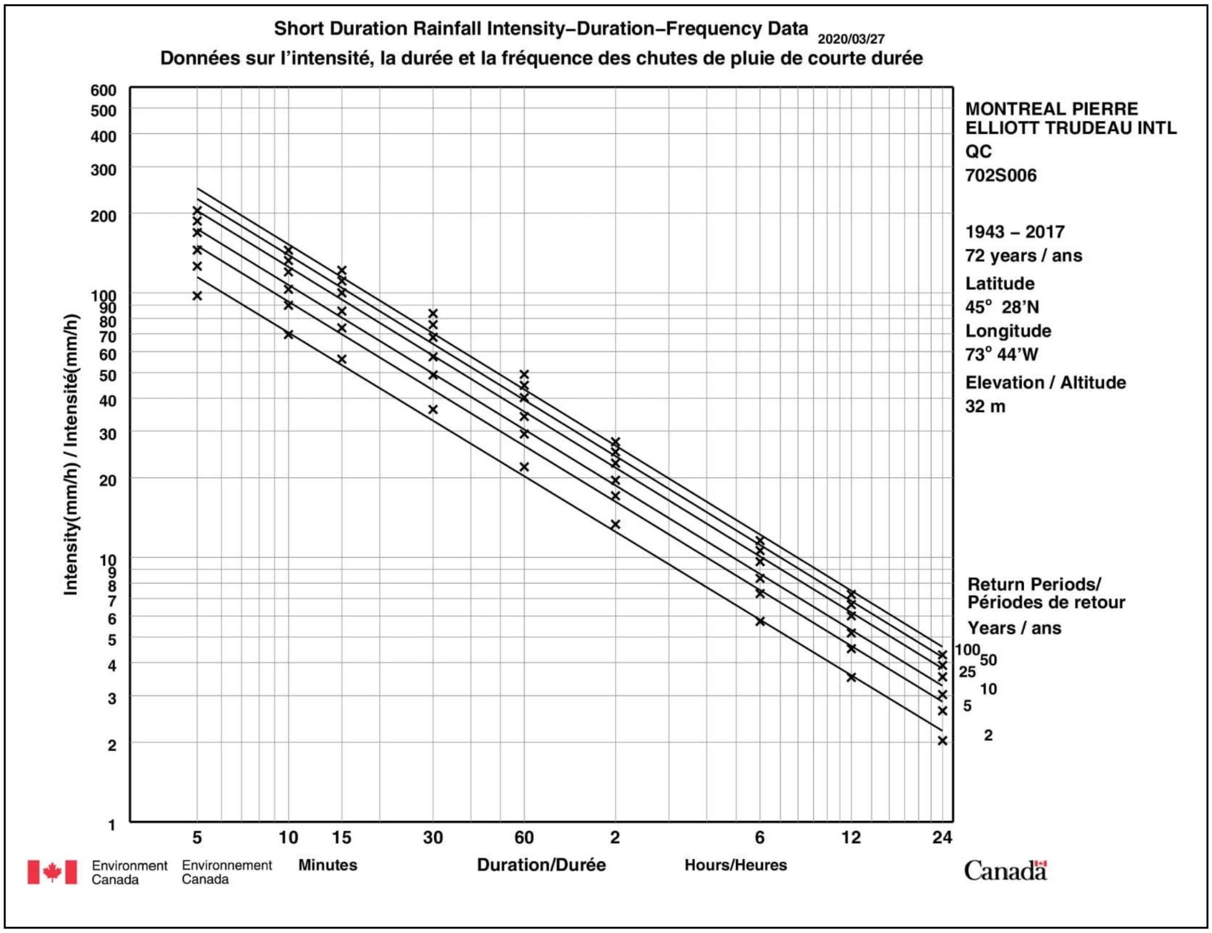
(a) and (b) are locality constants

And for duration greater than two hours

$$i = \frac{c}{t^n}$$, where (c) and (n) are locality constants.



1.14 Intensity- duration- frequency curves: for particular location it is possible to draw a series of curves, the probabilities of various intensities of rain fall occurring at that place in given periods, such curves may be plotted from records with natural or logarithmic scales for ordinate and abscissa.



$$n = 1.25\, t\ (r + 0.1)^{-3.55} \qquad \ldots\ldots\ldots 1$$

n = number of rain fall within 10 years

r = depth of rain falls (in)

t = duration (hr)

in SI unit

$$n = 1.214 * 10^5 t (P + 2.54)^{0.282} \qquad \ldots\ldots\ldots 2$$

P = depth of rain falls (mm), n , t =same unit

To determine depth of precipitation from frequency and duration rearrange the eq. 2

$$P = \left[\frac{(1.214 * 10^5)\, N\, T}{600}\right]^{0.282} - 2.54 \qquad \ldots\ldots\ldots 3$$

P = precipitation depth (mm)

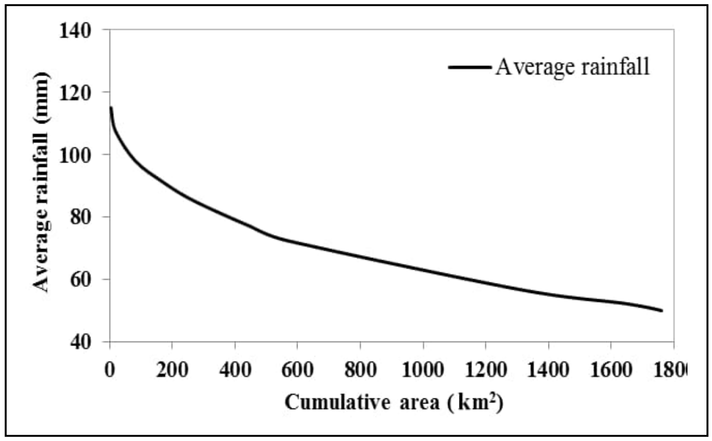
N = frequency; T = time duration (min)

By knowing P we can find i

$$i = \frac{60\,P}{T}\ ;$$

Where: i= intensity of precipitation in (mm/hr); P= depth (mm); T= duration (min)

1.15 Depth- Area- Time relationship: precipitation rarely occurs uniformly over an area. Variation in intensity and total depth of fall occurs from the center to the peripheries of storm.



Holland gave the following equation to define the relation point and real rain fall over area up to 10 Km^2 and for storm lasting from 2 to 120 minutes.

$$\frac{P^-}{P} = 1 - \frac{0.3\,\sqrt{A}}{t^*}$$

Where: P^- =average rain depth over the area.

P = point rain depth measured at center of the area.

A = area in Km^2.

t* = An inverse gamma function of storm obtained from table or figure.

t	2	6	8	10	20	40	60	100	200	300
t*	3	4	4.2	4.4	4.85	5.3	5.6	5.9	6.3	6.5

<u>**Example 1.9**</u>

Find average intensity in mm/hr over the area of 5 Km^2 with duration 60 minutes with frequency once in 10 years.

Solution:

$$P = \left[\frac{(1.214 * 10^5)\,N\,T}{600}\right]^{0.282} - 2.54$$

From the table or graph for 60 min. t* = 5.6

$$P = \left[\frac{(1.214 * 10^5)\,10 * 60}{600}\right]^{0.282} - 2.54 = 25\,mm$$

$$\frac{P^-}{P} = 1 - \frac{0.3\sqrt{A}}{t^*} \Rightarrow \frac{P^-}{25} = 1 - \frac{0.3\sqrt{5}}{5.6} \Rightarrow P^- = 22\ mm$$ average depth of rain falls over an area.

$$i = \frac{60\ P}{T} = \frac{60\ (22)}{60} = 22\ mm/hr$$ average intensity of rain falls.

Example 1.10 Determine the frequency of rain fall over a catchment area of 8 Km^2, within rain fall duration of 90 minutes, with average intensity of 30 mm/hr.

Solution: from the table for t = 90 min.; t* = 5.825

$$i = \frac{60\ P^-}{T} \Rightarrow 30 = \frac{60\ P^-}{90} \Rightarrow P^- = 45\ mm$$

$$\frac{P^-}{P} = 1 - \frac{0.3\sqrt{A}}{t^*} \Rightarrow \frac{45}{P} = 1 - \frac{0.3\sqrt{8}}{5.825} \Rightarrow P = 52.67\ mm$$

$$P = \left[\frac{(1.214 * 10^5)\ N\ T}{600}\right]^{0.282} - 2.54$$

$$52.67 = \left[\frac{(1.214 * 10^5)\ N * 90}{600}\right]^{0.282} - 2.54 \Rightarrow N = 82.615\ year\ frequncy$$

Example 1.11 the average intensity of a rain over a catchment area is 13 mm/hr with duration of 2hr with frequency of 1 in 15 years. Determine the catchment area.

Solution; find t* = 6.93 from table or graph.

$$i = \frac{60\ P^-}{T} \Rightarrow 13 = \frac{60\ P^-}{120} \Rightarrow P^- = 26\ mm$$

$$P = \left[\frac{(1.214 * 10^5)\ N\ T}{600}\right]^{0.282} - 2.54 \Rightarrow P = \left[\frac{(1.214 * 10^5)\ 15 * 120}{600}\right]^{0.282} - 2.54$$

$$P = 34.46\ mm$$

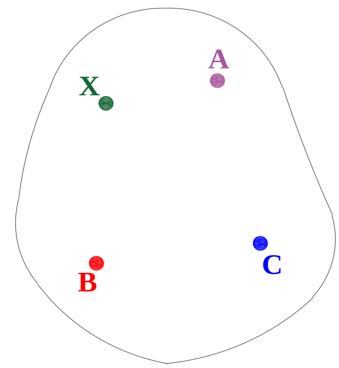
$$\frac{P^-}{P} = 1 - \frac{0.3\sqrt{A}}{t^*} \Rightarrow \frac{26}{34.46} = 1 - \frac{0.3\sqrt{A}}{6.98} \Rightarrow A = 32.63\ Km^2$$

1.16 Estimating Missing Precipitation Data:

Many precipitation station have short breaks in their records because of the absences of the observer or because of instrumental failures, in this case estimate is done for those periods, as follows.

1- U.S. Environmental data service method: for finding missing data at a station from other station.

a- use simple arithmetic average. If the normal annual precipitation at each of the index stations is within 10% of that for the station with the missing data.



% Different A-X = $\frac{P_{av.\ A} - P_{av.\ X}}{P_{av.\ X}} * 100$

% Different B-X = $\frac{P_{av.\ B} - P_{av.\ X}}{P_{av.\ X}} * 100$

% Different C-X = $\frac{P_{av.\ C} - P_{av.\ X}}{P_{av.\ X}} * 100$

If the parameters are less than 10 % then

$$P_X = (P_A + P_B + P_C + \cdots)/n$$

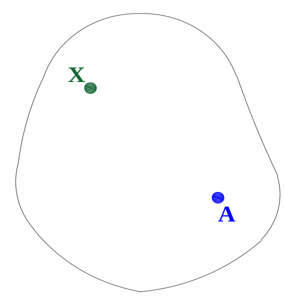
b- otherwise, use normal ratio method. Where the parameters are greater than 10% then,

$$P_X = \frac{1}{n}\left[\frac{P_{av.\ X}}{P_{av.\ A}} * P_A + \frac{P_{av.\ X}}{P_{av.\ B}} * P_B + \frac{P_{av.\ X}}{P_{av.\ C}} * P_C + \cdots\right]$$

Where; $P_{av.\ [A,B,C,X]} = average\ yarly\ precipitation\ at\ stations\ [A, B, C, \ldots, X]$

$P_{[A,B,C,X]} = depth\ of\ precipitation\ at\ stations\ [A, B, C, \ldots]$

2- If there are only two stations, and station X was in operative for a certain year, there is a gap in recording rain fall. Then,



$$P_X = \frac{P_{av.\ X}}{P_{av.\ A}} * P_A$$ this is simple proportion

<u>Example 1.12</u>: Estimate the missing precipitation data in X station, if the station reading are PA = 42mm, PB = 35mm, PC = 48 mm and average yearly precipitation are $P_{av.X}$ =385mm, $P_{av.A}$ = 441mm, $P_{av.B}$ =368mm, and $P_{av.C}$ =472mm.

<u>Solution:</u>

% Different C-X = $\frac{P_{av.\ C} - P_{av.\ X}}{P_{av.\ X}} * 100 = \frac{472 - 385}{385} * 100 = 22.5\% > 10\%$

$$\therefore P_X = \frac{1}{n}\left[\frac{P_{av.\ X}}{P_{av.\ A}} * P_A + \frac{P_{av.\ X}}{P_{av.\ B}} * P_B + \frac{P_{av.\ X}}{P_{av.\ C}} * P_C\right]$$

$$P_X = \frac{1}{3}\left[\frac{385}{441} * 42 + \frac{385}{368} * 35 + \frac{385}{472} * 48\right] = 37.5\ mm$$

<u>Example 1.13:</u> A rain over a catchment area with 120 minutes duration, the rain gages distributed according to following information find;

1- Average intensity over the catchment in mm/hr for the gap.

2- Occurrence frequency of the basin.

3- Area of the basin.

P_A=37mm, P_B = 42mm, P_C = 35 mm, and P_D = 40 mm station X is at the center of the basin. $P_{av.A}$ = 38.5 mm, $P_{av.B}$ = 44.1 mm, $P_{av.C}$ = 36.8 mm, $P_{av.D}$ = 41.7 mm, and $P_{av.\ X}$ = 47.2 mm.

Solution:

% Different C-X = $\left|\frac{P_{av.\ C} - P_{av.\ X}}{P_{av.\ X}}\right| * 100 = \left|\frac{36.8 - 47.2}{47.2}\right| * 100 = 22.\% > 10\%$

Then $P_X = \frac{1}{4}\left[\frac{47.2}{38.5} * 37 + \frac{47.2}{44.1} * 42 + \frac{47.2}{36.8} * 35 + \frac{47.2}{41.7} * 40\right] = 45.12\ mm$

P average for the basin = $P^- = \frac{P_A+P_B+P_C+P_D+P_X}{N} = \frac{37+42+35+40+45.12}{5} = 39.82\ mm$

$i_{average} = \frac{60\ P^-}{T} = \frac{60*39.82}{120} = 19.91\ mm/hr$

$P = \left[\frac{(1.214 * 10^5)\ N\ T}{600}\right]^{0.282} - 2.54 \Rightarrow 45.12 = \left[\frac{(1.214 * 10^5)\ N*120}{600}\right]^{0.282} - 2.54$

N = 36 years

$\frac{P^-}{P} = 1 - \frac{0.3\sqrt{A}}{t^*}\ ; from\ the\ table\ t^* = 5.98$

$\frac{39.82}{45.12} = 1 - \frac{0.3\sqrt{A}}{5.98} \Rightarrow A = 5.48\ Km^2$