Some questions of the Mathematics - Department Geology - first stage

Question one: Find the **Domain** and **Range** of these functions.

1. $f(x) = \sqrt{4-x}$
2. $f(x) = \frac{x+3}{x^3+1}$
3. $f(x) = (x+4)^2$
4. $f(x) = \sqrt{1-x^2}$
5. $f(x) = \sqrt{x-3}$
6. $f(x) = x^2 + 1$
7. Prove that $\boldsymbol{\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}}$
8. Prove that $\boldsymbol{\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}}$
9. prove that $\sin 2\alpha = 2\sin\alpha\cos\alpha$

Question two: Find **limit** of the following functions.(**with out using L-Hopital**)

10. $\lim_{x\to 3} \frac{x^3-2x^2}{x^2+2}$
11. $\lim_{h\to 0} \frac{(h+1)^2-1}{h}$
12. $\lim_{x\to 3} \frac{\sqrt{x+6}-3}{x-3}$
13. $\lim_{x\to 0} \frac{x^2+x}{\sqrt{x^4+2x^2}}$
14. $\lim_{x\to 1} \frac{x^2-1}{x^2+1}$
15. $\lim_{x\to -2} \frac{x^3+8}{x^2-4}$
16. $\lim_{x\to -3} \frac{\sqrt{x^2+2x+1}}{8+2x}$
17. $\lim_{x\to 3} \frac{\sqrt{x+6}-3}{x-3}$
18. *Find* $\boldsymbol{f^{(5)}(x)}$ *,where* $\boldsymbol{f(x) = e^{-x}}$
19. **Question three :** Suppose $f(x) = \begin{cases} \frac{x^2-9}{x-3} & , \quad x \neq 3 \\ 6 & , \quad x = 3 \end{cases}$, this function is **Continuous** or **Not at x=3 ?**

Question four : Find the **derivative** of the following functions.

20. $f(x) = x^6 - \sqrt{x}$

21. $f(x) = \frac{\cos x}{1+\sin x}$

22. $f(x) = (2x^3 - 3x + 1)^9$

23. $f(x) = \frac{x^2}{3x-1}$

24. $f(x) = \sqrt{13x^2 - 5x + 8}$

25. $f(x) = 3\sin x - 4\cos x$

26. Find values of a,b where the function

$$f(x) = \begin{cases} ax^2 + 1, & x > 2 \\ -11, & x = 2 \\ x^3 + b, & x < 2 \end{cases}.$$

Is continuous.

27. **Example 2:** discuss the continuity at $x = 3$ for the following functions.

$$f(x) = \frac{x^2 - 9}{x - 3} \qquad g(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, x \neq 3 \\ 4, x = 3 \end{cases} \qquad h(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, x \neq 3 \\ 6, x = 3 \end{cases}$$

Compute

28. $\lim_{x \to -\infty} \frac{e^x}{4 + 5e^{3x}}$

29. $\lim_{x \to \infty} \frac{2^x}{3^x}$

30. $\lim_{x \to \infty} e^{2x-1}$

31. $\lim_{x \to 2} \frac{x^3 + x + 1}{x^2 + 2}$

32. $\lim_{x\to 1}\dfrac{x^2-1}{x^2+1}$

33. $\lim_{x\to 2}\dfrac{x^3+x+1}{x^2+2}=\dfrac{(2)^3+2+1}{(2)^2+2}=\dfrac{11}{6}$

34. $\lim_{x\to 1}\dfrac{x^2-1}{x^2+1}=\dfrac{0}{2}=0$

35.
Evaluate the following limit

$$\lim_{x\to 0}\frac{5\sin 3x+\tan 7x}{3x+x^2}$$

Prove that:

36. $\sin(\alpha \mp \beta) = \sin\alpha\cos\beta \mp \sin\beta\cos\alpha$

37. $\cos(\alpha \mp \beta) = \cos\alpha\cos\beta \pm \sin\alpha\sin\beta$

38. $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

39. $\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

40. $\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$

41. $\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$

42 $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha-\beta)+\sin(\alpha+\beta)]$

43. $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$

44. $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$

45. $\sin\alpha+\sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

46. $\sin\alpha-\sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

47. $\cos\alpha+\cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

48. $\cos\alpha-\cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

49. $\sin 2\alpha = 2 \sin \alpha \ \cos \beta$

50. $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$

51. $\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$**:**