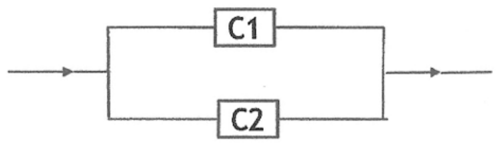
Parallel System (Redundant System):

A system is said to have n units which are reliability wise in parallel when only the failure of all n units in the system results in system failure. Conversely, for a parallel system to succeed at least one of the *n* parallel units in the system needs to succeed, or operate without failure, for the duration on the intended mission

Parallel System:



Generalization :

If we have n-unity working independently and they are connected in parallel. then the reliability of the system is :

$$R_s\ (t) = 1 - \prod_{i=1}^{n} [1 - R_i\ (t)]$$

Special case :-

If $R_i\ (t) = r(t)$ for all $i = 1,2,\ldots\ldots,n$

$$R_s\ (t) = 1 - [1 - r(t)]^n$$

Two- units cases

Let T_1 be the time to failure of unit one and T_2 be the time to failure of unit two and if the two units are functioning independently , then the reliability of the system Rs(t) is obtained as:

$$
\begin{aligned}
R_s(t) &= P(T > t) \\
&= 1 - P(T \le t) \\
&= 1 - [P(T_1 \le t \text{ and } T_2 \le t)] \\
&= 1 - [P(T_1 \le t) \,.\, P(T_2 \le t)] \\
&= 1 - [(1 - R_1(t))(1 - R_2(t)] \\
&= 1 - [1 - R_2(t) - R_1(1) + R_1(t)\, R_2(t)] \\
&= \not{1} - \not{1} + R_2(t) + R_1(t) - R_1(t)\, R_2(t) \\
&= R_1(t) + R_2(t) - R_1(t)\, R_2(t)
\end{aligned}
$$

Example// Assume two units connected in parallel and have constant failure rates λ_1 and λ_2 respectively find:

1- reliability of the system Rs(t)

2-MTBF

Solution:

$$
\begin{aligned}
1\text{- } R_s(t) &= R_1(t) + R_2(t) - R_1(t)\, R_2(t) \\
&= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}
\end{aligned}
$$

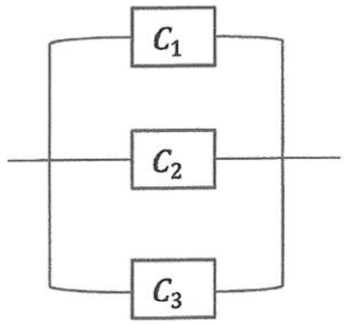
$$2\text{- } MTBF = \int_0^\infty R_s(t)\, dt$$

$$= \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2}$$

Example 2 H.W // if Ri(t) = R(t) for all i= 1,2,3

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda$$

Calculate :- $R_s(t)$ and MTBF of the following system.



Example// Let a parallel system be composed of n = 2 identical components, each with FR λ = 0.01 and mission time T = 10 hours, only one of which is needed for system success. Then, total system reliability, by both calculations, is:

$$R_i(10) = P\{X > 10\} = e^{-10\lambda} = e^{-0.1} = 0.9048; i = 1,2$$

$$R(10) = 1 - [1 - R_1(10)][1 - R_2(T)] = 1 - [1 - R_i(10)]^2$$

$$= 1 - (1 - 0.9048)^2 = 0.9909$$

$$R(T) = e^{-\lambda_1 T} + e^{-\lambda_2 T} - e^{-(\lambda_1 + \lambda_2)T} = 2e^{-\lambda T} - e^{-2\lambda T}$$

$$R(10) = 2e^{-10\lambda} - e^{-20\lambda} = 2e^{0.1} - e^{-0.2} = 0.9909; \text{for } T = 10;$$

Mean Time to Failure (in hours):

$$MTTF = \mu = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2} = \frac{2}{0.01} - \frac{1}{0.02} = 150$$