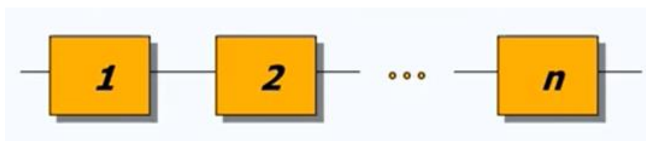
Reliability of system : for the time being we consider some simple type of system in which the parts are connected in series , parallel , or mixed.

- **Series System**
- **Parallel System (Redundant System)**
- **Combined Series-Parallel Systems**

1- connection in series

A system is said to be connected in series if one part of the system fails then the system fails :

Series system



$$R_s(t) = R_1(t)R_2(t)R_3(t)\cdots R_n(t) = \prod_{i=1}^{n} R_i(t)$$

Limit on System Reliability:

$$R_s(t) \le \min\{R_1(t), R_2(t), \cdots, R_n(t)\}$$

<u>hazard rate</u> *hs(t)* for series system:

$$h_s(t) = \sum h_i(t)$$

Special case

If the reliability of all units are the same or if

$R_i(t) = R(t)$ for all $i=1,2,\ldots\ldots,n$

$R_s(t) = [R(t)]^n$

Example 1:

Suppose a system consists of 4 components arranged **in series**. The first two components have reliabilities of 0.9 at time $t = 1$ year and the other two components have reliabilities of 0.8 at $t = 1$ year. What is the overall reliability of the system at one year?

$$R_s(t) = R_1(t)R_2(t)R_3(t)R_4(t)$$

$$= (0.9)^2(0.8)^2 =$$

$$= 0.5184$$

Example 2: Assume two units are connected in series and failure rates are λ1 and λ2 respectively find

1- reliability of the system

2- failure rate

3-MTBF

Solution:

1- reliability of the system

$$R_S(t) = R_1(t)\ R_2(t)$$

$$R_1(t) = P_r(T_1 > t) = e^{-\lambda_1 t}$$

$$R_2(t) = P_r(T_2 > t) = e^{-\lambda_2 t}$$

$$R_s(t) = e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} = e^{-(\lambda_1+\lambda_2)t}$$

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2- Failure rate

$$Z(t) = \lambda_S = \lambda_1 + \lambda_2$$

3-MTBF

$$MTBF = \frac{1}{\lambda_S} = \frac{1}{\lambda_1+\lambda_2}$$

Example 3//consider an electronic circuit 4 unit connected in series and each item of the above has exponential failure rate

$\lambda_1 = 4 * 10^{-5}$	$\lambda_2 = 2 * 10^{-5}$	$\lambda_3 = 2 * 10^{-5}$	$\lambda_4 = 2 * 10^{-5}$

Find: 1- Rs(t) 2- Rs(t=10) 3-MTBF

Solution :-

1-/

$R_s\ (t) = R_1\ (t) * R_2\ (t) * R_3\ (t) * R_4\ (t)$

$R_1\ (t) = e^{-\lambda_1 t} \quad = e^{-4 * 10^{-5} t}$

$R_2\ (t) = e^{-\lambda_2 t} \quad = e^{-2 * 10^{-5} t}$

$R_3\ (t) = e^{-\lambda_3 t} \quad = e^{-2 * 10^{-5} t}$

$R_4\ (t) = e^{-\lambda_4 t} \quad = e^{-2 * 10^{-5} t}$

$R_s\ (t) = e^{-4 * 10^{-5} t} * \ldots\ldots\ldots * e^{-2 * 10^{-5} t} = e^{-10^{-5}(4+2+2+2)\, t}$

$= e^{-10^{-5}(10)\, t} = e^{-10^{-4} t}$

$\lambda_s = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 10^{-5}\ (4+2+2+2) = 10^{-4}$

2- $R_s\ (t=10) = e^{\lambda_s t} = e^{-10^{-4}(10)} = e^{-10^{-3}}$

3-MTBF $= \frac{1}{\lambda_s} = \frac{1}{\lambda_1+\lambda_2+\lambda_3+\lambda_4}$

$= \frac{1}{10^{-4}} = 10000$ hours

Example 4: *<u>Exponential case</u>* H.W

Five components in series are each distributed exponentially with a hazard rate of 0.2 failures per hour. What is the reliability and MTTF of the system?