Salahaddin University-Erbil

College of Administration and Economics

Department: Statistics & information

Subject: reliability

Third stage

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RELABILITY : refers to the consistency and dependability of a system , process or measurement , it is an essential concept in various fields , including engineering , statistics ,psychology and more reliability can be understood in several different contexts .

Qualitative Definition: A reliability may be defined as the probability of a device to operate for a given period of time, without failure under a given operating conditions; for a given interval [0,t] .

Quantitative Definition:

Let T be a life length (or time to failure) of a component then the reliability is:

$$R(t) = P_r\ (T > t)$$

Properties of Reliability function:

1- Decreasing function.

2- $R(\infty) = 0$

3- $R(0) = 1$

4-$R(t)$ in terms of p. d. f f(t) may be written as

$$R(t) = \int_t^\infty f(u)du$$

5-R(t) in terms of c.d.f F(t) may be expressed as

$$R(t) = 1 - F(t)$$

6-Unreliability function Q(t)

$$Q(t) = P_r(T \le t)$$

$$R(t) + Q(t) = 1$$

What it means to say R(t)=0.90 ?

It means that 10 percent will fail of the identical items under the same working condition in the interval [0,t].

Reliability function or survival function:

The life time (T) is non negative r.v.

$$R(t) = P_r(T > t)$$

$$= P_r(\text{a component survives after age t})$$

$$R(t) = \int_t^{\infty} f(u)du$$

Distribution Function:

$$F(t) = P_r(T \le t)$$

$$= P_r(\text{failure of a compenent time} \le t)$$

$$F(t) = \int_0^{t} f(u)du$$

$$F(t) = Q(t) = 1 - R(t)$$

Probability of Failure (Failure Density function) :

The probability of failure may be defined as the unconditional probability of failure in the time interval (Δt) between (t) and ($t + \Delta t$).

$$f(t) = P_r(t < T \le t + \Delta t)$$

Hazard function:

The hazard function may be defined as the conditional probability at a component to fail in the interval (Δt), between (t and $t + \Delta t$) given that it has not failed until time (t) this may be expressed as:.

$Z(t) = P(t < T \leq (t + \Delta t)/T > t)$

$$Z(t) = \frac{P(t<T\leq(t+\Delta t))}{P(T>t)}$$

$$Z(t) = \frac{f(t)}{R(t)}$$

General Expression of Reliability

$$\boldsymbol{Z(t) = \frac{f(t)}{R(t)}}$$

$$Z(t) = \frac{-\acute{R}(t)}{R(t)}$$

$$\int_0^t Z(s)ds = \int_0^t \frac{-\acute{R}(s)}{R(s)} ds$$

$$\int_0^t Z(s)ds = -\ln R(s) \Big|_0^t$$

Special case

$if\ Z(s)\ is\ Constant \quad \rightarrow \quad Z(s) = \lambda$

$$R(t) = e^{-\int_0^t Z(s)ds} = e^{-\int_0^t \lambda\, ds} = e^{-\lambda t}$$

$$f(t) = Z(t) * R(t)$$

$$f(t) = \lambda e^{-\lambda t}$$

Empirical Reliability:

N_O : Number of a component under the test.

$N_S(t)$: Number of survival component up to time t.

$N_F(t)$: Number of failure component up to time t.

$\widehat{P}\text{Survival} = R(t) = \frac{N_S(t)}{N_O}$

$$\widehat{P}\text{Failure} = Q(t) = \frac{N_F(t)}{N_O}$$

$$N_O = N_S(t) + N_F(t)$$

Mean time to failure (MTTF):

Is the mean time to the first failure (non repairable device).

$$\text{MTTF} = \text{E(t)} = \int_0^\infty \text{tf(t)dt} = \int_0^\infty \text{R(t)dt}$$

Mean time between failure (MTBF):

Is the expected time between two successive failure (repairable component).

$$\frac{1}{MTBF} = \sum_{i=1}^{n} \frac{1}{MTTF_i}$$

$$MTBF = MTTF = \frac{1}{\lambda}$$

Theorem:

$$\boldsymbol{E(T) = \int_0^\infty R(t)\,dt}$$

Proof: Left hand

$$E(t) = \int_0^\infty tf(t)dt$$

$$= -\int_0^\infty t\,\acute{R}(t)\,dt \qquad Let:\ u = t \qquad du = dt$$

$$dv = -\acute{R}(t)dt \qquad v = -R(t)$$

$$E(t) = -t\,R(t)\Big|_0^\infty + \int_0^\infty R(t)\,dt$$

$$E(t) = -\infty\,R(\infty) - 0\,R(0) + \int_0^\infty R(t)\,dt$$

$$\therefore E(t) = \int_0^\infty R(t)\,dt$$

Right hand

$$\int_0^\infty R(t)\,dt = \int_0^\infty \left[\int_t^\infty f(s)ds\right] dt$$

$$Let:\ u = \int_t^\infty f(s)ds \qquad du = -f(t)dt$$

$$dv = dt \qquad v = t$$

$$\int_0^\infty R(t)\,dt = t\int_t^\infty f(s)ds\Big|_0^\infty + \int_0^\infty tf(t)dt$$

$$= 0 + \int_0^\infty tf(t)dt$$

$$\therefore \int_0^\infty \mathrm{R(t)dt} = \int_0^\infty \mathrm{tf(t)dt} = E(t)$$

In special case

If $Z(t) = \lambda$

$R(t) = e^{-\lambda t}$

$f(t) = \lambda e^{-\lambda t}$

$$E(t) = \int_0^\infty R(t)\,dt$$

$$E(t) = \int_0^\infty e^{-\lambda t} \quad dt = \frac{-1}{\lambda} e^{-\lambda t} \Big|_0^\infty$$

$$= \frac{-1}{\lambda} e^{-\lambda(\infty)} + \frac{1}{\lambda} e^{-\lambda(0)} = \frac{1}{\lambda}$$

$$E(t) = \frac{1}{\lambda} = MTTF$$

<u>Theorem:</u>

Let T be a time to failure of an item then the distribution of T is exponential, If and only if the failure rate is constant? $\mathbf{1} -$
$\boldsymbol{T} \sim \mathbf{exp}(\boldsymbol{\lambda}) \quad \rightarrow \quad \boldsymbol{Z(t) is\ constant 2} - \boldsymbol{Z(t)} =$
$\boldsymbol{\lambda\ constant} \quad \rightarrow \quad \boldsymbol{T \sim exp(\lambda)}$

$1 - \boldsymbol{T} \sim \mathbf{exp}(\boldsymbol{\lambda}) \quad \rightarrow \quad \boldsymbol{Z(t) is\ constant}$

$$\mathrm{Z(t)} = \frac{\mathrm{f(t)}}{\mathrm{R(t)}}$$

$\mathrm{f(t)} = \lambda \mathrm{e}^{-\lambda \mathrm{t}}$

$$\mathrm{R(t)} = \mathrm{P(T \geq t)} = \int_{\mathrm{t}}^\infty \mathrm{f(s)ds}$$

$$= \int_{\mathrm{t}}^\infty \lambda \mathrm{e}^{-\lambda \mathrm{s}} \mathrm{ds}$$

$= -e^{-\lambda s}\Big|_{t}^{\infty}$

$= e^{-\lambda t}$

$Z(t) = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$ constant Z(t)

$2 - Z(t) = \lambda$ constant → $T \sim \exp(\lambda)$

$Z(t) = \lambda$ $\quad Constant$

$R(t) = e^{-\int_0^t Z(s)ds} = e^{-\int_0^t \lambda\, ds} = e^{-\lambda t}$

$f(t) = Z(t) * R(t)$

$f(t) = \lambda e^{-\lambda t}$

Example(1)

If $Z(t) = 3 * 10^{-5}$ find:

1- R(t) for 100 hours.

2- What is the reliability equal MTTF?

Solution

$Z(t) = \lambda = 3 * 10^{-5}$ constant

$1 - R(t) = e^{-\lambda t}$

$R(t = 100) = e^{-3*10^{-5}*100} = e^{-3*10^{-3}} = 0.99$

$2 - R(t = MTTF)$=?

$$MTTF = E(t) = \int_0^{\infty} R(t)\, dt$$

$$E(t) = \int_0^{\infty} e^{-\lambda t}\ \ dt = \frac{-1}{\lambda} e^{-\lambda t}\Big|_0^{\infty}$$

$$= \frac{-1}{\lambda} e^{-\lambda(\infty)} + \frac{1}{\lambda} e^{-\lambda(0)} = \frac{1}{\lambda}$$

$MTTF = \frac{1}{\lambda}$

$R\left(t = \frac{1}{\lambda}\right) = e^{-\lambda t} = e^{-\lambda * \frac{1}{\lambda}} = e^{-1}$=0.37

<u>Example(2)</u>

Assume that (5000) items are put under the test, and if failure rate is constant $5 * 10^{-4}$ find:

1- Reliability for 500 hour.

2- $N_S(t)$ for 500 hours.

3-$N_f(t)$ for 500 hours.

Solution/

$N_O = 5000 \ items$

$\lambda = Z(t) = 5 * 10^{-4}$

$1 - R(t) = e^{-\lambda t}$

$R(t = 500) = e^{-5*10^{-4}*500}$

$= e^{-0.25} = 0.78$

$2 - N_S(t) = R(t) * N_O$

$= e^{-0.25} * 5000 = 3900$

$3 - N_F(t) = N_O - N_S(t)$

$= 5000 - 3900 = 1100$

<u>Example(3):</u>

If λ=0.01 parameter of exponential distribution and R(t)=0.90,

Find: t [the number of hours as a system operated]

<u>Solution:</u>

$R(t) = e^{-\lambda t}$

$0.90 = e^{-0.01*t}$

$Ln(0.90) = -0.01 * t$

$$t = \frac{-Ln(o.90)}{0.01} = 10.5\ hours$$

Example(4):

If the Reliability for 100h equal to 0.99 find the failure rate:

Solution:

$R(t = 100) = e^{-\lambda t} = 0.99$

$e^{-\lambda 100} = 0.99$

$Ln(0.99) = -\lambda 100$

$$\lambda = \frac{-Ln(o.99)}{100} = 0.00003$$