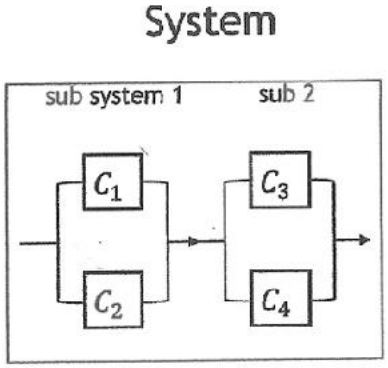
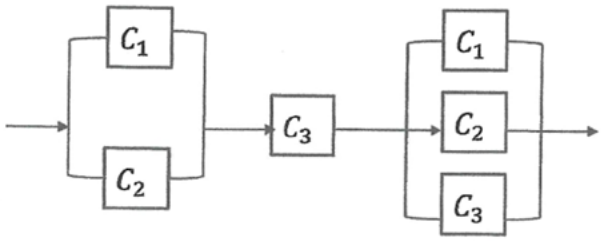
3- Mixed Connection

A- series-parallel



$$R_{s1}(t) = 1-(1- R_1(t))\ (1- R_2(t))$$
$$= R_1(t)+ R_2(t)- R_1(t)\ R_2(t)$$
$$R_{s2}(t) = 1-(1- R_3(t))\ (1- R_4(t))$$
$$= R_3(t)+ R_4(t)- R_3(t)\ R_4(t)$$

Example :- What is the probability that the system show below will operate with out failure for 10000 h.



Where :$\lambda_1=10^{-7}$, $\lambda_2 =0.5^* 10^{-7}$, $\lambda_3 =0.8^* 10^{-7}$

Solution:

$$R_{s1}(t)=R_1(t)+R_2(t)-R_1(t)\ R_2(t)$$
$$R_{s2}(t)=R_3(t)$$
$$R_{s3}(t)=1-\prod_{i=1}^{3}(1-R_i(t))$$
$$= 1-[(1-R_1(t))\ (1-R_2(t))\ (1-R_3(t))]$$

$$R_s(t)= R_{s1}(t) * R_{s2}(t)* R_{s3}(t)$$

$$= [R_1+ R_2- R_1 R_2]* R_3*[1-(1- R_1)(1- R_2)(1- R_3)]$$

$$= [e^{-\lambda_1 t}+ e^{-\lambda_2 t}- e^{-(\lambda_1+\lambda_2)t}]* e^{-\lambda_3 t}*[1-(1- e^{-\lambda_1 t})(1- e^{-\lambda_2 t})(1- e^{-\lambda_3 t})$$

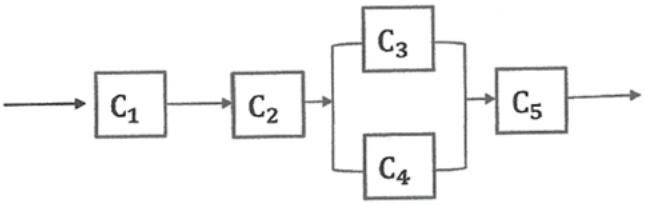
$$R_s(10000)= [e^{-10^{-7}*10^4}+ e^{-0.5*10^{-7}*10^4}- e^{-(10^{-7}+0.5*10^{-7})10^4}]\, e^{-0.8*10^{-7}*10^4}$$

$$*[1-(1- e^{-10^{-7}*10^4})\,(1- e^{-0.5*10^{-7}*10^4})(1- e^{-0.8*10^{-7}*10^4})\,]$$

$$= (0.999+0.9995-0.9985)*0.9992*(1-0.001*0.0005*0.0008)$$

$$= 0.9992=99.9\% \text{ approximately}$$

Example 2//



If $=\lambda_1= \lambda_2= \lambda_3= \lambda_4= \lambda_5=0.01$

Find $R_s(t)$, $R_s(t=20)$

Solution:-

$R_{s1}= R_1(t)$

$R_{s2}= R_2(t)$

$R_{s3}=1-\prod_{i=3}^{4} (1-R_i(t)) = 1-(1-R_3(t))(1-R_4(t))$

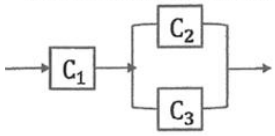
$= R_3(t)+R_4(t)-R_3(t)\, R_4(t)$

$$R_{s4}(t)=R_5(t)$$

$$\begin{aligned} R_s(t) &= R_{s1}(t)^* R_{s2}(t) * R_{s3}(t) * R_{s4}(t) \\ &= R_1(t)^*R_2(t)^*(R_3(t)+R_4(t)-R_3(t)\ R_4(t))^*R_5(t) \\ &= e^{-\lambda t}{}^* e^{-\lambda t}{}^* (e^{-\lambda t}+e^{-\lambda t}-\ e^{-(\lambda+\lambda)t}\ )\ ^*\ e^{-\lambda t} \\ &= e^{-3\lambda t}{}^*\ (2\ e^{-\lambda t}-\ e^{-2\lambda t})=2\ e^{-4\lambda t}-\ e^{-5\lambda t} \end{aligned}$$

$$\begin{aligned} R_s(t=20) &= 2e^{-4*0.01*20}-\ e^{-5*0.01*20} \\ &= 2e^{-0.8}-\ e^{-1} \end{aligned}$$

Example 3// three (3) independently working components are connected into a system as:



Suppose that the reliability for each component for an operation period of t-hour is given by $R(t) = e^{-0.03t}$ .If t is the time to failure find :

1. p.d.f of t 2. $R_s(t)$